MARKET MECHANISMS TO ALLOW TRADING OF IMPERVIOUS COVER

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To Ramy my wife,

Alfonso y Raquel my parents,

Patricio Contreras my cousin and F. Dioselinda

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Abstract

Problems with storm water runoff are becoming more frequent, and the main cause is the increase of impervious cover (IC). The imperviousness increases stream peak flows, changes peak times, and so changes the flood distribution. Several policies are used to manage flows and flooding; however most have been reported to be inefficient because land owners do not have correct exposure to price incentives and risk.

The main contributions of this thesis are an investigation into market mechanisms to price and allocate impervious cover allowances, while managing flood distribution. The market mechanisms are based on the electricity and gas markets which use linear programming formulations. This thesis develops three net pool market mechanisms: Det_MarketIC is a capped and deterministic market for IC, and Sto_MarketIC and Sto_MarketIC_Risk are stochastic market models with flood component penalties and risk positions representing the desired risk from the community respectively. Additionally, a gross pool market was extended under rainfall uncertainty, Gross_MarketIC.

The market design is an auction system with operational constraints and bids for IC allowances from participants. The system relates physical routed flows at nodal or control points to these bids.

The models clear the market by creating a demand (supply) curve for increments (reductions) in flows at specific places, and accounts for marginal changes in the expected flood damage and flood damage components. The market formulations estimate efficient allocations and prices. Decomposed prices from the market models are shown based on duality, as applied in electricity markets. The dual prices show spatial and temporal effects of flows, which impact at flooding areas. With Sto_MarketIC and Gross_MarketIC, prices account for changes in flood distribution.

With Sto_MarketIC_Risk, prices also account for the risk as CVaR in flooding areas. Thus, prices increase as binding risk conditions are tightened.

Finally, the net pool models are illustrated using hydrological and hydraulic simulators based on a small catchment located in Canterbury, New Zealand. Allocations and prices varied with the different models. Participants would face increasing prices in their IC allowances due to increments in flood damage.

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Terms

AEP – Annual exceedance probability

AMC – Antecedent moisture conditions

BMP – Best management practices

CDF – Cumulative distribution function

CN – Curve number

CV – Control volume

CVaR – Conditional value at risk

EP – Exceedance probability

EV – Extreme value

GIS – Geographic information system

HHI – Herfindahl-Hirschman Index

HIRDS – High intensity rainfall design system

IC – Impervious cover

LHS – Left hand side of an equation

LP – Linear programming

RHS – Right hand side of an equation

SM – Smart market

SO – System operator

TDR – Transferable development rights

TSSP – Two stage stochastic programming

VaR – Value at risk

Chapter 1

1 MECHANISMS FOR MANAGING IMPERVIOUS COVER AND RUNOFF

1.1 Introduction

In the last decades, severe damage caused from storm water runoff is more frequent and occurs in places that were commonly free of such serious problems (E.P.A. 1993; Strappazzon et al. 2003; Walls and McConnell 2004; Eigenraam et al. 2005; Jason and Jim 2005; Tang et al. 2005; Westra et al. 2005; Bradshaw et al. 2007; Hill et al. 2007; Pappas et al. 2008; Schielen 2009). The increased frequency of disaster has been mainly a consequence of human activities, changes in land management and development in hazard areas. One contributing factor is the increase of impervious cover (IC) due to economic development. The increase in IC has increased stream peak flows and the risk of flooding.

Authorities have looked at different mechanisms for controlling and minimising flood damage. Some of these mechanisms have been command and control by planning policies, taxes, insurance, and market-based instruments. This thesis extends the market approaches with a view to finding an alternative solution to the runoff problem and related downstream floods. This thesis deals with designing and modelling a smart market for IC allowances for managing runoffs and flood damage. The framework of this market approach has been used in other areas such as electricity and gas.

The market is designed under a storm scenario selection (Det_MarketIC) and under a stochastic approach (Sto_MarketIC and Gross_MarketIC) with the goal to internalise the price. Additionally, the market design will incorporate a risk profile (Sto_MarketIC_Risk) to hedge against changes in damage to encourage participants to internalise the costs generated by their runoffs.

The smart market models calculate prices and allocations based on auction bids and desirable environmental standards and risks. Price signals would create incentives to improve individual site-specific management to manage flood damage. In addition, when using the market design, lower transaction costs and more efficient allocation outcomes for society are expected.

1.2 The runoff-flood problem

Rogers and Defee II (2005) reported that impacts on catchment outflow arose with increasing development, imperviousness and edge density of roads. Those changes result in more frequent flooding and threatened natural habitat, in rural and urbanized areas. Jason et al. (2005) pointed out that in England and the European Continent, water levels had risen to the highest recorded levels. Similar issues have been reported by Kron (2005; 2007), Kazama et al. (2009), Middelmann-Fernandes (2010) and Yamada et al. (2010).

In New Zealand, the 2004 North Island floods were estimated to cost about NZ$300 million (Environment 2008). In the U.S.A. the Western flood in 1993 caused US$16 billion damage in the Mississippi and Missouri rivers. In Australia, annual flood damage in urban and rural areas reach AU$350 million on average; about 40,000 properties are under flooding risk with 1% annual exceedance probability (AEP) from storm events, and 160,000 urban properties with 1% AEP from storm flood events, i.e., in places such as banks of creeks or rivers (CSIRO 2000). In the UK, since 1990, weather-related insurance claims have cost £825 million per year, and the areas at risk cover assets estimated to be worth £200 billion with over 5 million people, 1.3 million homes and 130,000 business (Harman et al. 2002; Treby et al. 2006). Around the world, between 1990 and 2000, nearly 100,000 people were killed and 320 million people were displaced due to flooding; moreover, the damage to properties and infrastructure by floods cost over US$1,150 billion (Bradshaw et al. 2007).

Impervious surfaces alter the natural hydrology, reduce water infiltration in the ground and concentrate runoff over the landscape. As the imperviousness is directly related to runoff, increasing imperviousness will increase storm water runoff in the catchment. Consequently, the possibility of flooding increases as well as pollutant and contamination of drinking water, streams and aquifers. This effect is also stressed by Schielen (2009), who mentions that changes in land use affect water levels and flooding, and also points out that the effects of changes in land use are larger than the effects of climate change.

1.3 Mechanisms to deal with flooding

Damaging runoff flows have motivated governments to create mechanisms and policies to achieve economic development with minimum environmental impact. Non-market and market-based mechanisms have been proposed and applied to manage runoff and so flooding. Most policies relate to the traditional government role for protecting areas rather than preventing and managing flooding damage.

1.3.1 Summarising physical measures

Protection policies are widely used to manage flooding. Loucks et al. (2005) pointed out that flood storage capacity in reservoirs and channel enhancement are used to control peak flood and reduce damage. These controls are also linked to plans to decrease settlement in flooding areas which can feed into an integrated program between government, mitigation policy, insurance, and floodplain management (Burton et al. 1968; White 1994; Loucks et al. 2005; White 2011).

In the Netherlands, flooding is managed via protection systems such as levees, dikes, retention ponds and sewer systems, with protected areas (zones) as well as regulated plans and suitable land use to control the development of new projects (Loucks et al. 2005; Schielen 2009).

In England, the Planning Policy Guidance and the Planning Policy Statements establish the bases for local policies of land use (Richards 2007). The plans discourage development in floodplain areas –as a non-structural method such as command and control policy– and define flood risk zones in terms of frequency and return periods. Any application from developers has to present control and mitigation plans to manage runoff flows.

Public projects in Europe aim to establish storage and transport space for rivers, which have been shown to be effective but inefficient. Thus, people in flood areas could feel safe, but may not have reduced exposure to flooding, and those who increase flooding are not internalising the incremental damage of their decision. However, with a proper market system, incentives are clear and landholders can internalise the incremental damage. Thus, price signals may help to make decisions about protecting areas.

Australia uses structural and non-structural floodplain management and infrastructure policy to manage flooding problems. Previously, the plans were focused on command and control policies via protecting policies via infrastructure, typically levees, zoning, voluntary incentives to move out from flood areas, and flood emergency. Now plans address economics for future growth, current and future infrastructures, resources management, risk management, flood emergencies, and land use (CSIRO 2000). The idea of land use management proposed in the plan is an interesting point, in particular if allocations are via a market which accounts for flood limits and risk as proposed in this thesis.

In the USA, structural and non-structural as well as market-based systems are used to manage flooding. The National Flood Insurance, The Flood Control Act, and the National Environmental Policy Act were created to deal with flooding. Additionally, local plan requirements include zoning and land use changes in flooding areas. Non-structural options limiting development in upstream areas were used as a way to reduce future damage and flooding cost at Chester Creek (Olsen et al. 2000). Jason et al. (2005) pointed out that American flood policies rely mainly on infrastructure for flood control. Credits for reducing runoff have not obtained good results (Doll et al. 1998). This result has been for the voluntary participation of some programs, and people located in upstream areas that do not internalise the flooding cost nor the risk. Thus, a market based mechanism should induce conditions for encouraging participation, transferring the cost-risk of flood damage to each property-owner, simplifying bargaining between the authority and each participant (reduce transaction cost), and avoiding extreme payment of the authority for any private change in flood damage. These points will be discussed in next paragraphs and in Section 2.2 in Chapter 2 and Section 4.9 in Chapter 4.

Researchers have proposed flood management decision models based on optimisation. Olsen et al. (2000) proposed a dynamic model for floodplain management to choose between levee options (structural flood protection) and economic development. Lund (2002) used stochastic programming to evaluate hypothetical structural options to manage floodplain plans. Ford (1985) used a branch and bound algorithm to plan flood damage mitigation when the plan accounts for a combination of flood changes, flood damage and loss burden. Karamouz et al. (2009) evaluated structural options of flood protection and land use by a genetic algorithm. They chose a combination of flood damage, plan for managing flooding and land use at different points of the catchment. Piantadosi et al. (2008) proposed stochastic dynamic programming to manage storm water, and explicitly included a downside risk measure (Conditional Value at Risk) for environmental damage.

1.3.2 Policies to manage flooding

Proposed market-based systems consider the taxes, fees and rebates for voluntary programs to use best management practices (BMPs) (Thurston et al. 2008), tradable allowances of ICs (Thurston et al. 2003), zoning with IC thresholds and Transferable Development Rights (TDR) (Kauffman and Brant 2000; McConnell et al. 2006; Walls and McConnell 2007), insurance and risk analysis (Arnell et al. 1984; Harman et al. 2002; Purnell 2002; Sayers et al. 2002; Ermolieva 2005; Treby et al. 2006; Roche et al. 2010), or merely command and control (Parker 1995). Despite the theoretical plausibility of these methods, empirical evidence has shown problems with efficiencies, prices, allocations and, especially, transaction costs.

Insurance is widely used to recover cost for flooding damage (Arnell et al. 1984; Harman et al. 2002; Purnell 2002; Sayers et al. 2002; Ermolieva and Ermoliev 2005; Treby et al. 2006). Treby et al. (2006) pointed out that in the insurance system in UK, the government deals with insurance companies to provide flood insurance to household contents. In the USA, the National Flood Insurance Program charges premiums to encourage the reduction of private exposure to flooding. The system provides comprehensive buffer against flood risk, but has not reduced the vulnerability to flood. Indeed, those who contribute to flooding are not internalising the damage nor the incremented risk. Arnell et al. (1984) noted that the insurance system seems inconsistent, as it encourages people to stay in hazardous places. Roche et al. (2010) pointed out that where government becomes insurer without guidance on land use insurance, taxes and residual market mechanisms, usually fail to provide long term solutions. White (2011) noted that incorrectly applied insurance could exacerbate flood losses and discourage adoption of long term mitigation measures.

The use of Transferable Development Rights (TDRs) is another market-based instrument which has the potential to preserve undeveloped land, transferring development to other places where land can be made dense (Chomitz 2004; Chomitz et al. 2004; Walls and McConnell 2004; Kopits et al. 2005; McConnell et al. 2005; McConnell et al. 2006;

Walls and McConnell 2007). TDRs are intended to promote suitable land uses, BMPs and control of environmental damage caused by demographic density, runoff and sedimentation. However, TDR programs are more complex than other market-based instruments because they combine a market system with local legislation. TDRs works by defining rights and rights equivalences between multiple zone and objectives, which are hard to establish; users may not internalize the flooding cost, and TDR programs face high transaction costs (Walls and McConnell 2004; 2007).

Thurston et al. (2003) stated that tradable allowances for impervious surfaces should create incentives to incorporate BMPs to reduce storm-water runoff. Parikh et al. (2005) compared storm water control policies from a hydrologic and economics perspective. They noted that a tradable allowance policy should be a better solution than runoff fees and voluntary programs. Allowances should be recognised as property rights, avoiding the open access to impervious cover. Additionally, individual entitlements should be defined, allocated and recorded to allow monitoring. Enforcement can then borrow from civil processes.

An allowance market should allow reaching a least cost solution and managing flooding in the catchment. Additionally, an allowances market should use a suitable framework to clear exchanges between participants, pollutant modelling to improve accuracy, and a market model which takes into account exchanges. This thesis will propose such a market design.

Some storm water utilities offer credits for reducing peak flows based on a specific storm design (Doll et al. 1998). Landowners who reduce runoff will receive compensation by receiving an equivalent reduction in taxes. The credit system can encourage the adoption of best-practice, storm water practices, lessening the cost of mitigation and potential damage. However, this instrument has serious problems with lack of participation given its voluntary nature.

Ribaudo et al. (1999) pointed out that a market based on runoff may work under average runoff levels, but trade between point and non-point sources could be possible only by defining ratios. The authors noticed that a market based on runoff allowances could face issues related to monitoring, information availability, modellers, and legal problems based on differences between the average and actual runoff.

Despite the theoretical plausibility of these market-based methods, the empirical evidence has shown some problems such as inefficient outcomes, poor environmental efficacy, and especially high transaction costs. These points will be further discussed in the following sections and in next paragraphs.

Cost efficiency is lost in the presence of transaction costs, market power, externalities, and incomplete information (Jaffe et al. 2005; McCann et al. 2005; Stavins 2008; Peace and Stavins 2010; Jarrow and Larsson 2012). For instance, Jaffe et al. (2005) pointed out that incomplete information and uncertainty discourage investments in new technologies, which makes environmental policy more expensive for society. Environmental inefficiency could be noticed if participants do not internalise the damage cost. Any market based mechanism should address arbitrage opportunities for environmental efficiency (Jarrow and Larsson 2012). Moreover, a market based mechanism should enforce property rights and their identification with low transaction costs and externalities (Coase 1960).

Non-supporting prices result for several reasons. One is when the allocation mechanism fails to achieve an optimum allocation for society (Dinar et al. 1997). Dinar et al. (1997) point out that market power can cause such inefficient allocation and non-supporting prices. Or, another reason is when the allocation is optimum but the settlement prices are miscalculated. A third instance is when the optimization problem is non-convex, in which case supporting prices might not exist (O'Neill et al. 2005). Non-supporting prices may also result when transaction costs are significant. In contrast, Fama (1970) noticed that efficient prices could be obtained if the price "fully reflect" all information, which is available to all participants.

Transaction costs are high in most environmental market-based instruments (McCann and Easter 1999; McCann et al. 2005). McCann et al. (2005) identified possible reasons that can cause market-based policies fail due to transaction costs. The authors classify transaction costs into categories such as research and information, litigation, design and implementation, administration, contracting, monitoring, and enforcement and prosecution. Netusil and Braden (2001) evaluated the transaction costs in a bilateral market and noticed that the total cost rose to attain an overall load target of sediment control. The authors also noticed that in environmental market such as for CO_2 and water pollution, transaction costs could be high, raising costs for society.

As we shall see later in Section 1.4 and 2.2, the market design proposed here is likely to handle these issues and to have a range of other good features.

1.4 Smart market

A smart market could be an efficient solution to efficiently manage the storm water runoff and flooding. A smart market (SM) is an auction system assisted by mathematical tools which manage complexities and externalities. Such effects are impossible to handle with ordinary auctions. The SM would enable efficient allocations and prices for society by trading via the auction in a centralised pool, and through a system operator (SO) (McCabe et al. 1989; 1991; Ring 1995; Hogan et al. 1996; Alvey et al. 1998; Read and Chattopadhyay 1999; Murphy et al. 2000; Plagmann and Raffensperger 2007; Prabodanie and Raffensperger 2007; Raffensperger 2007; Pinto et al. 2008b; Murphy et al. 2009; Prabodanie and Raffensperger 2009; Raffensperger et al. 2009; Prabodanie 2010; Prabodanie et al. 2010; Raffensperger and Cochrane 2010; Pepper et al. 2012; Pinto et al. 2012).

McAfee and McMillan (1987) defined an auction as a “market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants”. Efficient rules ensure market discipline and allow obtaining policy goals. An auction could raise revenues for the authority, but also reveal the bidder’s valuation of the resource. Hence, when using a smart market, the regulator may be able to make efficient decisions about allocations and prices. Additionally, auctions increase transparency, fairness, competition, and efficiency, and reduce transaction costs (Vickrey 1961; Wilson 1979; McAfee and McMillan 1987; Latacz-Lohmann and Hamsvoort 1997; Strappazzon et al. 2003; Eigenraam et al. 2005; Chu and Shen 2007; McAdams 2007; Plagmann and Raffensperger 2007; Montero 2008).

Ordinary auctions systems are used in a diversity of public issues and areas. Governments have successfully completed auctions to privatise state-owned firms, and to franchise infrastructure and public natural resources (Afualo and McMillan 1996). For instance, a simple reverse auction was applied by the USA Environmental Protection Agency in The Shepherd Creek, Cincinnati, Ohio, to encourage landowners to reduce imperviousness by BMPs in their properties (Thurston et al. 2008). The auction mechanism chooses according to a merit order that relates bid prices, investment costs and an environmental index. This system could be difficult to clear in a multidimensional system with many control points and landowners impacting in different places and levels. Other auctions have been used for licences to extract oil in USA, minerals in England, social food provision in Chile, and electromagnetic spectrum in Australia. However, ordinary auctions have shown some disadvantages because they may be vulnerable to collusion and strategic bidding. Thus, these may lead to higher social costs and inefficiencies in the allocation (Wilson 1979; Riley and Samuelson 1981; McAfee and McMillan 1987; Rothkopf et al. 1990; McCabe et al. 1991; Guasch and Glaessner 1993; Latacz-Lohmann and Hamsvoort 1997; Latacz-Lohmann and Schilizzi 2005; Rothkopf 2007; Montero 2008). Rothkopf et al. (1990) and Rothkopf (2007) noted that a Vickrey auction has difficulties in practice, because the dominant strategy equilibrium is a weak equilibrium, the auction could be vulnerable to collusion, participants could manipulate their bids and do not reveal their real cost, and because of low revenues for participants and the SO. Milgrom (2000), Schummer and Vohra (2003), and Montero (2008) also noted that when using ordinary auctions to allocate public resources, participants try to manipulate their bids and do not always reveal their real costs. Thus, participants may inflate or deflate their real costs with the purpose of paying a price below the social price or obtaining more rights, respectively. Nevertheless, auctions with pertinent corrections and rules are effective to implement (Vickrey 1961; Kwerel 1977; Riley and Samuelson 1981; McAdams 2007; Montero 2008).

Compared to ordinary auctions, a smart market is able to manage hydrological and an appropriately defined smart market could manage hydraulic effects from trading among point and non-point sources (Pinto et al. 2008a; Pinto et al. 2008b; Prabodanie and Raffensperger 2009; Raffensperger et al. 2009; Prabodanie 2010; Prabodanie et al. 2010; Raffensperger and Cochrane 2010; Pinto et al. 2012). In addition, a SM can use and be updated with all available relevant information, which is difficult to handle with simple auctions or pair-wise trading, but flexible with a SM.

In contrast to pair-wise trading in a traditional market, the smart market reduces transaction costs because users do not need to search for trading partners, bargaining is simpler, price information can be made available, and the manager ensures market discipline. However, both designs require expenses for setting up the market, monitoring and enforcement.

Some real market designs apply optimization techniques to allow allocating and pricing under market efficiency criteria (Bohn et al. 1984; Ring 1995; Hogan et al. 1996; Miller 1996; Alvey et al. 1998; Read et al. 1998; Read and Chattopadhyay 1999; Hogan 2002).

Bohn et al. (1984) showed that prices in the electricity market represent the fundamental physical and engineering properties of electricity. The authors maximised the consumers' and generators' surplus in a constrained system with demand and supply spatially located in a fixed network with flow capacities and transmission losses. Hogan et al. (1996) presented a linear programming model (LP) to obtain spot prices in an electricity market. The market operator calculates the optimum dispatch, based on bids from participants. Prices vary spatially as regard to the network and specific constraints in the power system. They noticed that a computer-based market using linear models encouraged users to obtain "beneficial trades" while reducing coordination problems and transaction costs. Similarly, Murphy et al. (2000; 2009) presented a design of a water market to achieve efficient allocation and pricing. McCabe et al. (1989; 1991) designed a computer-assisted market for natural gas as a sealed bid auction. In that case, the wholesale market coordinated the trading from sources in a pipelines network to delivery points. Thus, participants would submit locational offers and demands and the model would clear the market, maximizing the total surplus of trade, with transport capacities as constraints. The model was able to calculate efficient allocations and prices for all participants. Gallien and Wein (2005) reported a smart market for electronic trade in real time for industrial procurement with capacity constraints. The market framework would enable complex transactions in real-time.

Raffensperger et al. (2009) and Plagmann and Raffensperger (2007) demonstrated that a market could be an efficient mechanism to allocate rights for managing ground water, and that transaction costs would be reduced. In addition, the market design allows handling regulator criteria as well as quickly incorporating new information in the system.

Prabodanie and Raffensperger (2007) and Prabodanie et al. (2010) proposed a deterministic market for licences to discharge nitrates, while maximum quantities of nitrates are limited by location. Those papers developed centralized system in a common pool, controlled by a market manager, who would use a linear model to achieve maximum trade surplus. The linear program would allow trading within environmental thresholds over time, while nitrate is leaching from each property.

A market based on trading runoff could have serious issues as was observed by Ribaudo et al. (1999). Ideally, a market would allocate the actual discharges and impacts. However, these discharges are uncertain, and the auction manager's selected runoff model may be incorrect. Hence, the market could be fundamentally about participants' land use and management practices; the market must enforce the agreed behaviours in terms of IC of the contract holders. This point enforces the idea of a market which allocates IC while reaching optimum land use in the long-term with flooding according to desired standards and conditions.

A smart market for IC would consider the users' willingness to pay for impacting flows at flooding places and the environmental thresholds established by the authority via a centralised system for processing the information. Sayers et al. (2002) claimed that any integrated flood management system needs to be supported by a computer based system. This statement reinforces the idea of having a smart market to incorporate the consequences of changes in the catchment.

If the market were established to choose which technology each participant should use, by submitting 0 or 1 type bids, the market model would have integer decision variables. This market design would be non-convex, which probably results in non-supporting prices (O'Neill et al. 2005). Pinto et al. (2012) pointed out that participants' decisions about technologies are usually private; if participants offering to control obtain only a fraction of the equivalent technology, they may need to bid a high price in the next round to obtain the next part.

Pinto et al. (2008a; 2008b) and Pinto et al. (2012) developed a deterministic smart market for controlling externalities related to sediment discharge. Raffensperger and Cochrane (2010) proposed a deterministic smart market for IC to control problems from excess runoff. In this market, the participant trades the consent to change IC as measured by the infiltration capacity of the land while the authority specifies maximum capacity of storm water runoff at channel control points. Although this study introduced the idea of smart market for IC, it did not consider the stochastic nature of rainfall and the risk if an extreme event occurs. The current proposal extends the research of Raffensperger and Cochrane (2010) to a market that incorporates the rainfall uncertainty.

Hydrological phenomena such as rainfall events are complex and impossible to predict with certainty. Calculations about rainfall are often based on simple averages which create problems when designing infrastructure and developing policy instruments which depend on rainfall events. Ignoring the stochastic nature of the rainfall may lead to poor decisions and inefficient outcomes. Malcolm and Zenios (1994) suggested incorporating uncertainty into planning and infrastructure design, and Shortle and Horan (2008) pointed out that uncertainty about sources in water quality trading could lead to market failure.

Under uncertainty of rainfall events, the regulator should make decisions about design, planning and environmental thresholds to keep society safe from flooding harm at reasonable cost (James and Lee 1971; Loucks et al. 2005; Cowdin 2008; EFTEC 2010). James and Lee (1971), and Loucks et al. (2005) presented theoretical benefit-cost analyses, which could be applied to analyse flood control. The decision is influenced by many factors such as the rainfall distribution, risk, and the cost of extreme events. The regulator must deal with these concerns in the design of a market to encourage hedging against a range of events in the market, i.e., to lower flood damage for a range of storms, or if extreme storms affect the catchment.

A flood *hazard* is the occurrence of a flow event with a predetermined flow *exceedance probability*. Flood *vulnerability* is the susceptibility to *damage* from extreme storm events. *Risk* is the result of the combination between an *event's probability* and *vulnerability*, resulting in the potential damage. *Disaster* is the realisation of the risk, i.e., this is the flooding damage. Kron (2007) pointed out that probability of occurrence and vulnerability should be estimated in levels of damage for a whole range of events. Thus, a functional relationship can be identified between flood components; this relationship will be included in the market clearing formulation.

This thesis develops a market to trade IC allowances for managing runoff and so flooding. This market will be approached first as a deterministic storm design based on extreme storm events and then as a market with risk positions for disaster and uncertain rainfall.

1.5 Scope

The manuscript is organized as follows. Chapter 2 presents a framework runoff hydrographs, flood damage estimates and the smart market. Chapter 3 presents a deterministic net pool market for IC allowances under an extreme event. Chapter 4 describes a net pool market with rainfall stochasticity. Chapter 5 extends the previous market and proposes flood penalties. Chapter 6 describes a net pool market which accounts for an acceptable risk position that the community desires to face in the catchment. Chapter 7 presents a study case. Chapter 8 extends the market to a gross pool formulation, and Chapter 9 gives conclusions and proposes future research.

Chapter 2

2 MODELLING FLOWS, FLOOD, AND FLOOD DAMAGE

2.1 Introduction

This Chapter presents the conceptual framework for a market system based on impervious cover (IC[1]) allowances, which accounts for storm water runoffs. An IC allowance is a tradable permit to use a specific level of perviousness in a specific area (hectare). The chapter explains the smart market, IC, storm estimation, storm distribution, runoff hydrographs and runoff estimates, governing routed flows, flow estimates, flood damage function, flood components, and the expected flood damage. These concepts are part of the framework that will be used in the market formulations.

The market model assumes a linear system, which accounts for linear changes in flows for supper positions, or marginal changes from a status quo of flows across the catchment. The purpose is to combine the effects of IC allowances at control points across storm scenarios. Thus, imperviousness levels are transformed into linear effects on flows and flooding flows (as impact flow coefficients) at different places in the catchment. Hydrological and hydraulic models such as HEC-HMS (HEC 2008a) and HEC-RAS (HEC 2008b) are used to estimate these coefficients. Using linear assumptions is adequate for modest changes, but dramatic changes will require extra non-linear modelling efforts to update coefficients.

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[1] An empirical parameter directly connected with imperviousness is the curve number (CN) which was developed by the USDA Natural Resources Conservation Service (SCS 1985). CN measures the direct runoff from a rainfall event based on infiltrations and the area's hydrologic soil group, land use, soil moisture management and hydrologic condition.

Loucks et al. (2005) present a framework to estimate the expected flood damage and its changes. The framework accounts for peak flows, stage flood, flood and peak flow probability distributions and flood damage, and calculates the changes in the expected flood damage. The authors show, by graphical means, the calculations of the expected flood damage, Figure 2-1 illustrates those calculations. An equivalent framework will be used in this thesis; thus, the IC market estimates changes in the expected flood damage, which depends on the changes in the imperviousness levels in each property and consequently in the catchment.

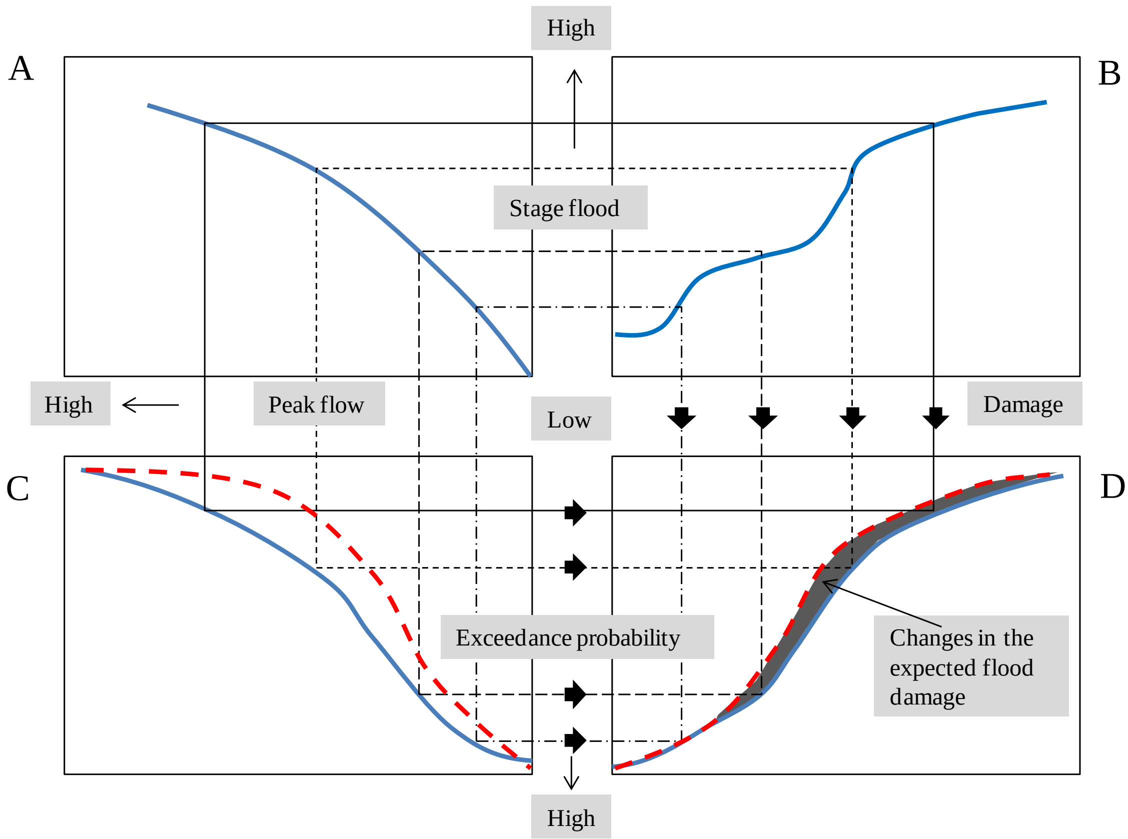

Figure 2-1 Changes in the expected flood damage. (A) corresponds to the relationship between peak flow and stage flood, (B) is the stage flood and flood damage relationship, (C) is the cumulative probability distribution of peak flow, and (D) is the cumulative flood damage distribution. Figure is based on Loucks et al. (2005)

Then, the smart market clears the participants' demand and supply, based on their preferences for impervious levels. The market model calculates how these changes impact flows and flood damage at control points. The different market formulations in this thesis account for changes in flood damage and the flood damage components such as peak flow and stage flood, rapid inundation and duration. Additionally, the flood damage functions and their components are used to create demand curves of IC allowances for participants with the Sto_MarketIC.

Therefore, the IC market design will deal, for instance, with hydrograph curves, with their flows at specific places in the catchment, routing flows and the governing hydraulic equations, storm probability distribution, and flood damage that are conceptualised as follows.

2.2 Smart market for IC allowances

In many parts of the world, deregulated markets are used to solve complex problems such as the buying and selling electricity and gas (Bohn et al. 1984; McCabe et al. 1989; 1991; Read and Chattopadhyay 1999; Pepper et al. 2012). Most of these established markets are centrally controlled and enable multilateral trading in a common pool; many permit bilateral trading on the outside as well. Commonly, an optimisation model calculates dispatch schedules and prices based on the economic maximisation of trading, for submitted linearised demand and supply bids from participants.

These markets allow for management of complicated interactions and externalities by trading. A mathematical programming model clears the market. Read and Chattopadhyay (1999) present an overview of an electricity clearing market based on a LP model which maximises the economic surplus from generators (sellers) and loads (buyers) at specific places (nodes), with a specific set of: constraints about operation, transmission capacities, consumption, and system security and reliability. Thus, a specific network with nodes (regions), demands, and supplies are balanced to estimate a set of nodal prices and the best set of the dispatch schedules. Additionally, the government oversees the trading system through an authorised institution called the Electricity Commission. McCabe et al. (1989; 1991) called this type of market a “smart market” (SM) and an equivalent conceptual framework is proposed for the market for IC allowances.

As stated in the previous Chapter, the proposed market for IC allowances corresponds to an auction system where equilibrium prices and optimum allocations are found by using mathematical optimisation. This market is operated by an auction manager or system operator (SO) who can be a governmental agency. Trades are not bilateral between market users; rather they trade through the SO, buying and selling these allowances.

IC allowances can be decomposed into a set of rights to impact flows at control points. These impacting flow rights represent physical flow properties related to capped points and flood components (such as the stage flood or duration of a flood). IC allowances in the property determine specific flooding conditions across control points and scenario(s) with the stochastic formulations. This point will be discussed further in the thesis. Thus, when participants trade IC allowances, they are actually bidding to change the set of flow impacts across control points which, in turn, change the flood physical distribution in these areas. This will be discussed in Chapters 3, 4, 5, and 6. Chapter 8 extends the IC market to a gross pool formulation.

Trading IC allowances

The SM allows bidding (offering) in steps, or tranches of packages of IC allowances. Participants could bid (offer) more than one quantity (package). Bids and offers could be submitted to the market online. The market is cleared for buyers and sellers simultaneously. This market has a uniform price "pool" market arrangement per control point (node), and individual prices are adjusted according to specific impacts in the catchment. Thus, users with similar IC allowances and at the same place would face the same price.

Alternatively, participants could express their bids as wedges or proportions of changes for IC allowances in the property. This idea has been used in electricity markets for ancillary services, where a set of incremental reserve proportions of generation are established (Read et al. 1998). Thus, a participant may express their desired conditions as an incremental proportion of changes for IC allowances in their property, related to an established reference usage for IC allowances. From this reference, participants could bid for the incremental wedges of changes in IC allowances in their property. We will not pursue this idea further.

In the market, participants (landholders) trade IC allowances in their areas (property) which would change runoff patterns and so the flows at control points. It is important to differentiate between "land use" from "IC allowance". Land use is what physically happens on a piece of land, the "IC allowance" the runoff flow rights owned or "anticipated impacting flows" at control points the measure of flows that are traded. The supply is characterised by participants who are willing to reduce or avoid runoff and can thus sell the option to reduce IC allowances. To reduce imperviousness, participants can change impervious allowances and use various Best Management Practices[2] (BMPs) or install control technologies. Participants who desire to increase the IC allowances of their land represent the demand. Thus, they have the willingness to buy the IC allowances for the area; buyers alternatively could install control technologies and BMPs.

If a participant does not desire to change IC allowances, he/she has the option of either selling at a high price or buying alternatives in a net pool at a low price. If a participant changes their land use in a way that does not affect their anticipated runoff and flows at control points, they do not need to participate in the market. This will assist the market operation when complex combinations of IC allowances prevail on the property. The SO could estimate the anticipated runoff and flows for the property if the participant wanted to change IC allowances. This would particularly assist with projects such as real estate development. Reducing anticipated runoff should also not require participation (but it could be beneficial).

The model will calculate market prices that signal resource shortage. The system would incentivise improved management of runoff control and use of BMPs, especially near sensitive areas. Thus, the system may incentivise IC changes to reduce stage-flood and to move flows from the peak time at control points. This means that a participant could be paid to reduce IC allowance, if the change in flows avoids peak flow time. Furthermore, the geography and runoff patterns could mean that a participant might increase their IC allowance and be paid if the new flows were to reduce the peak of the flood or the length of time the flood was at its peak. Most cases of increasing IC allowance would increase peak flows.

An IC market covers a physical area (geography and catchment size) with its connected network of channels and rivers. Thus, relevant inflows should come from the physical area and be accounted for the market. Similarly, a power transmission network connects adjacent electricity markets, and enables interactions between markets for transmission capacities. Participants in an IC market would interact only if common rivers or channels exist. Prices associated with control points common to adjacent catchments would signal

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[2] For example, in urban and rural places, technologies may include detention ponds, wetlands, infiltration tranches, porous pavement, swales, vegetative detainments, mulches, agricultural conservation practices, etc (Loucks et al 2005).

capacity constraints and capacity violations, and each participant whose flows go through these points would face these prices in their IC allowances.

Trade should lead to the lowest social cost, if participants bid truthfully and rationally, as would be expected under perfect competition. Strategic decisions are outside the scope of this thesis. (For more details on setting prices, innovation and strategic decisions, see Laffont and Tirole (1996a; b), Kwerel (1977) and Montero (2008). The goal of such a market would be to enable society to satisfy a range of desired environmental outcomes at minimum cost).

Role of the market manager

The functions described here are all required for the market and while there are many regulatory options that would suit the purposes of the market, for ease of discussion most of the roles will be attributed to the SO.

The SO manages the imperviousness in the catchment. In this way the SO manages the risk of flooding, and the flood damage, to hedge against a range of extreme storm events. If the SO does not control the imperviousness, the impervious level may increase in the catchment and the areas threatened by flood will enlarge, which will produce greater flood damage. Figure 2-2 illustrates the effects of different levels of IC in the catchment and their equivalent flood levels with a similar storm. The level of imperviousness increases from IC allowance *i* to *v*. In this illustration, the SO could desire to maintain a safe flow level by establishing and limiting an equivalent total flow *ICL iii*. Therefore, the SO may allow IC allowances to trade as long as these flow levels in hazard areas are not violated (capped design). The IC allowances *i* and *ii* do not produce flood problems, while the flows from IC allowances *iv* and *v* result in flooding.

The SO could be concerned about the risk of flooding and would desire to limit the frequency of flooding and hedge against a range of flooding events. Given data on these events and the affected areas, the SO could estimate flood damage and use this information to design the market. The SO should not encourage development in hazard areas. The SO should define zoning where floods need to be managed. Additionally, building codes and infrastructure requirements should be defined for the flood areas. Roche et al. (2010) stressed that regulatory power should focus on zoning and building codes in flood areas. The SO or some other regulatory body (e.g., Regional Councils) already place considerable resources and funding to mitigate and manage the risk of flooding. Such risk management is outside the scope of this thesis, but the implementation of the various Regional Council flood mitigation works, e.g., installing “stop banks”, will (positively) impact on the modelled flood damage and associated costs.

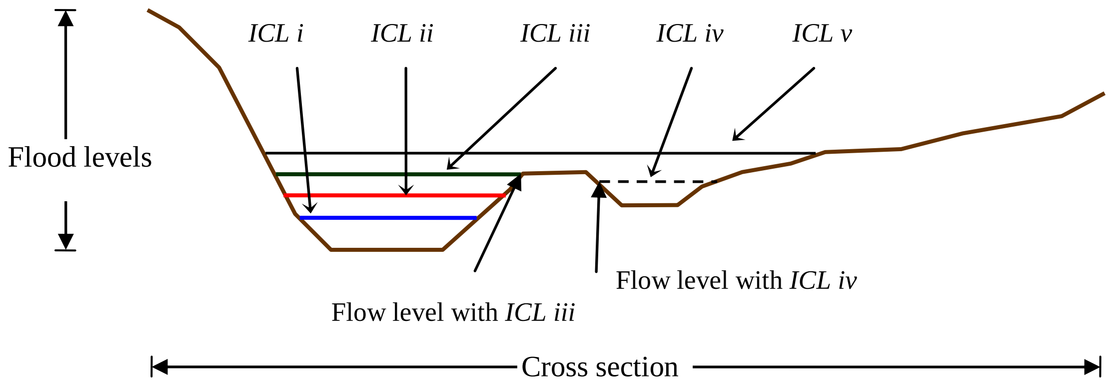

Figure 2-2 Flood levels reached with different IC allowances (*ICL*) in the catchment with a similar storm. Level of imperviousness increases from *i* to *v*. The ground profile represents the cross section in the flood area. Note that impervious level *ICL iii* is the maximum flow level reached before flooding occurs.

The authority could define flow thresholds in the catchment, especially in hazard areas where flooding is or could become a problem. Thus, it is possible to use thresholds that will account for limiting flows at specific places along channels, streams, rivers, and lakes in a deterministic market formulation, or limits that will be violated in a stochastic market formulation. Different criteria could be used to establish thresholds in the catchment (for more details see Brabec et al., 2002), but the research presented in this thesis will focus on runoff flows and capacities in streams, channels, and floodplains only.

The flow thresholds should be defined according to a "storm design" and infrastructure design, for instance a design for a 100 year flood event. This capacity in terms of year events is measured in terms of stage flood and peak flows, such a maximum flow of 200 m^3/sec. The thresholds also relate to the risk cost analysis used for designing the infrastructure capacity, which relies on the expected flood cost that the authority desires to face in the area.

To estimate the individual impact in the catchment at hazard points and places, the SO could use hydrological and hydraulic models (Ormsbee et al. 1984; Dutta et al. 2000;

Mascarenhas and Miguez 2005; HEC 2008a; b; USDA-SWAT 2008; USDA-WEPP 2008). These models enable the calculation of runoffs in a routed channel and floodplains for specific rainfall events. In predicting impacting flow from property owners, the SO would choose one rainfall event "design storm" to incorporate in the market design, for instance, a single 200 mm storm for a deterministic market, or several events for a stochastic market. Consequently, impacts of participants can be measured as impact coefficients by time and storm scenario.

The storm design affects the demand and supply from participants and their related impacting flows. As stated in previous paragraphs, the IC allowance is a set of impacting flow rights at different points in the catchment, which are estimated based on storm designs. Thus, the IC allowances will produce different impacting flows according to the storm design; for instance, total flows from a storm design 200 mm in 24 hours are greater than flows from a storm design 50 mm in 24 hours. These flows are constrained by threshold capacities in a deterministic formulation, or will violate the capacity under extreme storm events with a stochastic formulation. Because of the relationship between storm design, flows, and threshold capacities, the model solution could be feasible or infeasible in the market, and consequently the SO could face different revenues. This issue there will be further discussed in Subsection 3.4.6 in Chapter 3, Section 4.9.1 in Chapter 4, and Section 8.8 in Chapter 8.

The SO should use monitoring to obtain an accurate measure of individual flow impact along channels and rivers, which translates IC at a particular site into flows and the flood stage at a location downstream under a designed storm, especially environmentally sensitive places (common flooding areas). These coefficients should be evaluated and periodically controlled to update the market model. Note that the SO independently monitors market development. For example, in Michigan the monitoring is done by the State of Michigan Department of Environmental Quality for non-point source trading (Woodward and Kaiser 2002).

The SO guarantees the operation of the system by defining rules and enforcing them. These trading rules guarantee transparency, fairness, efficiency, and competitiveness in the market. In addition, these rules promote confidence among market participants. These rules establish: (i) the definition of rights, (ii) which rights are going to be traded, (iii) how the participants will obtain these rights, (iv) how long rights will last, (v) registration (enrolment requirements), (vi) how the market will operate, (vii) penalties for participants who break rules, (viii) flood damage costs and conditions which are related to violated boundaries established in the market, and (ix) procedures for verification of rights and trades. These rules must be known to all participants.

Auction rules require that all participants abide by land uses and management practices (IC allowances) for which their bids were accepted. Following market clearing, the SO must enforce the agreed behaviours of the contract holders. Ideally, the SO would enforce the actual discharges and impacts. However, these discharges are uncertain, and the auction manager’s selected discharge model may be incorrect. Hence, the auction is fundamentally about participants’ land use and management practices, and flood modelling.

2.3 Rainfall events (Storms)

Many hydrologic studies discuss rainfall and flood frequency and their probability distributions (Pearson 1991b; a; Haan et al. 1994; Ramachandra Rao and Hamed 2000; Mays 2001; Koutsoyiannis 2007; Kron 2007; Pasche 2007). Probabilistic modelling is key to estimating damage and flood risk due to extreme storm and flooding events. Because the storm probability distribution and subsequent flooding are often unknown, simple storm distributions are used to describe the hydrological phenomenon (Ramachandra Rao and Hamed 2000; Koutsoyiannis 2007). However, rainfall events are difficult to estimate, due to their spatial distribution. Thus, the proposed markets could consider a storm distribution, at a specific place, to design storms, under the assumption that similar storms would affect the whole catchment.

Singh and Strupczewski (2002) summarised the methods used to estimate the statistical parameters of the storm probability distribution into four groups: (a) empirical, (b) phenomenological, (c) dynamic and (d) stochastic with Monte Carlo simulation. The authors suggested that the empirical methods would be most suitable and accurate to estimate exceedance probabilities (EP) at extreme events, particularly the at-site frequency analysis method.

The at-site frequency method requires data collection about storms, plotting and estimating probabilities as well as modelling the probability distribution and testing goodness of fit. Accordingly, literature suggests different extreme probability distribution types for both storms and floods. These are extreme values type I (EV1) such as exponential, lognormal and Weibull which belong to the Gumbel distribution, and type II (EV2) such as Pareto and log-Pearson type 3 (three parameter distribution) and Wakeby (four-parameter distribution) (see Ramachandra Rao and Hamed 2000; Koutsoyiannis 2007). From a theoretical and practical point of view, Koutsoyiannis (2007) reviewed the applicability of rainfall distributions and pointed out that the popular Gumbel distribution and EV1 underestimate events at the tail distribution. The author also presented theoretical and empirical arguments to support the use of EV2 type distribution. For an accurate rainfall distribution estimate in the catchment, annual exceedance probabilities (AEP) can be calculated for different storm intensity (depth) events.

The exceedance probability (P) corresponds to the probability that a storm will meet or exceed a particular event in a period, $P\left(S \geq s_T\right)=1-P\left(S < s_T\right)\ =1-F\left(s_T\right)$, where $S \geq s_T$, is an event or rainfall in volume with a specific magnitude across time, for instance 100 mm in 24 hours, and $F\left(\cdot\right)$ is the cumulative distribution function (CDF). For instance, F(S<100) = 0.975, then P(S≥100) = 0.025, which represents a storm of 100 mm in 24 hours. Figure 2-3 illustrates an AEP and storm intensity curve reached in a hypothetical catchment.

The storm probability distribution is continuous and multidimensional with dimensions corresponding to ‘storm parameters’ such as: intensity, duration, total precipitation and geographic distribution. For this thesis, it is determined by a number of representative storm events. Each representative storm represents a subset of the storm distribution and this storm will be called “storm event” or “event”. The single event could occur with a probability 0.025; however, this storm event could occur during a given period or not at all.

Alternatively, a discrete probability can be obtained using the “basic stage method” which categorizes storms with similar parameters of magnitude and intensity. Hence, it is possible to obtain a storm magnitude probability, $P\left(S \geq s_{(m)}\right)=\frac{m}{N}$, where m is the ranking in a descending order, $s_{(m)}$ is the mth largest storm in volume, and the series N is the total scale storm associated with the magnitude and ranking. This probability is an estimator of the cumulative probability distribution $F\left(S\right)=\int_0^S f\left(s\right)ds$ of the sample into a rankordered probability. Hence, it is possible to use statistical methods to estimate probabilities of different events in a range $P\left(a \le S \le b\right) = \int_a^b f\left(s\right)ds$. For more details, see Tung et al. (2006), Haan et al. (1994) and Mays (2001).

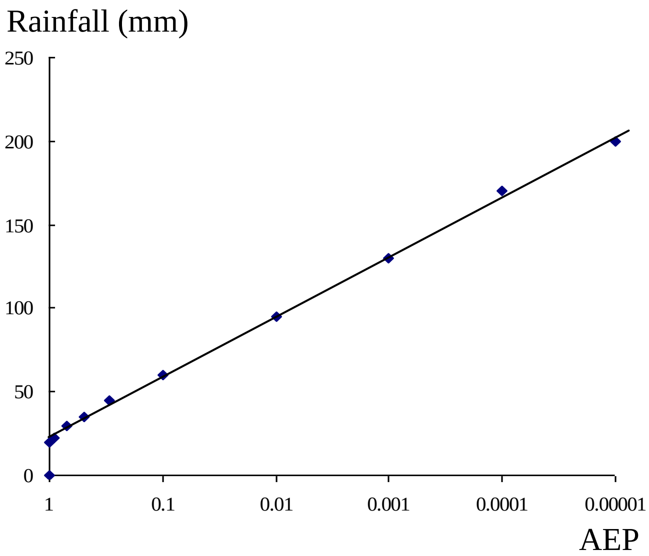

Figure 2-3 Hypothetical annual exceedance probability (AEP) and rainfall

In Chapter 3, the proposed determinist IC market formulations will be based on a single "design storm". This worst case will define the possible runoff from the properties, the status quo of flows (based on the current IC allowances) and impact flows at control points. In Chapters 4, 5, 6 and 8 a range of events will be accounted for based on stochastic market formulations. These events will represent the storm distribution within the catchment.

Different probability representations can be used to evaluate the relationship between rainfall and floods. However, Koutsoyiannis (2007) noticed that rainfall and floods have similar probability distributions and Loucks et al. (2005) pointed out that the probability to reach a peak flow that causes damage is the same as the probability of flood damage. Thus, different combinations of rainfall depths, flow peaks, and flood levels would present similar distributions in the catchment. This could also be represented in terms of recurrence intervals using High Intensity Rainfall Design System for New Zealand HIRDS (NIWA 2002; 2008). However, as to be discussed further in this thesis, flooding components could be managed by changing impervious cover (Olsen et al. 2000); consequently, a particular flooding condition could change in probability of occurrence as well as the floods physical distributions. Figure 2-4 illustrates the stochastic nature of rainfall on impervious cover, and its effects on flows and flood distributions in flood areas. ϕ^{s} is the probability of a storm scenario *s*, CP1 and CP2 are control points 1 and 2 respectively, and ICL is the imperviousness level in the area.

The rainfall scenarios' probability (A) will relate the resulting runoff hydrographs due to the different land uses, BMPs or control technologies from properties (B). These flows from point and non-point sources go through channels and streams, impacting different places with different magnitudes and intensities, according to IC allowance levels, catchment conditions and storm scenarios (C). Changes to IC allowances (ICL) will change the flows and resulting flood damage distributions (at control points) (D). Thus, under the same storm distributions, changes in the IC allowance modify the flooding distribution and damage.

Schielen (2009) pointed out that changes in IC allowances can significantly affect flooding, but climate changes would have minimal on flows. Although other stochastic sources affect runoff from properties and transport of runoff flows, this research will focus on the stochasticity related to storms that affect the catchment (Ribaudo et al. 1999; Shortle and Horan 2001; Shortle and Horan 2008). The proposed Det_MarketIC market model will use a design storm as the basis for trading, and the issue of defining the storm will not be discussed further in this research.

2.4 Storm water runoff

Storm water runoff is the excess water from a rainfall event that does not infiltrate into the soil and is discharged from a land surface. This flow is part of the total flow, which corresponds to the runoff and groundwater (base flow). This research will focus on the runoff and the base flow, estimated as part of the status quo of imperviousness level in the catchment and storms. We assume a base flow from which changes in imperviousness will change only the runoff from the properties.

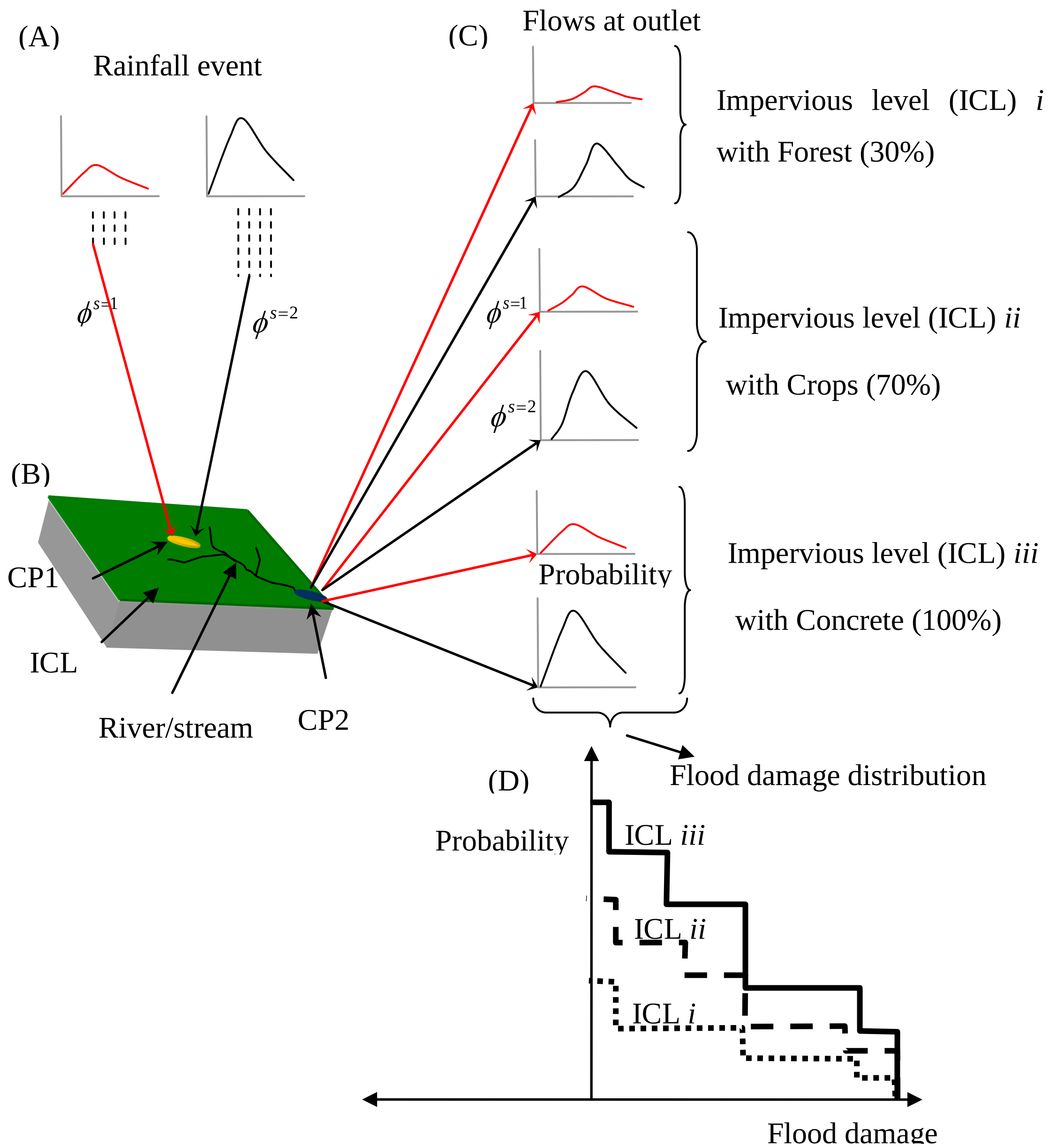

Figure 2-4 Relationship between (A) rainfall events, (B) IC allowances, (C) flows at control points, and (D) flood damage distributions in the catchment. CP1 and CP2 are control points, ϕ^s storms probabilities, and *i*, *ii* and *iii* are final IC allowances (ICL).

The runoff flow may be connected to channel systems, storm sewers, pipes, and streams; thus, these flows convey and reach different points in the catchment such as a reservoir, lakes, rivers, estuaries, and the sea. These flows may exceed threshold capacities in channels and streams, producing flooding problems.

The overland runoff flows can be quantified through monitoring or modelling; however, monitoring is expensive due to the logistics needed in the field to estimate flows. Alternatively, modelling is widely used to estimate runoff flows, and the rational method is probably the simplest runoff model available to determine peak flow discharges from a drainage area based on runoff coefficients, rainfall intensity and surface area. This method, however, does not allow estimating the flow movement in routed channels and streams, nor allows estimating runoff according to different impervious levels (assumes 100% of imperviousness). Actually, the method[3] does include a level of imperviousness through the loss coefficient and the calculation is only for estimating peak flows. Complex models estimate runoff by using components of a hydrological cycle. This is called flood routing. These models are also able to approximate flows in channels and streams. In addition, hydrological models can be linked to geographic information systems (GIS) such as HEC-HMS (HEC 2008a) and SWAT (USDA-SWAT 2008) in order to improve the spatial data management and visualisation (Pasche 2007). A certain method may be more appropriate than another under specific scenarios of location, vegetation, topography, catchment size, weather, and land uses; and without such information it is difficult to determine the best method (Haan et al. 1994; Pasche 2007).

A runoff hydrograph is a record of runoff flows by time. Flows depend on upstream land characteristics such as land cover (IC), soil moisture, soil type, management practices, and the rainfall distribution, which affect the infiltration patterns and groundwater (Pasche 2007). Figure 2-5 illustrates runoff hydrographs from two types of hypothetical rainfall distributions categorized as type I and II (whose peaks occur at 1/3 and ½ duration of the storm, respectively) as well as storm events with different probabilities. Changes in rainfall distribution and probability affect the concentration time, peak time, and total flows discharged.

Figure 2-5 illustrates runoff hydrographs resulting from different IC allowances A, B and C. Increasing IC decreases infiltration, increasing the total volume of flows (area under curve) and possibly reduces the time of concentration[4]. For instance, increasing IC, by

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[3] The rational method calculates peak flows (Q) as follows: $Q = C \times i \times A$, where C is loss coefficient, i is rainfall intensity, and A is area.

[4] The time of concentration (tc) is closely related to discharge and infiltration. This time corresponds to the time from exceeding runoff from the most remote point to the outlet of a property or catchment. The tc

changing the land use from forestry to horticulture land would reduce infiltration and accelerate concentration time. Consequently, peak flows at channels and flooding control points could occur earlier.

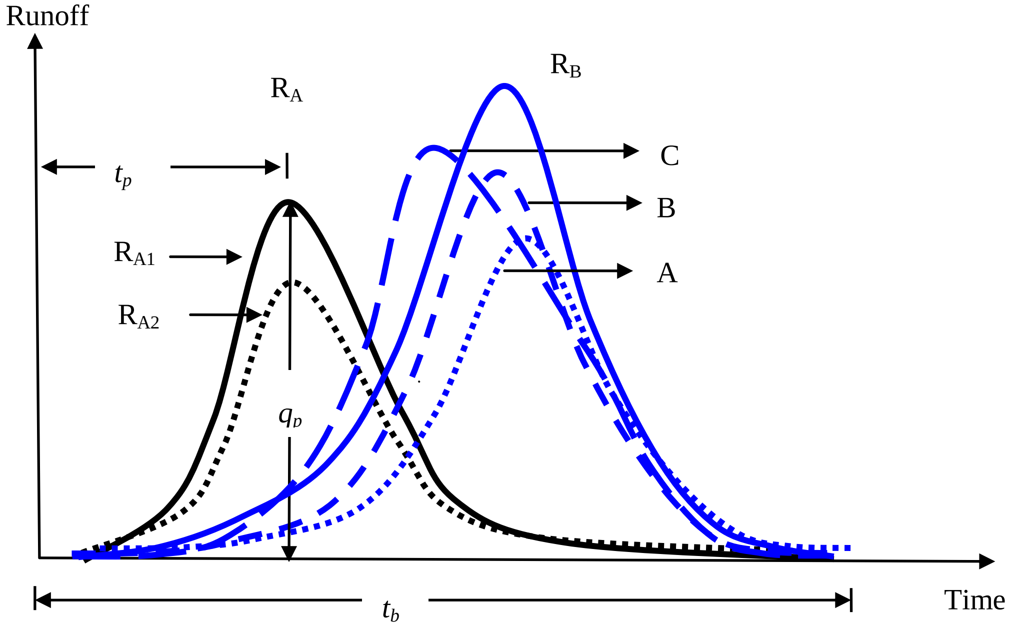

Figure 2-5 Runoff hydrograph curve of two rainfall distributions. R_A is a type *I* storm and R_B is a type *II*. R_{A1} rainfall event 1 and R_{A2} rainfall event 2, $R_{A1} > R_{A2}$. A, B and C are runoff hydrographs with different land uses or IC allowances (A<B<C) under similar rainfall events. q_p is the peak flow, t_p is the time peak, and t_b is the total time duration of runoff.

BMPs and technologies will modify the runoff hydrographs. For instance, retention ponds slow the release of water. In this case, A, B and C represent different runoff control practices (for a specific land use). Changes in land use, implementation of runoff control technology, and other BMPs may be approximated to an equivalent IC allowance in the catchment (Haan et al. 1994). Such practices may change the peak time as well as the total runoff flows (Figure 2-5) from the property, and consequently the routed flows and peak

quantifies the time from the overland flow and the conveyance time in the channels, streams, junctions, rivers, street gutters and storm sewers, etc. Similar to runoff estimations and routing, different methods can be used to quantify. The *tc* is mainly affected by overland and channel factors such as surface roughness, slopes, sectional shape, length, and storms (NRCS 1975; SCS 1985).

flows at flooding areas. These changes in IC have a spatial and temporal flow effect at channel and flooding areas.

Hydrological models such as HEC-HMS (HEC 2008a) and WIN-TR55 (NRCS 1975) simulate runoff processes and flow hydrographs. Hydraulic models such as HEC-RAS (HEC 2008b), SHE model, MIKE SHE and MIKE11 (Pasche 2007) are used to simulate changes in water levels in rivers and streams. Flows obtained from hydrological models can be used in hydraulic models to simulate flooding. This thesis will use HEC-HMS and HEC-RAS to illustrate a market application in Chapter 7.

2.5 Hydraulic modelling: governing equations

This section presents 1-dimensional governing equations used for flooding in channels. The main equations account for overland and routing flows which constitute the black box in the formulation of the clearing model. These equations are used by HEC-HMS and HEC-RAS to estimate runoff and flows in channels.

Governing equations for hydraulic flows may be included in the market by discretising and linearising simulated transport coefficients from each participant, or implicitly by using available hydraulic models in the clearing formulation. For instance, the Victorian gas market in Australia uses implicit governing equations to simulate gas flows and pressures in pipes (Pepper et al. 2012). Other proposed markets have used simulators to obtain impacts and transport coefficients, which are included in the market clearing formulation. Examples of these are the proposed groundwater and nitrate markets by Raffensperger et al. (2009) and Prabodanie et al. (2010).

The governing equations for simulating flooding are the mass and momentum conservation equations, known as Saint Venant equations in one-dimensional flow (Pasche 2007). These equations predict unsteady flow routing in channels, streams, and floodplain areas. Figure 2-6 illustrates sectional shapes of channels and presents the main flow components included in these equations. Views (A), (B) and (C) respectively correspond to longitudinal, plane and transversal views of a control volume (*CV*) for the momentum balance.

Mass conservation represents the moving volume between adjacent locations, accounting for the changes in water storage, inflows and outflows, evaporation, etc. A control volume, *CV*, of the flow with density ρ and lateral inflows q corresponds to the sum of the mass variation by unit of time. The mass variation accounts for the spatial flow change in the direction *x* in the time *t* as shown in Equation [2.1].

$$\rho \frac{\partial Q}{\partial x} dx + \frac{\partial}{\partial t}(\rho A dx) = \rho q \, dx \qquad [2.1]$$

where *Q* is the flow discharge and *A* the wetted area. Since water is incompressible, flows are uniform and constant in direction *x* (*dx*), the mass equation is as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad [2.2]$$

The LHS in equation [2.2] corresponds to changes in flow along *x* and changes in wetted area *A* and by time *t*, which equals the lateral inflows (RHS). Equations [2.1] and [2.2] represent one dimensional flow continuity. Using a similar control volume, the linear variation in time momentum conservation[5] (*Ml*), $\frac{\partial (Ml)}{\partial t}$, accounts for the forces acting in the fluid, and is measured in terms of mass and velocity, *v*, as follows:

$$\frac{\partial (Ml)_{cv}}{\partial t} = \rho \frac{\partial}{\partial x}\left(v^2 A\right) + \rho dx \frac{\partial Q}{\partial t} = F_x \qquad [2.3]$$

In Equation [2.3], F corresponds to the force in the *x* direction. Pressure forces change in depth and space between $F_{pl}\big|_{x1}$ and $F_{pl}\big|_{x2}$. Compression forces F_P and $F_P^{'}$ for lateral slope at channels as depicted in Figure 2-6 (B). Mascarenhas et al. (2005) presented and illustrated the changes in forces as follows:

$$F_{pl}\big|_{x1} - F_{pl}\big|_{x2} = -\rho \, g \, A \frac{\partial h}{\partial x} dx - \rho g \int_0^{h(x)} \left[h(x) - \xi\right] \frac{\partial}{\partial x} B(x,\xi) \, d\xi dx \qquad [2.4]$$

Where $B(x,\xi)$ is the channel or stream width, ξ is the height from the bottom of the channel, $[h(x)-\xi]$ is the distance of the centroid to the free surface, and $A = \int_0^{h(x)} B(x,\xi)\, d\xi$ is the wetted area; accordingly $dA = B \, d\xi$.

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[5] Ml = mass × velocity = ρ *Adx*

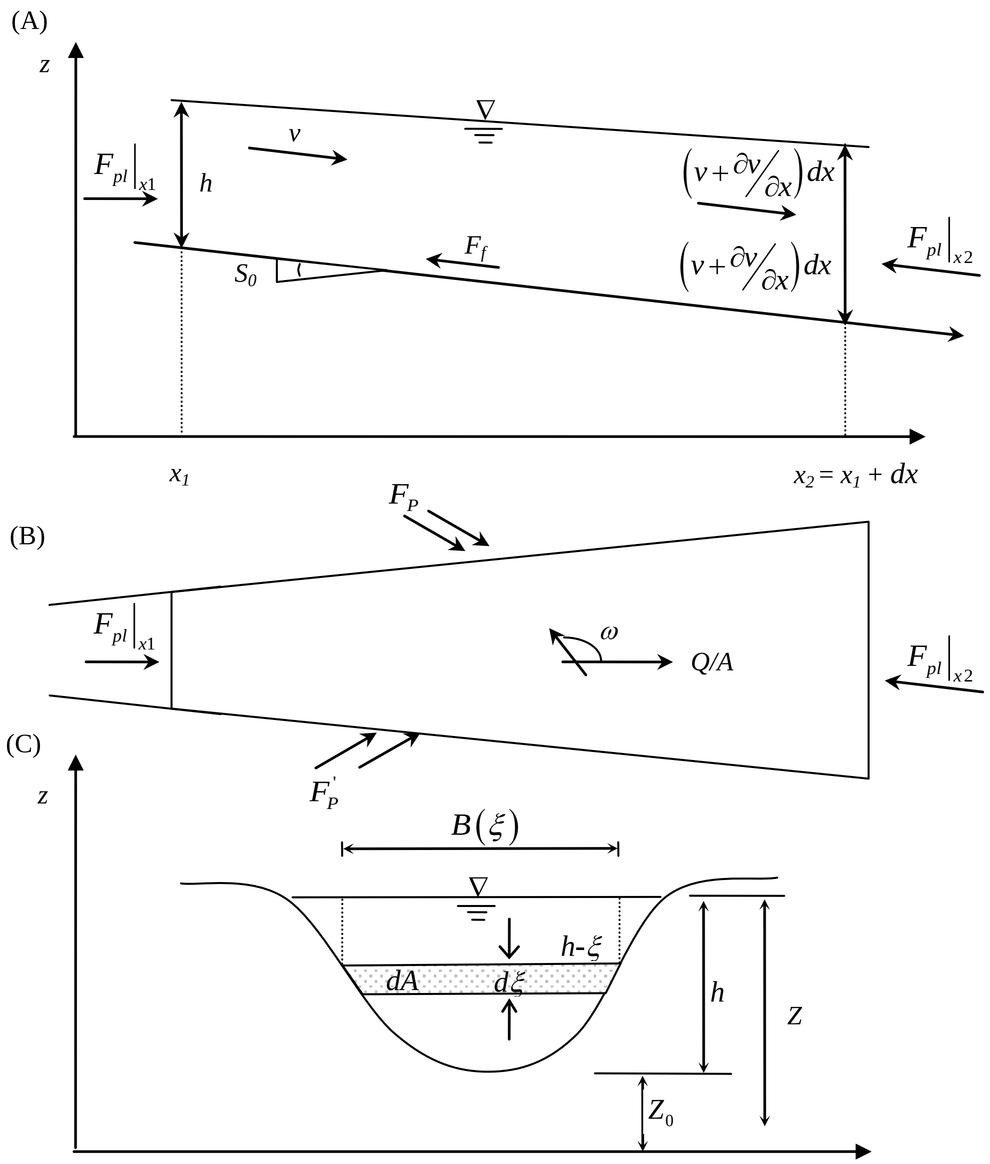


Figure 2-6 Sectional shape with channel factors and physical components. A, B and C are longitudinal, plane and transversal views respectively of a control volume (*CV*) for the momentum balance. Individual factors are presented in equations 2.1 to 2.5.

The LHS (Equation [2.4]) is the change in pressure between points x_1 and x_2. The RHS accounts for the changes in force in the *x* direction. But this formulation does not account for changes in force induced by the lateral channel or river slope. Changes in the sectional width $F_s|_x$ implies changes in forces, according to the wetted area $dBd\xi$, given depth *h* and the distance $\left[h(x)-\xi\right]$ of the centroid to the free surface (see Figure 2-6 C). The changes in forces also account for the bottom slope force $F_s\big|_x$ and the friction force $F_f\big|_x$; hence, the resulting momentum condition becomes:

$$F_x = F_{pl}\big|_{x1} - F_{pl}\big|_{x2} + F_p\big|_x + F_s\big|_x - F_f\big|_x = -\rho\, g\, A dx \frac{\partial h}{\partial x} + \rho\, g\, A dx\, S_0 - \rho\, g\, A dx\, S_f \qquad [2.5]$$

where S_f is the energy line slope related to friction head loss of the flow, and S_0 is the bottom slope. Finally, substituting F_x from Equation [2.5] into the momentum balance of equation [2.3] and rearranging, the momentum balance equation becomes:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial h}{\partial x} - g\, AS_0 + g\, AS_f = 0 \qquad [2.6]$$

From left to right, the terms in Equation [2.6] are the local acceleration $\frac{\partial Q}{\partial t}$, the conservative acceleration $\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$, the pressure force $A\frac{\partial h}{\partial x}$, the gravity force or inertia $g\, AS_0$, and the friction force term $g\, AS_f$.

From the governing equations, it is possible to obtain relationships in terms of velocity and depth under different conditions and assumptions (Haan et al. 1994; Mascarenhas and Miguez 2005). Such relationships include i) discharge and mean depth; ii) discharge and water surface elevation; iii) mean velocity and depth; iv) mean velocity and water surface. These relationships are obtained from Equations [2.2] and [2.6], given assumptions about forces and sectional topography. The relationship used to model flow and flood components will depend on particular conditions in the catchment (Mascarenhas and Miguez 2005). Alternatively, the governing equations can be represented in terms of depth and velocity without lateral inflows. This relationship would depend on the sectional shape across the channel and flooding area. The mathematical complexity to obtain a feasible solution increases with more irregular sectional shapes. For instance, assuming a rectangular sectional shape, the depth, *h*, and velocity, *v*, conditions from balance and momentum will be as follows:

$$\frac{\partial h}{\partial t} + h\frac{\partial v}{\partial x} + v\frac{\partial h}{\partial x} = 0 \qquad [2.7]$$

$$\frac{1}{g}\frac{\partial v}{\partial t}+\frac{v}{g}\frac{\partial v}{\partial x}+\frac{\partial h}{\partial x}=S_0-S_f \qquad [2.8]$$

$$S_f=\frac{n^2v^2}{h^{4/3}} \qquad [2.9]$$

$$h=h(x,t)\text{, and } v=v(x,t) \qquad [2.10]$$

Equation [2.7] is the dynamic equation in terms of depth and velocity. Equation [2.8] is the continuity equation. Equation [2.9] corresponds to the resistance based on the Manning-Chezy formula. Equation [2.10] represents the depth and velocity which depend on the position in the channel and time. Thus, a flow Q^* could produce a specific h^* and v^*, and any change from this baseline of flow Q^* will produce changes in h and v. Mascarenhas et al. (2005) pointed out that the main equations can be discretised; therefore, velocity and depth can be discretised for a section x at time t. Although it is possible to obtain many relationships, the mathematical complexity increases as well as problems with stability, consistency and convergence.

2.6 Linear impacting flows

At this stage, the governing routing flows were included to clarify the hydraulic black-box. This box represents the main physical flow variables which can be used explicitly in a market clearing formulation. Mascarenhas and Miguez (2005) pointed out that the governing equations can be discretised and linearised for which flows could be calculated based on sectional shapes. The authors noticed that problems with stability, consistency and convergence could be observed when simulating flows. Pepper et al. (2012) pointed out that the operational gas market model relies on non-linear governing relationships which are linearised. The authors showed how governing relationships are discretised, such as the relationship between flow rate and the pressure change across a pipe segment. The authors presented employed linearisations for both piece-wise and successive linear representations. However, rather than explicitly simulating flows and variables, simulators are used for the purpose of obtaining impact coefficients for the IC market. Flow variables related to flooding components will be used to model flood damage as further explained in the following section.

The market for IC allowances account for the changes in impacting flows at control points from a status quo of flows related to the initial imperviousness levels in the catchment. The flows and their changes due to trading are linearised to be included in the market formulations. This linear system translates imperviousness levels into linear effects in flows, and the linear effects can be estimated, for instance, by superposition of individual hydrographs or by a Taylor expansion from the status quo of flows and their translated initial imperviousness levels.

Because, flows are capped in the deterministic market, the market model includes constraints. This constraint representation is illustrated in Figure 2-7. The illustration shows two flow levels related to initial IC allowances; $\bar{g}_{i,j}$ and $\tilde{g}_{i,j}$ correspond to the initial IC allowances *j* of a participant *i*, and $Q^0_{k,t}$ and $\bar{Q}^0_{k,t}$ are the two flow levels related to initial IC levels G_0 and G_1 at time *t* and control point *k* in the catchment respectively; and $h^t_{0,k}$ and $\bar{h}^t_{0,k}$ are the calculated flows of the linearisations. L_k is the threshold capacity at control point *k*. This is just one dimension of a multi-dimensional surface of flows and IC level patterns at a control point. Thus, any change in IC allowances would change flows from the initial flow conditions by $H^{t-u+1}_{i,j,k} g_{i,j}$.

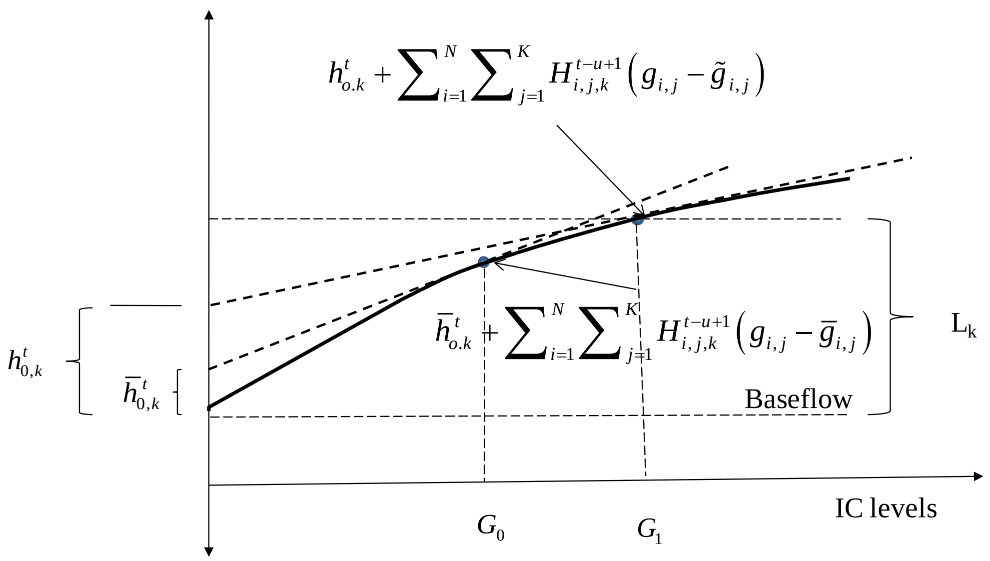


Figure 2-7 Impact flow linear approximations

The proposed market model aims to avoid flooding by limiting this flow as described in the Chapter 3. However, extreme storm events may generate flows over channel capacity, which violate the limits and produce flood problems at control points. For these “exceeding flows" it is necessary to model flooding for the purpose of predicting flood damage costs. The expected flood damage cost can then be determined as a function of channels flows (over capacity) at the various control points over time.

2.7 Flood modelling

Modelling of flooding requires quantification of the maximum flood stage that may take place in the catchment, given an extreme storm event. In many places, flooding is also affected by snowmelt (Kattelmann 1997). Snow pack would simply be modelled as a different physical system (Walter et al. 2005; Kerkez et al. 2010). Walter et al. (2005) pointed out that snowmelt modelling is well-established and based on energy budget models. Extra modelling would be required for the market formulation. Although this issue is outside the scope of this thesis, it would be make an interesting topic for future research. Maximum flooding can be estimated from models by using historical data supported by GIS (Guganesharajah et al. 1985; Tineke De Jonge and Hogeweg 1996; Dutta et al. 2000; 2003). Mathematical formulations to approximate flooding patterns, based on sectional shapes in channels and floodplains with routed flows, can be devised using Manning's equation, the Muskingum-Cunge method, and the Saint Venant equations (Ghosh 1997; Mays 2001; Mascarenhas and Miguez 2005; Kron 2007). These equations represent the movement of fluid (water) in routed rivers, channels, and reservoirs using parameters such as inflow, outflow, sectional shape, depth, and velocity, using hydraulic principles. But, obtaining precise estimates is complex. Thus, it is necessary to simplify the analysis by making assumptions about sectional shapes, roughness, slope, etc. (further detail see e.g., Ormsbee et al. 1984; Ghosh 1997; Mujumdar 2001; Knight and Shamseldin 2005; Mascarenhas and Miguez 2005; Kron 2007). For instance, in the Neckar river, Germany, one dimensional flood modelling is used for linking flood parameters with risk analysis and mapping inundation areas (Kron 2007).

Accurate spatial and temporal information is required to model flooding and also flooding components, which will be used in the market clearing formulation. Flooding depth, $G(x)$, depends on the total flows x at control points, which are conditioned to the channel capacity or flow capacity x^{*}_{cap}, the IC and soil moisture, the storm event s, and topography p in the catchment. Thus, these flows above capacity (exceeding flows) would generate a flooding condition, $g(\cdot)$, which is represented as follows:

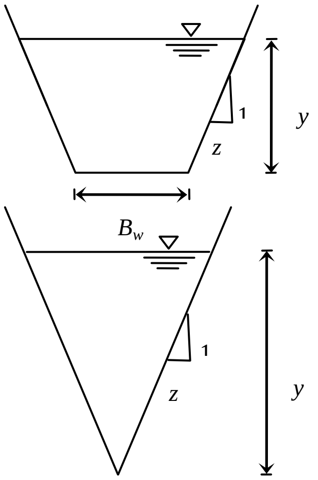
$$G(x) = g\left(x \,\middle|\, IC, s, ..., p, x^{*}_{cap}\right) \qquad [2.11]$$

Routing flows at control points are unsteady, changing over time; thereby, parameters such as depth, velocity, duration, and sediment load, will also change over time (Ghosh 1997; Dutta et al. 2000). In this research, HEC-HMS is used to model steady state inflows and HEC-RAS is used to model unsteady flows at control points among the channel. However, changes in some factors such as peak and velocity are quite slow, particularly under extreme storms (Haestad et al. 2003).

Using Manning's Equation, Equation [2.12], it is possible to approximate flooding depth and velocity under assumed steady flows. Equation [2.12] estimates the channel flow Q in m^3/sec, where A is the channel cross sectional area below the water level in m^2, R is the hydraulic radio which can be represented as $\frac{A}{P}$ where P is the wetted perimeter in *m*, S is the slope, and n is Manning's roughness coefficient.

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_o^{\frac{1}{2}} \qquad [2.12]$$

Additionally, from Manning' equation velocity $v = \frac{Q}{A}$, with cross sectional shape and flow information, it is possible to estimate changes in flooding depths for a particular area (Figure 2-8). Figure 2-9 illustrates changes in depths for trapezoid sectional shape channels.



$$Q = \frac{\left(B_w + zy\right)^{\frac{5}{3}} y^{\frac{5}{3}}}{n\left(B_w + 2y\sqrt{1+z^2}\right)^{\frac{2}{3}}} S^{\frac{1}{2}}$$

$$Q = \frac{z^{\frac{5}{3}} S^{\frac{1}{2}}}{n\left(2\sqrt{1+z^2}\right)^{\frac{2}{3}}} y^2$$

Figure 2-8 Sectional channel shapes and relationship between flow and depth based on Manning's equation (Mays 2001), where z corresponds to channel bank slope in meters, y is depth in meters, n is manning's coefficient, S is the channel slope, and B_w is channel base in meters.

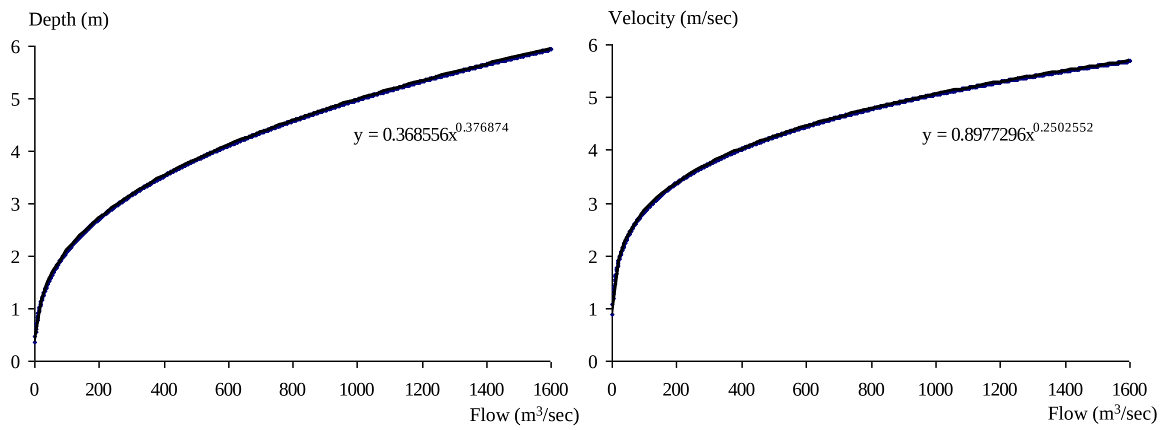


Figure 2-9 Functional depth vs. flow and velocity vs. flow in a trapezoidal shaped channel. Sectional parameters are B_w = 10, z = 500, S = 0.004, and n = 0.023.

Channel and stream cross-sections are, however, not uniform trapezoid or triangular shapes; in fact, most of them are irregular, as illustrated in Figure 2-10. This non-uniformity produces non-linear and non-convex depth.

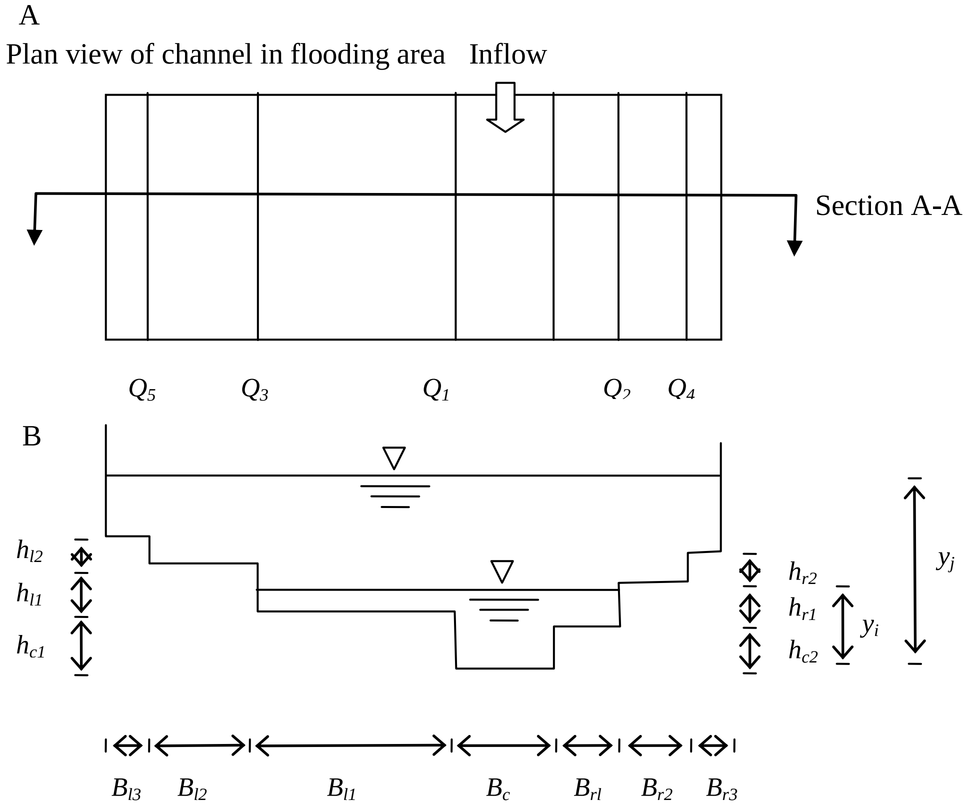


Figure 2-10 (A) Plan view of channel (stream) in flooding area; where Q_1, Q_2, Q_3, Q_4, and Q_5 are flows at different levels in the area. (B) Sectional shape A-A; B_{l3}, B_{l2}, B_{l1}, B_c, B_{r1}, B_{r2}, and B_{r3} are base widths; h_{l2}, h_{l1}, h_{c1}, h_{c2}, h_{r1}, and h_{r2} are the maximum depth levels for each section; y_i and y_j are maximum depths related to flows Q_i and Q_j.

Depth levels, at different sectional places and flows can be observed in an example application in Figure 2-11. The example uses the Manning equation with Manning's coefficient 0.002, channel slope = 0.0004, B_{l3} = 400 m, B_{l2} = 250 m, B_{l1} = 200 m, B_c = 15 m, B_{r1} = 300 m, B_{r2} = 200 m, B_{r3} = 400 m, h_{c1} = 1.2 m, h_{c2} = 0.9 m, h_{l1} = 0.35 m, h_{r1} = 0.55 m, h_{l2} = 0.55 m, and h_{r2} = 0.4 m. The changes in depth level at different section levels can be estimated by varying flows between 0 and 5,000 m^3/sec.

The total simulated depth corresponds to the maximum depth level reached at the middle of the channel or stream as well as the flow depth reached at different areas across the channel sectional shape. For instance, flooding starts at B_{l1} level when flows are greater than 80 m^3/sec, reaching a 1.5 m depth with 3,310 m^3/sec. At B_{r3} level with flows above 680 m^3/sec, the depth reaches 1 m with approx 4,010 m^3/sec.

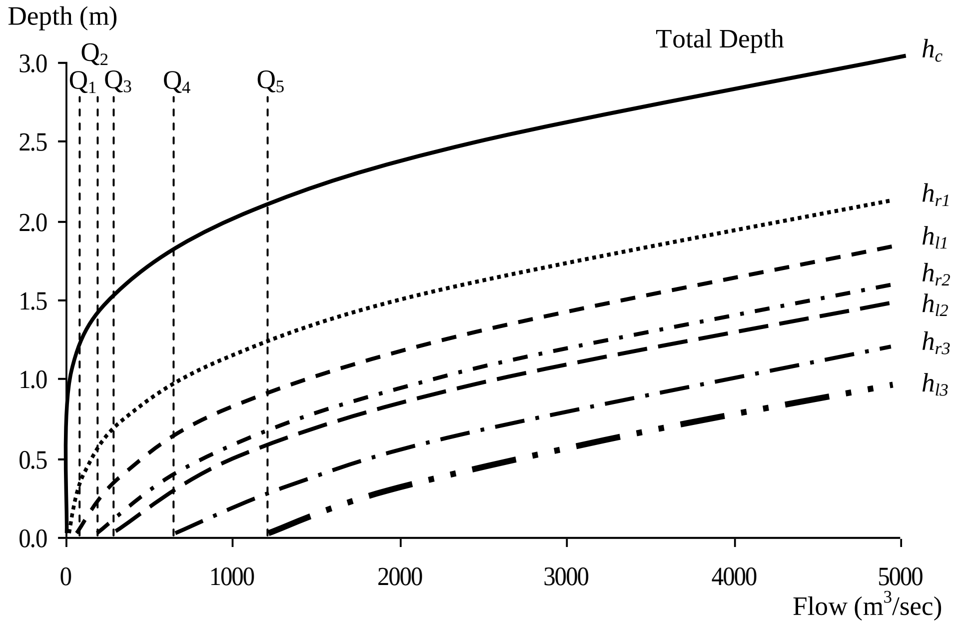


Figure 2-11 Changes in depth with different flows across the sectional shape A-A; Q_1, Q_2, Q_3, Q_4, and Q_5 are flows that reach different levels in the section; and h_{l1}, h_{l2}, h_{l3}, h_{r1}, h_{r2}, h_{r3}, and h_c are the maximum depth levels for each section.

In floodplain areas, flow depth and flows are modelled using water mass balance and continuity equations (e.g., Ormsbee et al. 1984; Ghosh 1997; Philbrick Jr and Kitanidis 1999; Knight and Shamseldin 2005; Mascarenhas and Miguez 2005). Figure 2-12 illustrates routing relationships between inflow and flood parameters such as depth, outflow, and storage in the routing process at specific locations. In addition, it is possible to estimate other relationships from the flooding area such as depth vs. flood area, and maximum flow vs. flood area. These relationships depend on topography, soil, land use (vegetation), and infrastructure in the flood area; accordingly, they should be estimated at each channel, river and floodplain area segment.

Flood routing processes are non-linear, which may imply a need for piecewise approximations in the market formulation. Ormsbee et al. (1984) utilised piecewise approximations for routing models in channels and basins when modelling the design of a detention catchment for storm water management in Glen Ellyn, Illinois. A comparable approach could be used for the IC market formulation; however, the market clearing model could include the flow-flooding relationships explicitly in the damage function.

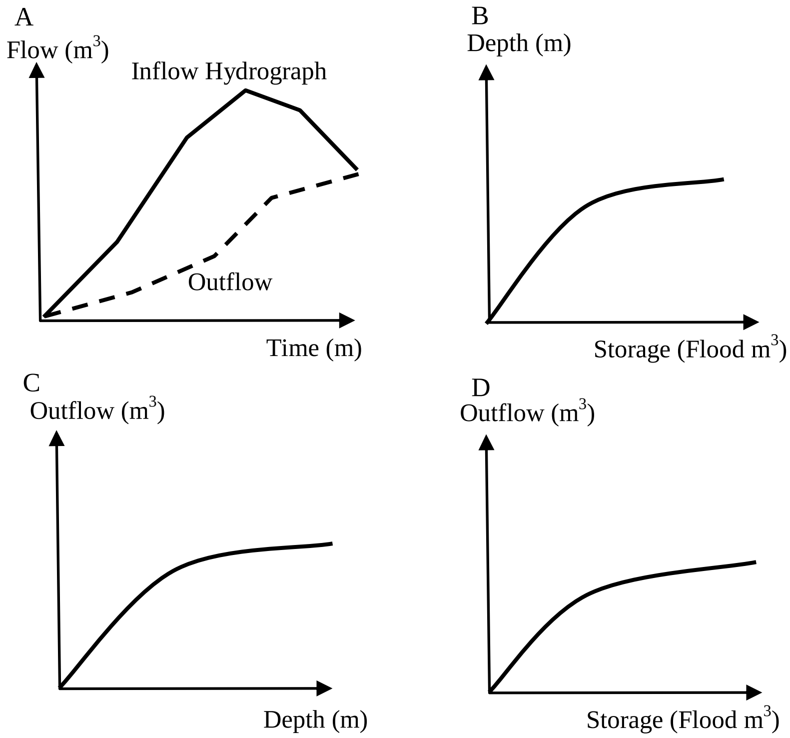


Figure 2-12 Graphical depiction of flood routing. Panel A is a relationship between inflow and outflow by time, B is a relationship between storage and depth, C is a relationship between depth and outflow, and D is the relationship between outflow and storage.

Flood damage could be represented by the violations to boundaries at channels, streams, pipes, and floodplain areas, related to cost penalties for each flooding component. Thus, it is possible to assume maximum thresholds in the channel's capacity in order to estimate the cost damage. This issue will be presented in Chapter 3 when dealing with a deterministic formulation that incorporates maximum channel capacities in the market clearing model. However, given the stochastic nature of rainfall and the violations of channel capacity, it will be necessary to estimate flooding parameters in channels and floodplains.

2.8 Flood damage estimation

With the aim of considering flood damage, the proposed Sto_MarketIC and Sto_MarketIC_Risk clearing models in Chapters 4, 5, 6 and 8 link flooding components to penalties in each control point. Flood damage is used for participants to create a demand function for IC (see Section 2.2 in this Chapter, Section 3.2 in Chapter 3, and Section 4.6 in Chapter 4), which depends on the selected storm scenarios used to establish the markets (see Section 3.4.9 in Chapter 3 and Section 4.3 in Chapter 4). The market models account for the storm scenarios at each control point, and the models calculate the expected flood damage based on the changes in imperviousness and a flood damage function.

Damage depends on the spatial area where flooding may occur. Therefore, cost penalties in the market formulation account for mitigation and damage based on maximum flooding depth, hastened flood which increases damage due to abrupt increases in flow velocity and shear stress in the river's topographic cross-sections, and slow reductions in flow rates which lengthen flood durations. Thus, the accurate estimation of damage function is important to the market design.

The damage estimate considers direct and indirect effects (Smith 1994; Parker 1995; Penning-Rowsell and Green 2000; Veerbeek 2007). The former accounts for the direct contact damage of flood on populations and properties, while the second accounts for flood damage induced on services, traffic problems and economic losses in surrounding areas.

Penning-Rowsell and Green (2000) presented an update of the flood damage estimation theory, pointing out that damages are too complex to estimate due to the indirect and secondary damage. The importance of such damage depends on the specific area. For instance, in the lower Thames area, the authors pointed out that the indirect damage would represent only 4% of the total damage. Additionally, they suggested periodically calibrating the total damage due to more expensive contents derived from economic growth. The authors also noticed that damage estimation had shifted recently toward replacement cost rather than depreciated values.

To assess the cost of flooding, the SO (authority) could use simple approximations with category-unit loss function, which accounts for the stage-flood damage function (Krzysztofowicz and Davis 1983a; Ormsbee et al. 1984; Hannan and Goulter 1985; Kron 2007; de Moel and Aerts 2011) or complex methods by using GIS (Smith 1994; Tineke De Jonge and Hogeweg 1996; Dutta et al. 2000; 2003; Herath 2003; Kazama et al. 2009). The unit loss estimates the potential damage per property or per economic sector ("the loss unit"), related to maximum flood depth in an ex-ante flood situation or under pre-established flood damage relationships.

Penning-Rowsell and Chatterton (1977) estimated stage-damage curves for urban areas in the UK. They modelled a depth damage function for different residential and building properties based on property conditions. They used these curves to assess flood damage and evaluate flood mitigation options. Tineke De Jonge and Hogeweg (1996) noted that the flood cost depends on the land uses, flooding depth and spatial distribution. Embrechts and Schmidli (1994), and Ermolieva and Ermolie (2005) pointed out that costs depend on the timing of flows (flooding). Dutta et al. (2003) and Herath (2003) recommended estimating damage with detailed GIS information. Additionally, based on a spatial distribution, those authors modelled inundation depth with a stage-damage function. When doing a conventional valuation, Tung et al. (2006) noted that a flood damage function can be linked with the physical characteristics of the installed infrastructure; thus, cost damage could be estimated according to violations in constraints associated with infrastructure capacity. De Moel and Aerts (2011) focused on flood damage estimates under uncertainty based on depth-damage functions at different flood areas with different land use. They accounted for possible damage based on assessments and land uses.

Smith (1994) reviewed some urban flood damage methods, noting that finding a stage-damage curve (flooding depth) is a key step in assessing the flooding cost; however, this is only a first approximation to the flooding cost, and the assessment requires field surveys and spatial risk analysis. The author suggested relating hydrological information such as probability of storm occurrence, peak flow, flow duration, and flow velocity with detailed flooding cost to predict cost damage of flooding.

The market models Sto_MarketIC and Sto_MarketIC_Risk will thus relate the flooding cost to hydrological information and flooding components, and so creating a demand curve for IC. The main factors linked to flood damage are the peak flow f (closely related to the maximum depth flood), the time to reach a flood level h (related to flash flooding and changes in velocity), inundation period d (duration), and sediment load e (Smith 1994; Parker 1995; Ghosh 1997; Dutta et al. 2000; 2003; Middelmann-Fernandes 2010; de Moel and Aerts 2011). Factors such as availability of information, and external response to flooding (Kron 2007) is not accounted for in our analysis, nor sediment load (sediment load is covered in Pinto et al. (2012)). Figure 2-13 illustrates damage cost $\tilde{C}(f)$ in terms of inundation depth and inundation period (duration) at a given level of depth. A deeper inundation and a longer duration produce greater cost. In addition, the cost of damage changes with the place; however, most of the damage follows similar nonlinear trends. Herath (2003) noted that the Japanese Ministry of Construction obtained an exponential function of damage to crops for flood duration, and for damage to urban places, a fourth polynomial function with depth. Kron (2007) pointed out that square root stage damage, power, and polygon functions are used according to the contents, inventory, and property condition. Penning-Rowsell and Green (2000) noted that a single flood-depth-damage could be over and under estimated due to the property levels, which would require to work with an expected damage function.

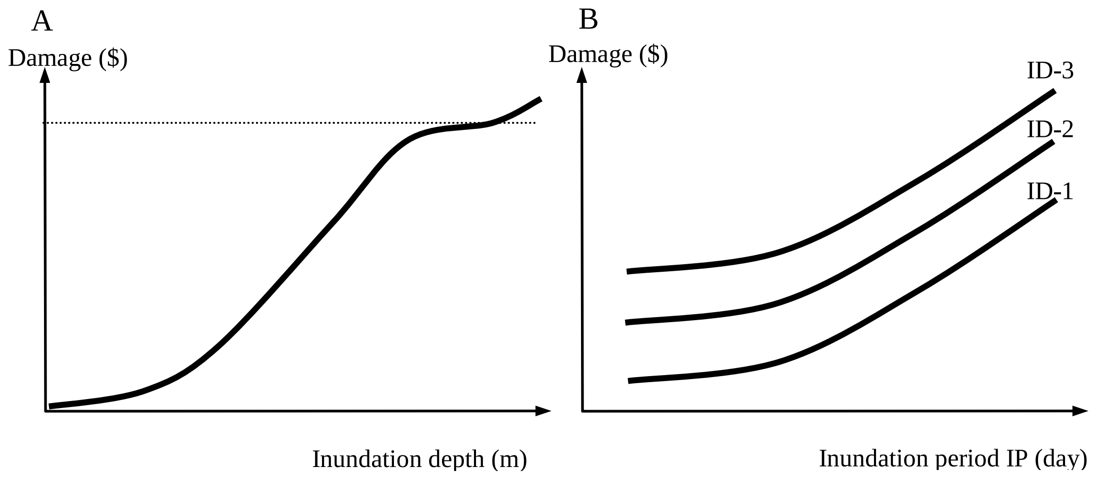


Figure 2-13 (A) Expected relationship between damage and inundation depth. (B) Relationship between inundation period and damage rate under different inundation depths, ID-1 < ID-2 < ID-3.

Figure 2-14 illustrates the effect of hastening peak flood or increase the rising limb (slope) of the hydrograph curve, which represents the fast increase of flow and flood. The stage-flood is reached previously for increasing IC allowances in the catchment and consequently the damage is increased from C(t*) to C(t**). Thus, a participant would pay the equivalent cost for increasing flows as $F_{t^{**}}'$ or would receive for reducing flows as $F_{t^{**}}''$.

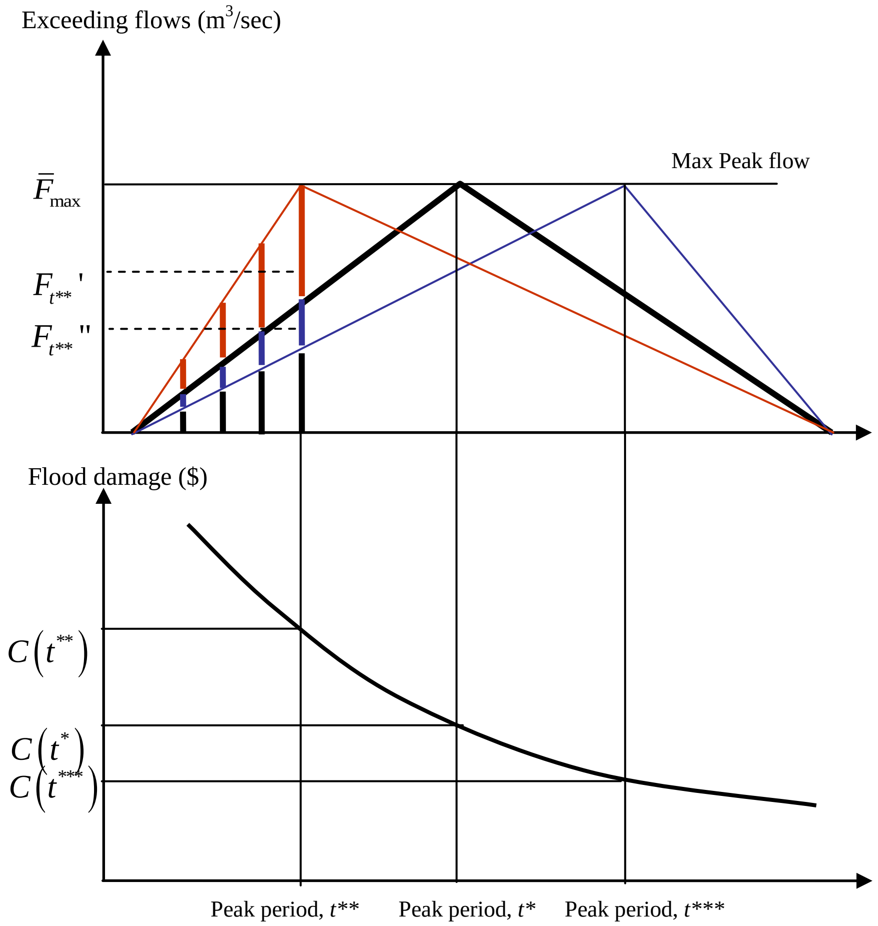


Figure 2-14 Changes in peak time and flood damage in a storm scenario.

Most flooding relationships, such as flow versus velocity and flow versus maximum depth, are non-linear and non-convex. However, electricity and gas market-clearing models use piecewise linear approximations to handle these issues (Hogan et al. 1996; Hogan 2002). In the Sto_MarketIC and Sto_MarketIC_Risk models, the damage cost function could be convexified for each control point using piecewise approximations. Alternatives for linearising and convexifying the flood damage function will be further discussed for those models. Convex curves guarantee an efficient solution (allocations and pricings) since a global maximum can be reached (Kall and Wallace 1994; Birge and Louveaux 1997; van der Vlerk 2002).

Flooding components

Flooding components such as peak flow depth *h* and hastening peak flood *t* (which account for raising the slope of the rising limb of the hydrograph curve), duration *d* and sediments *e* can be linked to flood patterns, which generate different levels of damage. For instance, a high peak flow in a short time may produce flood damage greater than the damage generated from the same peak, but later as illustrated in Figure 2-14. Similarly, lengthened duration could produce more damage than a shorter duration.

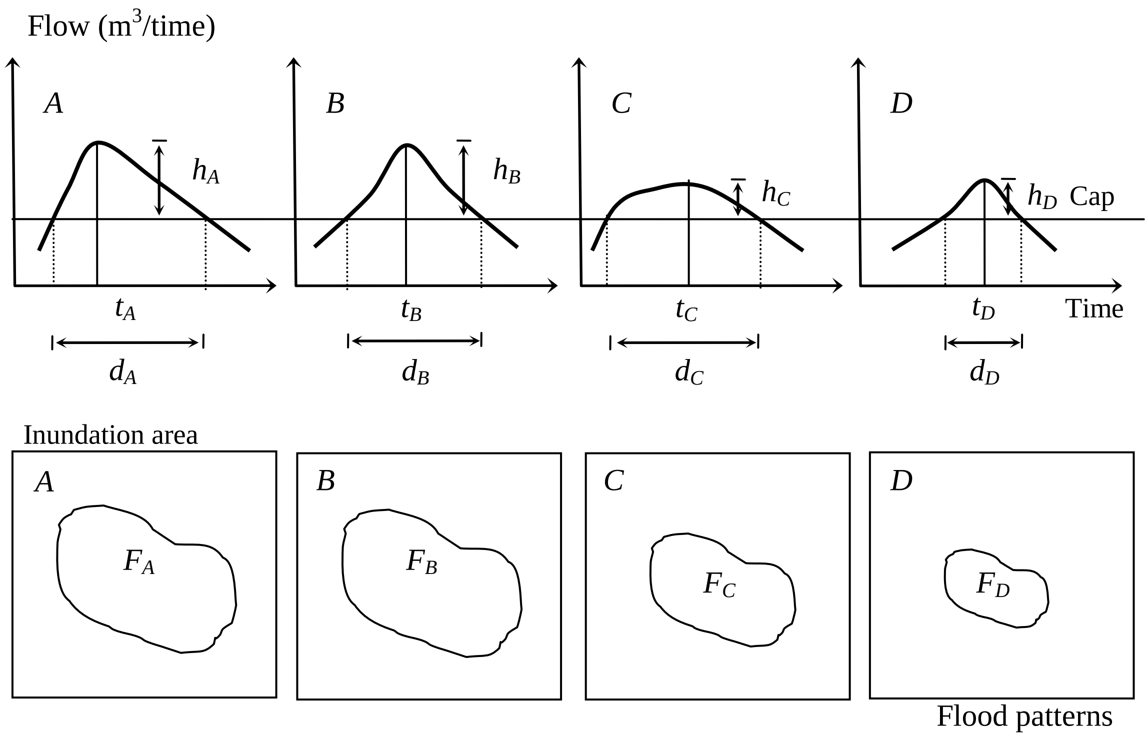


Figure 2-15 Flow hydrographs *A*, *B*, *C* and *D* at control point and distributional differences in floods; h_A, h_B, h_C, and h_D are peak flows at control point; F_A, F_B, F_C, and F_D are inundation areas; t_A, t_B, t_C, and t_D are the time peaks or peak flows; and d_A, d_B, d_C, and d_D are the lengths of the flow which contribute to the flooding.

Figure 2-15 shows four different flow hydrographs linked to hypothetical floods in the floodplain area. For instance, flows *A* and *B* could receive similar total inflows (runoff), reaching a similar peak flow (related to maximum flooding depth) $h_A \approx h_B$, duration $d_A \approx d_B$, and flooded areas $F_A \approx F_B$; however, the damage could be greater for *A* because the peak flow was reached sooner, $t_A < t_B$. A shorter time increases the damage due to the extra cost in terms of evacuation and warning (Krzysztofowicz and Davis 1983b).

Comparable analysis can be done with patterns *B* and *C* where even though they have similar total inflows, the flooding duration in *C* is longer ($d_C > d_B$) and the peak flows in *B* is greater than in *C* ($h_B > h_C$). In this case, a trade-off is expected between depth peak and duration. However, if the main damage component is related to flooding peak (maximum depth), the flooding pattern in *B* will generate more damage than *C*. Finally, pattern *C* may produce more damage than *D*, although peak flows and peak depth are similar $h_C \approx h_D$, because the flood duration in *C* is longer than in *D* ($d_C > d_D$). In brief, different flood patterns produce different flood damage.

As was previously pointed out, in modelling flood damage different functional cost relationships can be obtained. For example, in the flooding area there is a quadratic depth peak function (f), Equation [2.13], and an exponential damage function $\tilde{C}\left(f\right)$, equation [2.14], where x is the flows above boundary (channel capacity).

$$f = \alpha_0 \, x^{\alpha_1} \qquad [2.13]$$

$$\tilde{C}\left(f\right) = \beta_0 \, e^{\beta_1 \times f} \text{, and so } \tilde{C}\left(x\right) = \beta_0 \, e^{\beta_1 \left(\alpha_0 \times x^{\alpha_1}\right)} \qquad [2.14]$$

To ensure convexity in the cost damage function, if required, sufficient conditions are to have the parameters α_0, α_1, β_0 and β_1 > 0. Thus, $\frac{\partial \tilde{C}\left(f\right)}{\partial f} \geq 0$ and $\frac{\partial^2 \tilde{C}\left(f\right)}{\partial f^2} \geq 0$, and also $\frac{\partial \tilde{C}\left(f\left(x\right)\right)}{\partial f\left(x\right)} \frac{\partial f\left(x\right)}{\partial x} dx \geq 0$ and $\frac{\partial^2 \tilde{C}\left(f\left(x\right)\right)}{\partial f\left(x\right)^2} \frac{\partial^2 f\left(x\right)}{\partial x^2} dx \geq 0$. For instance, if cost and depth-flow parameters were β_0 = 0.5, β_1 = 0.8, α_0 = 0.32 and α_0 = 0.44, and those represent the maximum flood level that could be reached in the flooding area (or its equivalent peak flow), the cost damage function with flooding depth and total flooding flows would have trends such as those presented in Figure 2-16.

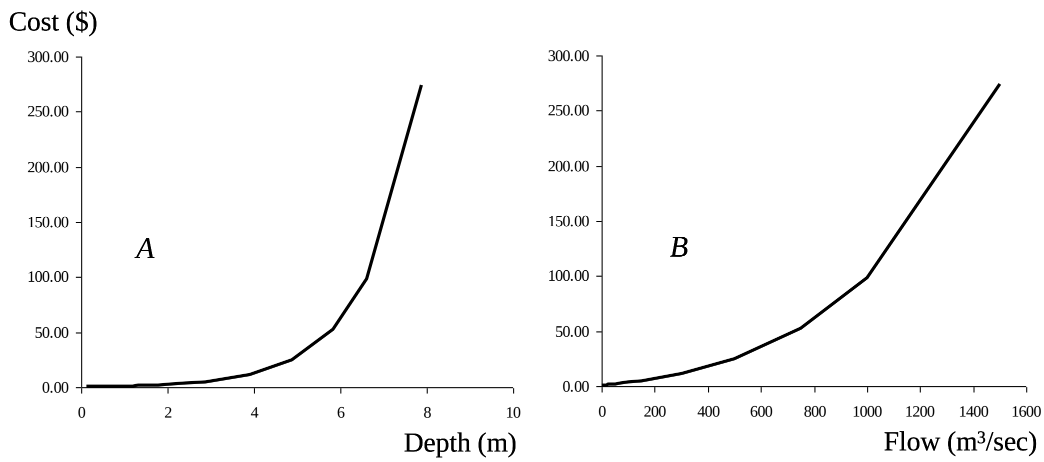


Figure 2-16 Cost related to maximum depth at control point. Panel A is the cost relationship with maximum depth and B is the cost relationship with peak flow.

For non-uniform channel shape, Figure 2-10 illustrates the relationship between damage and flow and Figure 2-17 illustrates flood damage estimates related to maximum stage-flood with an exponential damage function when reaching a flood depth at each level in the area (levels could correspond to levees). Despite the convex damage function at sections, the final flood damage function could be non-convex with respect to the peak flow over capacity. This is because of the non-uniform channel shape. In Chapter 4, the Sto_MarketIC clearing model will deal with a non-convex damage cost function, choosing scenarios and convexification of the flood damage function.

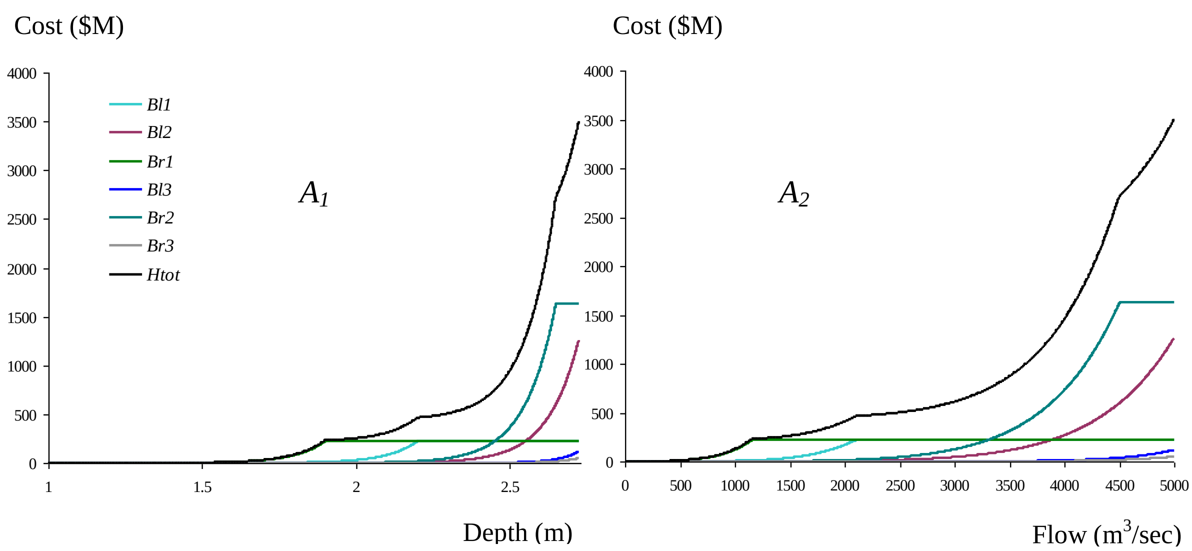


Figure 2-17 Flood damage with exponential damage at each section. A_1 and A_2 are flood damage relationships with depth levels and total flows respectively.

2.9 Flood damage function

In a specific place, rural or urban areas from source and non-source points, the flood damage could be represented as $\tilde{C}\left(x \middle| x_c^*\right)$ or $\tilde{C}\left(f, h, t, d, e \middle| x_c^*\right)$, *x* is the flows, *f* peak flow, depth *h*, hastening peak flood *t*, duration *d* and sediments *e*, and x_c^* is the flow boundary or capacities.

As previously stated, flows $x\left[IC, s\right]$ depend on the impervious cover (*IC*) and storm event $s \in S$, for a given infrastructure and topography; thus, a general representation of flood damage, in terms of the maximum peak flow *f*, is as follows:

$$\tilde{C}\left(f\left(x\left[IC, s\right]\right) \middle| x_c^*\right) \qquad [2.15]$$

A flood damage model similar to Equation [2.15] will be included in Sto_MarketIC in Chapter 4. This market model will also take into account the flood damage related to maximum flooding depth (maximum flow) along scenarios, which will be affected by IC trade.

Changes in IC allowances change runoff hydrographs, which may change peak flows and volumes at control points, resulting in changes to the flood distribution in the flooding areas (Figure 2-4 (D)). Figure 2-18 illustrates hypothetical changes of IC allowances, and the corresponding changes in flood damage from storm scenarios. The solid arrow represents the damage effects resulting from changes in IC (A). This damage function is non-linear; however, a piecewise linear approximation could be applied in the market clearing formulation.

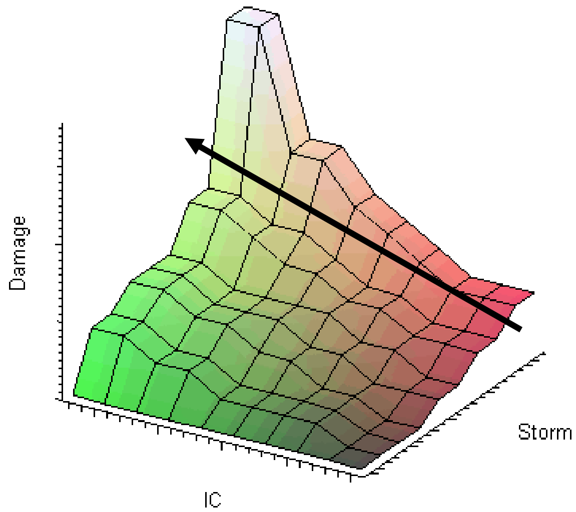


Figure 2-18 Final damage and changes in IC and storms at a hypothetical control point.

A discretised change in damage under different IC allowances by scenario can be represented as follows:

$$\tilde{C}\left(f\left(x[IC_i,s]\right)\middle|x_c^*\right)-\tilde{C}\left(f\left(x\left[IC_j,s\right]\right)\middle|x_c^*\right),\ \forall \text{ s} \in \text{storm scenario} \qquad [2.16]$$

Because the change in damage depends on the change in IC allowance from *i to j*, the marginal value varies in accordance with this level. Thus, Equation [2.16] will be greater or equal than zero when $i > j$ and lower or equal than zero when $i < j$. The last would imply that changes in damage, when increasing IC, would be greater than zero. This will be extended with a theoretical example to show that changes in IC allowances are convex in expectation.

The flood damage can be represented in each scenario *s* and control point *k* as $\tilde{C}_k\left(f,h,d,e\middle|x_c^*\right)$. The damage function can be expressed in terms of flows (or exceeding flows) x as $\tilde{C}_k\left(f(x),h(x),d(x),e(x)\middle|x_c^*\right)$, given the hydraulic conditions of the catchment. Then, the marginal changes in damage, at control point *k* and scenario *s*, for increasing maximum depth as $\frac{\partial\tilde{C}_k\left(f,h,d,e\middle|x_c^*\right)}{\partial f}$ can be calculated. Given that depth depends on exceeding flow, the resulting marginal change in damage is

$\frac{\partial \tilde{C}_k\left(f,h,d,e \middle| x_c^*\right)}{\partial f\left(x\right)} \frac{\partial f\left(x\right)}{\partial x} \frac{\partial h\left(x\right)}{\partial x} \frac{\partial d\left(x\right)}{\partial x} \frac{\partial e\left(x\right)}{\partial x}$. The change in flood damage can also be estimated as a function of with the flooding duration and hastening peak flood components. Finally, the total expected damage is $\int_s^S \tilde{C}_k\left(f_k\left(x\right), h_k\left(x\right), d_k\left(x\right), e_k\left(x\right) \middle| x_c^*\right) g\left(x\left(s\right)\right) ds$, where $g\left(x\left(s\right)\right)$ is the probability distribution of the rainfall event *s*.

2.10 Expected flood damage

The proposed market models Sto_MarketIC, Sto_MarketIC_Risk and Gross_MarketIC will be formulated as two-stage stochastic programming (TSSP) models. The recourse flood cost in the models includes a flood damage cost function, and changes in IC allowances result in changes in the expected flood damage. Kall and Wallace (1994), Birge and Louveaux (1997) pointed out that the theoretical foundation of stochastic programming is around properties of the expectation, and averaging has a convexifying effect, so the expectation is “usually” convex (Wets 1998). For the approximation, the recourse cost may have a convex hull. Our market design could use the same argument to model the expected flood damage as a convex function.

Despite a flood damage function obtained from the expectation, the flood damage could be non-convex, but monotonically increasing, $C\left(x, \xi\right)$. Figure 2-19 illustrates a flood damage condition that may be faced in the flood area. Figure 2-19A shows hypothetical flood damage conditions under different storm scenarios, and Figure 2-19B shows an approximation of the flood damage function, which assumes that the flow impact is normally distributed. An approximation to the flood damage function may be required if the changes in expectation are not convex. However, changes in IC allowances will usually increase flows at peak times and produce changes in expected flood damage. Because the flood damage function is monotonically increasing, the changes in the expected flood damage could be convex when flows are increased.

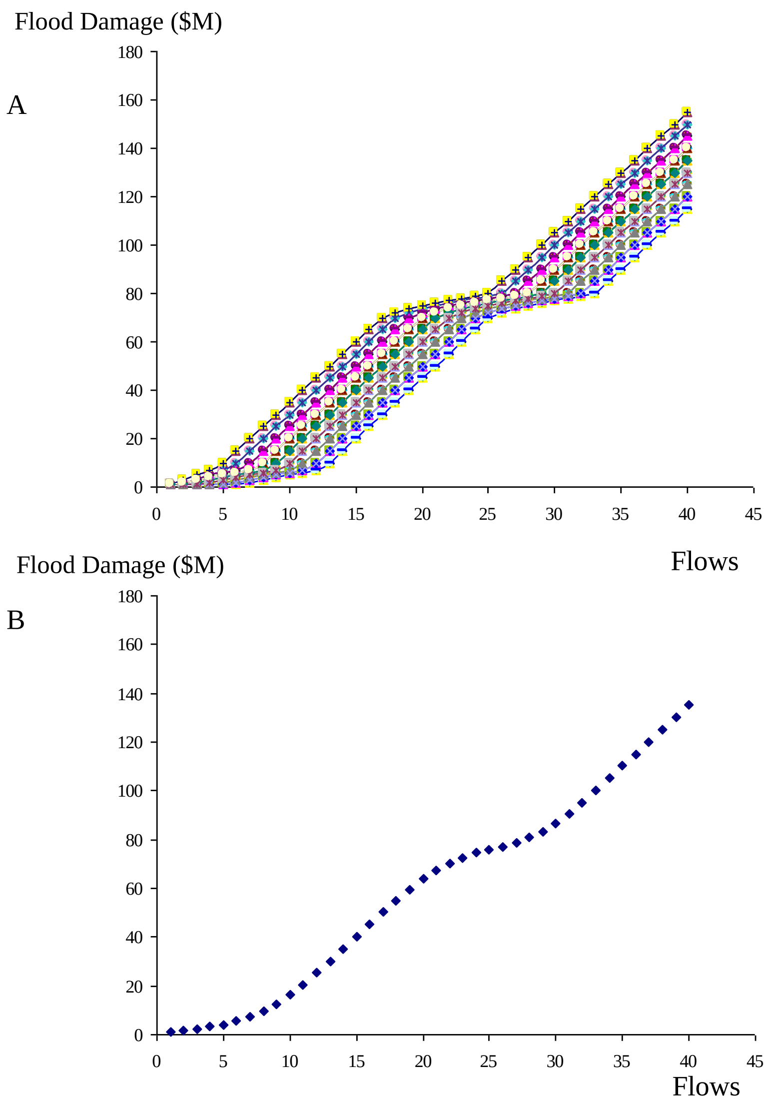


Figure 2-19 (A) Illustration of flood damage condition under different damage-storm events and (B) an approximation of the floods damage function, which assumes that the flow impact is normally distributed

The expected flood damage for maximum depth (peak flow), $E\left[\tilde{C}\middle|x\right]$, can be represented in a continuous or in a discrete formulation. Thus, if flood damage $\tilde{C}\left[f\left(IC,\cdot\right)\right]$ for changes in flows is monotonically increasing, $\frac{\partial\tilde{C}\left[f\left(IC,\cdot\right)\right]}{\partial f}\frac{\partial f\left(IC,\cdot\right)}{\partial IC}dIC \geq 0$, the changes in the expected flood damage with increasing IC allowances could be convex

$E\int_{s}^{S} \frac{\partial\left(\frac{\partial \tilde{C}\left[f\left(IC,\cdot\right)\right]}{f}\frac{\partial f\left(IC,\cdot\right)}{\partial IC}dIC\right)}{\partial IC}\phi^{s}ds \geq 0$. Thus, it would be reasonable to assume that changes in the expected flood damage for changes in flow would be convex.

The proposed market models use discrete storm scenarios and focus on such storms as the main source of uncertainty. The flood damage is represented by the flood cost function in the TSSP market model, which may be non-convex. A piecewise approximation is calculated for the damage function. Furthermore, the market can deal with a non-convex damage function (monotonic increment condition) as will be presented in Chapter 4 and Appendix B

Chapter 3

3 DETERMINISTIC MARKET FOR IC ALLOWANCES

3.1 Introduction

This Chapter presents a smart market design (Det_MarketIC) for IC allowances which is based on one extreme storm event. This market is cleared by a LP model which incorporates capped points at stream and channel locations as desired thresholds in rural and urban places, and impact coefficients from each participant.

The market is a net pool formulation. In the net pool formulation, participants specify demands and offers based on their initial positions which correspond to a status quo of current IC allowances in their property. Thus, participants are bidding from an initial IC allowance to a desired IC allowance.

Participants trade in IC allowances by land use, such as 1 ha of mixed forests and crops. An IC allowance is a tradable permit to use a specific IC allowance in a specific area for the allowance term ($/ha for a specific impervious cover). The area is the continuous decision variable. The IC allowance terms may be long or short. We assume IC allowances last for a short time such as 1 or 2 year, however, the optimum IC allowance period should be evaluated for the SO. Much of the analysis in this thesis can also apply for longer periods. Participants can bid for changes in advance, which allows them to plan their IC allowances throughout a year or longer. Those actions can be linked to contracts, which stipulate future physical actions on the land regarding IC allowance, or equivalent imperviousness at a specific time. Participants whose IC allowance may not match the current land use (based on a reference usage discussed in Chapter 8) as well as those who desire to develop new projects can also participate in the market. Thus, participants will trade IC allowances which correspond to specific land use (imperviousness) or BMPs and area (for instance, 1 hectare) which is part of a specific region (e.g., their farm) across time periods. The catchment moisture condition affects runoff; thus, the antecedent moisture condition (AMC) has to be assumed. The IC would be estimated based on a uniform moisture condition AMC II (SCS 1985; Haan et al. 1994).

The number of periods in a given auction, the length of each period, and the timing of the auction will be called the ‘auction schedule’. The storm period corresponds to elapsed time during a storm. The auction period corresponds to elapsed time between opportunities to trade land use. The IC allowance period corresponds to the length of time of the associated right. The thesis will focus on one auction period, which would depend on the hydrological seasons and the main economic activities in the catchment. For instance, in a catchment comprised mainly of farms, the market would run with regard to the agricultural seasons, i.e., two to four times per year; while in an urban area, the market may run monthly. Determining the best action time period is likely to be based on regular review by the authority. A full discussion of this appears later in this chapter.

Equivalent IC allowances can be estimated for runoff controlling technologies and/or BMPs. If the market were to choose which technology each participant should use, the model would have integer decision variables and would be non-convex, probably resulting in non-supporting prices (O'Neill et al. 2005). However, decisions about technologies and IC allowances are private, so we allow any bid to be fractionally met. Therefore, if a participant offering to control runoff using BMPs obtained only three quarters of the equivalent IC allowance, that would mean they had not offered a low enough price; however, this participant could sell the next quarter or sell back the unused allowance in the following auction period.

Outside the market, participants may apply different options to control their runoff, while satisfying the System Operator (SO) rules; thus, they would not worsen their runoff hydrograph. Where a participant did not meet their obligations, penalties would apply. These options could be expensive, which could encourage participating in the market. Thus, participants could trade in the market and buy (sell) impervious areas; however, participants could not buy (sell) more rights than their initial area, nor could they finish with an IC allowance different that their initial area.

A change of imperviousness and technology might increase or reduce the runoff from the property, raising or reducing penalties if a participant does not comply with obligations. To simplify calculations, the SO could develop a web site where participants could estimate their IC allowance for different land uses and options.

The SO will need to validate the impervious surface for each property and for each participant in the catchment. Different methods could be used for this purpose. For instance, the manager could use a satellite map to estimate the imperviousness of the area as was used by Dougherty et al. (2004). In any event, the manager needs to be aware of the specific land use to estimate the impact flow coefficients for each property. However, how the SO might go about this is outside the scope of this thesis.

The SO needs to specify the storm event upon which the market will be based. Different storms indicate the risks that the SO would address. For instance, 100 mm and 200 mm in 24 hours may be established in the dry and wet seasons respectively. Short IC allowance periods, i.e., the length of time of the associated right, would encourage participation. However, participants who desire to change their impacts permanently would need to acquire long allowance periods with impacts estimated across the whole allowance periods.

Section 3.2 discusses stepwise demand and supply. Section 3.3 discusses clearing the market. Section 3.4 presents a primal formulation of the market model, and analyses the market pricing and the trade conditions. Section 3.4.1 describes bid formation. Section 3.4.2 presents settlements for participants. Section 3.4.3 is price a analysis. Section 3.4.4 presents implications of trading. Section 3.4.5 describes locational prices. Section 3.4.6 discusses operator revenue. Section 3.4.7 discusses the frequency in the auction schedule. Section 3.4.8 analyses the planning horizon. Section 3.4.9 discusses selection of storms to establish the market. Section 3.5 presents an alternative market formulation, and includes price analysis and trade conditions. Section 3.6 presents settlements for participants. Section 3.7 compares two clearing model formulations. Section 3.8 presents a short discussion of competitiveness in the market for IC allowance. Final remarks and conclusions are presented in Section 3.9.

3.2 Demand and supply

Demand and supply describe the maximum willingness to buy or sell IC allowances. Demand and supply will be represented by tranches or steps, corresponding to prices associated with area of different impervious levels. Therefore, market bids are required to be defined in this form. By aggregating individual demands and supplies, total demand and supply can be characterised. Models with nonlinear demand and supply curves could be solved using nonlinear programming, avoiding the need for multistep supply and demand curves. If supply curves are not monotonically non-decreasing or demand curves re not monotonically non-increasing, however, nonlinear models may suffer from non-convexity and non-supporting prices. This would also be true with linear approximations that deal with the non-convexities issues using binary variables.

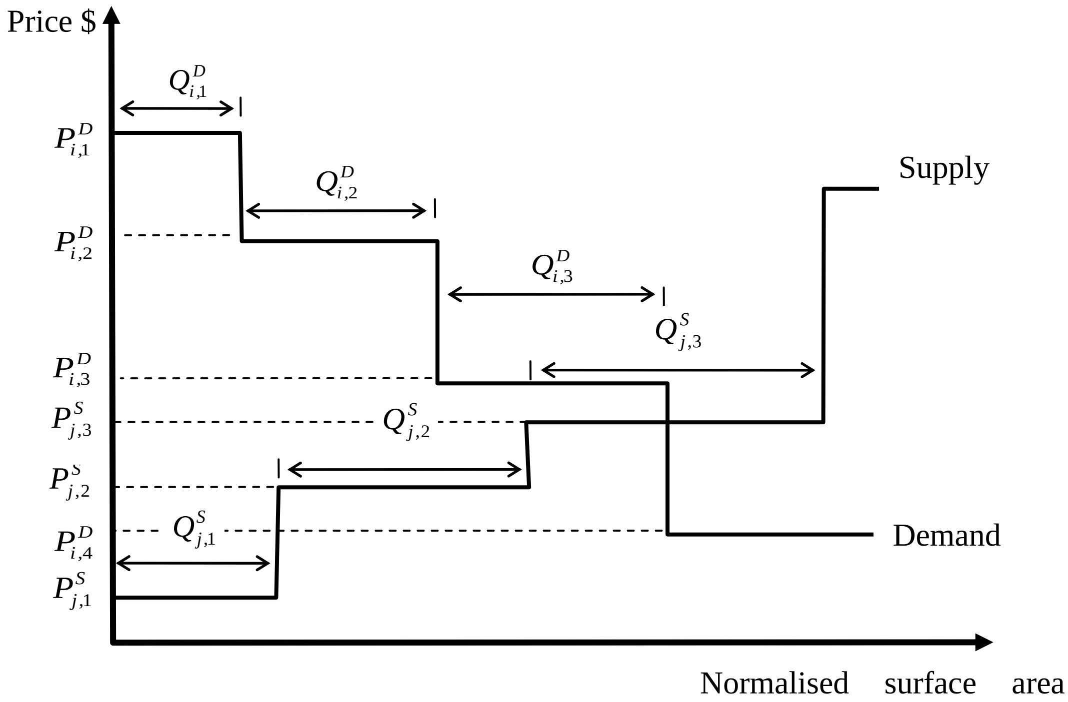


Figure 3-1 Piecewise linear approximation of demand and supply curve. $Q_{i,b}^D$ and $P_{i,b}^D$ represents the demanded area (ha) from participant *i* and price in tranche *b*; $Q_{j,b}^S$ and $P_{j,b}^S$ represents the supplied area (ha) from participant *j* and price in tranche *b*.

Figure 3-1 illustrates the stair-wise linear approximation of the demand and supply. Because properties located at different places in the catchment have different effects on the stage flow and stage flood (based on the linear coefficient that summarise the runoff and flow routing relationship), the demand and supply curves represent normalised areas (ha), which account for the flows at flooding areas and translate the prices and area into prices for an effective area. For instance, participant *i* desires to increase the impervious level to "crop" with CN 70 in the first 0.5 ha ($Q_{i,1}^D$) and bids at \$2/ha ($P_{i,1}^D$). This represents the first step or tranche *b* in the demand. Additionally, he/she demands the next 0.25 hectares ($Q_{i,2}^{D}$) to "crop" with CN 70 of area allowance at \$1.5/ha ($P_{i,2}^{D}$) in the second tranche, and so on.

3.3 Clearing the Det_MarketIC

Matching buy and sell bids requires a comparison between demand and supply, bid prices (buy and sell), and impacts (flows). The market objective accounts for the difference in the integrals of the demand *D*(*A*) and supply *S*(*A*) curves $\int_{Q}\left(D\left(A\left[IC\right]\right)-S\left(A\left[IC\right]\right)\right)$, where *A* is area (ha) with imperviousness levels related to IC allowances. These allowances and changes for trading are translated into aggregated flows by time *t* at different places *k* in the catchment, $Q_{t,k}\left(A\left[IC\right]\right)$. Thus, changes in imperviousness will change the status quo, $Q_{i,k}^{0}\left(A\left[IC\right]\right)$, of flows that could be noticed in the catchment. (These marginal changes can be represented by "*H*" (see Section 2.6 in Chapter 2 and Section 3.4)). The aggregated flows correspond to individual impacting flows (hydrographs) from participants located at different places in the catchment; these participants could notice different impacting flows related to their imperviousness level in their properties.

Nonlinear demand and supply curves could be solved using nonlinear programming, but possible problems with non-convexity and non-supporting prices could be faced. Participants could try to trade the same level of imperviousness with similar prices, but their bids may not match due to different impacting flows at control points. Figure 3-2 illustrates flows patterns with two levels of IC allowances (1 and 2) for different locations (*A* and *B*) and hence different flows from them. In the illustration, participant *A* wishes to reduce from level 1 to 2 and participant *B* wishes to increase from 2 to 1. Bid prices for reducing and increasing IC allowances would be the same; however, because the two participants have different flows by time, the trade could not be cleared between them. Participant *A* may reduce IC allowance, and participant *B* could probably not change IC allowances if flow capacities are bounded at their peak flow time 2.

Additionally, participants whose demand prices are too high or offer prices too low will not have their bids accepted. To clear the market, the SO must adjust the supply and demand according to the impacts, especially at points with flooding problems. Thus, prices will be associated with impacts at different places in the catchment.

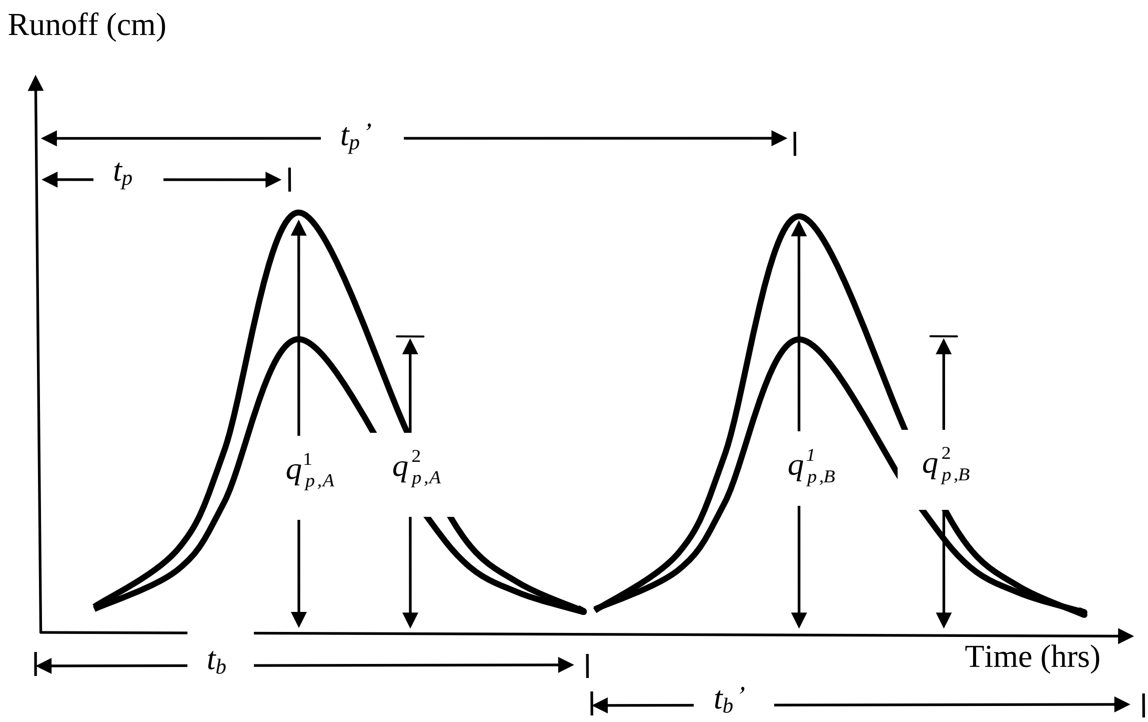


Figure 3-2 Flow patterns with locations (*A* and *B*) at control point. In the graph, $q^1_{p,A}$, $q^2_{p,A}$, $q^1_{p,B}$ and $q^2_{p,B}$ are the individual peak flows with $q^1_{p,A} = q^1_{p,B}$, and $q^2_{p,A} = q^2_{p,B}$; t_b and t_b' are the time peaks for location *A* and *B* respectively; and t_b and t_b' are the total time duration of flows for 1 and 2, $t_b = t_b$'.

To deal with the complexity, the SO uses a linear program that incorporates the hydraulic and hydrologic process in routed channels and streams. The linear program clears the market and estimates allocations and prices for each participant. The clearing prices depend on individual impacts at control points, and the nodal prices at control points, which are determined according to the supply and demand for changing IC allowances in the catchment.

In Det_MarketIC1, each participant sells their initial IC allowance and buys their desired final IC allowance condition. Thus, for a participant to buy 10 ha of concrete, they would simultaneously sell their initial 10 ha of grass. The total area of IC allowance purchased must equal the total area of IC allowance sold, even though the flows at control points are different.

As will be discussed in Section 3.4.2 and across this thesis, the settlement from each participant depends on the initial allocation or a specific reference usage [land use] $j = m$. Thus, participants will not face only the price for the final allocation,, $\sum_i^N \sum_j^J \sum_k^K \sum_t^T H_{i,j,k}^t \lambda_{t,k}$, the SO may not be revenue adequate in each trading auction. Actually, the settlement and the applied price for participant would account for their final and initial allowance $\sum_i^N \sum_j^J \sum_k^K \sum_t^T \left(H_{i,j,k}^t - H_{i,j=m,k}^t \right) \lambda_{t,k}$ and the SO is guaranteed to be revenue adequate or neutral only if the catchment is fully allocated. The SO´s rental will be analysed in Section 3.4.6, and the following section describes the market model.

3.4 A LP impacting/pricing model Det_MarketIC1

This section presents a mathematical model for a market associated with a single extreme storm event. The specified storm event establishes the impact coefficients at control points, with local linearisation of the channel flow capacities. The market is assumed to be perfectly competitive, i.e., participants do not exercise any market power. Additionally, the base flows[6] in channels and streams are kept relatively constant over time. This assumes that the SO defines thresholds at control points. Control points have no individual constraints other than capped threshold capacities. The thresholds are maximum flow levels in the channel or established maximum flood conditions related to the extreme storm. The model will be as follows:

Indices

i = Participant, 1,…,N.

j,m,n,p = Land use type (CN or imperviousness), j= 1,…,J.

b = Bid step, b=1,...,B.

k,l = Control point (node), k=1,...,K.

t,u,r = Storm time period, t=1,…,T.

Parameters

$A_{i,j}$ = Total area of IC allowance type j owned by participant i (ha).

[6] The flow of water entering stream channels from groundwater sources in the drainage (hydrology).

$D_{i,j,b}^{\max}$ = Maximum area in IC allowance type *j* (ha) that participant *i* in bid step *b* is willing to buy at price $P_{i,j,b}^{D}$.

$S_{i,j,b}^{\max}$ = Maximum area in IC allowance type *j* (ha) that participant *i* in bid step *b* is willing to sell at price $P_{i,j,b}^{S}$.

$P_{i,j,b}^{D}$ = Demand price ($/ha) for IC allowance type *j* from participant *i* and bid step *b*. This is the maximum that participant *i* is willing to pay for a specific IC allowance type *j* and bid step *b*.

$P_{i,j,b}^{S}$ = Bid price ($/ha) for IC allowance type *j* from participant *i* and bid step *b*. This is the minimum that participant *i* is willing to pay for a specific IC allowance type *j* and bid step *b*.

L_k = Maximum allowable flow (volume/time) at channel control point *k*.

$R_{i,j}$ = Total area with IC allowance type *j* from participant *i* that is not traded (ha).

$Q_{k,t}^{0}$ = Initial total flows at control point *k* and time *t*. These flows are related to the initial IC allowance and the chosen storm scenario.

$H_{i,j,k}^{t-u+1}$ = Marginal flow impact at control point *k* of IC allowance type *j* across time *t* from participant *i* at the end of time *t* –*u*+1. This corresponds to the hydrograph. *u* is the lag time between commencement of the storm and when the flow reaches the control point (volume/time ha). This coefficient relates conditions of the participant's property to the marginal flow impact at control points given the storm scenario long the time. This linear coefficient is likely to depend on the initial IC allowance conditions in the catchment, so it should be updated as IC allowances changes to improve accuracy. If participant *i* does not impact control point *k*, then $H_{i,j,k}^{t}$ = 0.

Decision variables

$qsell_{i,j,b}$ = Area in hectares of IC allowance type *j* and bid steps *b* sold by participant *i*.

$qbuy_{i,j,b}$ = Area in hectares of IC allowance type *j* and bid steps *b* bought by participant *i*.

$g_{i,j}$ = Total area of IC allowance type *j* for participant *i* (ha).

Dual variables

$\mu_{i,j}$ = Price for participant *i* and land of IC allowance type *j* ($/ha).

$\lambda_{t,k}$ = Clearing price or nodal price. Price to impact at control point *k*, time *t* ($/volume/time).

Model: **Det_MarketIC1**

$$\text{Maximize } \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{D}_{i,j,b}\ qbuy_{i,j,b} - \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{S}_{i,j,b}\ qsell_{i,j,b} \qquad [3.1]$$

Subject to

$$0 \le qbuy_{i,j,b} \le D^{\max}_{i,j,b},\ \forall\ i,j,b \qquad : \beta^{-}_{i,j,b}, \beta^{+}_{i,j,b} \qquad [3.2]$$

$$0 \le qsell_{i,j,b} \le S^{\max}_{i,j,b},\ \forall\ i,j,b \qquad : \gamma^{-}_{i,j,b}, \gamma^{+}_{i,j,b} \qquad [3.3]$$

$$\sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{b=1}^{B} qsell_{i,j,b} + A_{i,j} = g_{i,j},\ \forall\ i,j \qquad : \mu_{i,j}\ \text{(free)} \qquad [3.4]$$

$$\sum_{j=1}^{J}\sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{j=1}^{J}\sum_{b=1}^{B} qsell_{i,j,b} = 0,\ \forall\ i \qquad : v_i\ \text{(free).} \qquad [3.5]$$

$$Q^{0}_{k,t} + \sum_{i=1}^{N}\sum_{j=1}^{J} H^{t-u+1}_{i,j,k}\ g_{i,j} + \sum_{i=1}^{N}\sum_{j=1}^{J} H^{t-u+1}_{i,j,k}\ R_{i,j} \le L_k, \forall\ t, k : \lambda_{t,k} \qquad [3.6]$$

$g_{i,j}$ free. [3.7]

Explanation:

[3.1] The objective function maximizes the gains for trading IC allowance. Changes in the objective are the appropriate measure of changes in welfare (assuming the market is competitive enough that offer/bids reflect marginal costs). It is recognised that participants may attempt to bid strategically; but such an issue is beyond the scope of this thesis.

[3.2] Total IC allowance bought in each tranche is bounded by demand quantities.

[3.3] Total IC allowance sold in each tranche is bounded by bid quantities.

[3.4] The final area of IC allowance type *j* of participant *i* equals the total cleared (dual price $\mu_{i,j}$). Thus, if participant *i* desires to change to *j* and their initial condition is *m* this constraint will be $qbuy_{i,j} = g_{i,j}$ and $-qsell_{i,m} + A_{i,m} = g_{i,m}$.

[3.5] The total IC allowance sold for each participant must equal the total IC allowance bought. This constraint ensures that all participants keep IC allowance for their whole land area. That is, the number of hectares owned by participant *i* is constant. For instance, if a participant sold 0.25 hectares with 40% IC allowance, she/he should buy 0.25 hectares with other IC allowance.

[3.6] The total flow at control point *k* in time *t* must be less than the target flow in the channel. This is only one of the many representations that could be employed in this constraint. The left side of the constraint assumes a reference point related to the actual existing IC allowances (land uses). Thus, if participants are not changing IC allowances, the initial flows at control point *k* and time *t* is:

$$\sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} A_{i,j} + \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} \ R_{i,j}$$

This constraint also accounts for net changes in impacting flows from participant *i*. The net change in flow is a consequence of the changes between the impact from the participant's final IC allowance of type *j*, $\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} g_{i,j}$, and the impact from their initial IC allowance type *m*, $\sum_{j=1}^{J} H_{i,m,k}^{t-u+1} g_{i,m}$. For instance, if participant *i* is not changing IC allowance, then their impacting flow is $\sum_{j=1}^{J} H_{i,j=m,k}^{t-u+1} A_{i,j=m}$. If participant *i* is changing part of the IC allowance, then their impact will be $\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} \, g_{i,j}$ for the new IC allowance type *j*, plus $\sum_{j=1}^{J} H_{i,m,k}^{t-u+1} \left(A_{i,m} - g_{i,j} \right)$ for the IC allowance that remains with type *m*. Thus, the net change in total flows from participant *i* is $\sum_{j=1}^{J} H_{i,j=m,k}^{t-u+1} A_{i,j=m} - \left(\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} g_{i,j} + \sum_{j=1}^{J} H_{i,m,k}^{t-u+1} \left(A_{i,m} - g_{i,j} \right) \right)$. Equivalent notation about this condition was proposed by Raffensperger et al. (2009), Raffensperger and Cochrane (2010), Prabodanie et al. (2010) and Pinto et al. (2012). The positive or negative net changes will be discussed next in the dual price analysis and trade implications.

[3.7] Buying and selling quantities of IC allowances are non-negative, and these non-negativity constraints will limit the final allocation of IC allowance, $g_{i,j}$. So imposing additional limits on $g_{i,j}$ will over-constrain the problem, creating a degeneracy which could compromise prices.

The lower bounds from equations [3.2] and [3.3] have negative dual variables, but such duals are represented in a canonical way, with duals with positive signs, i.e., $-qbuy_{i,j,b} \leq 0$ and $-qsell_{i,j,b} \leq 0$, and so $-\beta_{i,j,b}{}^{(-)} \geq 0$ and $-\gamma_{i,j,b}{}^{(-)} \geq 0$ respectively. Positive $\beta^{-}_{i,j,b}$ and $\gamma^{-}_{i,j,b}$ will be conveyed.

3.4.1 Bid formation

The model formulation is based on bids from participants wanting to make changes to their IC allowances. Thus, participants trade from their initial IC allowances to the final, and for doing that they need to bid simultaneously for both buying and selling conditions. This condition of buying and selling forces the participant to face a price difference from their initial IC allowance. The change in IC allowance considers pairs of these clearing prices $\mu_{i,j} - \mu_{i,m}$, representing a change of 1 unit from type *m* to type *j*.

Participants located at similar places that are changing similar IC allowances could present different bids, such as from $1,000/ha to $1,050/ha, and another participant from $0/ha to $50/ha. The market model accounts only for the differences, and not the absolute bids.

To simplify bids, the SO could require that bid prices for the initial condition be zero, so participants would bid only for their increment or reduction of IC allowance from the initial allowance. Participants that want to increase allowances should bid a high positive price and those who desire to reduce negative prices. This price implication will be discussed in following sections and in Chapter 8.

3.4.2 Settlement with market model Det_MarketIC1

The model will allocate efficiently with prices $\mu_{i,j}$ for each participant. The settlement, r_i, for participant *i* for the changes in IC allowances is $r_i = \sum_{j=1}^{J} \mu_{i,j} \left(\sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{b=1}^{B} qsell_{i,j,b} \right)$. The settlement considers that participants have rights for the initial IC allowances. Participant i is a net payer if $r_i > 0$, and a net receiver if $r_i < 0$.

Because, the participant i has an initial IC allowance allocation $j=m$, which can be decomposed in terms of impacting flows $H_{i,m,k}^{t-e+1}$, by time t and control points k, and after clearing the market, could obtain the IC allowance $j=p$ with their impacting flows $H_{i,p,k}^{t-u+1}$, the settlement becomes as follows:

$$\sum_{m,p\in j}^{J}\sum_{k=1}^{K}\sum_{t=u,e}^{T}\left(H_{i,p,k}^{t-u+1}g_{i,p}-H_{i,m,k}^{t-e+1}g_{\,i,m}\right)\lambda_{t,k} \qquad [3.8]$$

3.4.3 Price analysis

The dual variable $\mu_{i,j}$ in constraint [3.4] is the marginal value to the market for another unit of IC allowance type j for participant i. This value corresponds to the cost to the rest of the system, when the SO allows an additional unit of IC allowance type j to participant i. As would be expected, the LP dual shows that prices are determined by the flow impacts at control points, where limits are binding.

The IC allowances for participant i are valued at price $\mu_{i,j}$ and consequently the price can be re-expressed as an equivalent set of “constraint rights”, each one with a particular shadow price, $\lambda_{t,k}$, where individual flows are impacting at time t in control point k. However, the price used to charge each participant depends on the price differences $\mu_{i,j}-\mu_{i,m}$ as will be analysed in the following section.

The dual price $\lambda_{t,k}$ in constraint [3.6] represents the improvement in economic surplus if the SO allowed another unit of capacity in the channel at control point k. This shadow price also represents the congestion or capacity price, as in the electricity and the gas markets (Hogan 2002; Nesbitt and Scotcher 2009).

Because there are no other constraints, the dual can be represented as $\mu_{i,j}=\sum_{t=u}^{T}\sum_{k=1}^{K}H_{i,j,k}^{t-u+1}\ \lambda_{t,k}$. Although this price considers only a simple catchment market, the price could be decomposed across sub-catchments. In this case, the price for each participant will be $\mu_{i,j}=\sum_{k=1}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t,-u+1}\lambda_{t,k}+\sum_{k=z}^{Z}\sum_{t=u}^{T}H_{i,j,k=z}^{t-u+1}\eta_{t,k}$, which represents the value for impacting the catchment plus the value for impacting the common control points by time[7] (see the full dual formulation Det_MarketIC1.2 in Appendix A).

In general, this price $\mu_{i,j}$ will not match the demand and supply price offered by participant *i*, on any step of their discharge demand and supply curve. (Bids are supra or infra "marginal" bid steps, discussed later in the chapter.)

The dual variable $\beta^{+}_{i,j,b}$ in constraint [3.2] represents the marginal consumer surplus to participant *i* who is demanding a new IC allowance type *j*. This dual can be decomposed as follows (see Appendix A for the full dual formulation in a canonical representation):

$$\mu_{i,j} = P^{D}_{i,j,b} - \beta^{+}_{i,j,b} + \beta^{-}_{i,j,b} - \nu_i \qquad [3.9]$$

If $qbuy_{i,j,b} > 0$, then by complementary slackness, $\beta^{-}_{i,j,b}$ = 0, so the dual price $\mu_{i,j}$ becomes $\mu_{i,j} = P^{D}_{i,j,b} - \beta^{+}_{i,j,b} - \nu_i$. This is the marginal value to participant *i* from another unit of land with IC allowance type *j*, given the opportunity cost of the land area ν_i. The dual price ν_i corresponds to the value of the area balance of IC allowance for participant *i*, or the opportunity cost for their marginal land use. The value $P^{D}_{i,j,b} - \mu_{i,j} - \nu_i$ is the marginal surplus that participant *i* would have from an additional unit, minus the opportunity cost for a land area balance. Following trading, the participant's land must retain IC allowances.

The dual variable $\gamma^{+}_{i,j,b}$ in constraint [3.3] represents the competitive rent from participant *i* who offers a specific IC allowance. The dual can be decomposed as follows:

$$\mu_{i,j} = P^{S}_{i,j,b} + \gamma^{+}_{i,j,b} - \gamma^{-}_{i,j,b} - \nu_i \qquad [3.10]$$

If $qsell_{i,j,b} > 0$, then by complementary slackness, $\gamma^{-}_{i,j,b}$ = 0, and $\mu_{i,j} = P^{S}_{i,j,b} + \gamma^{+}_{i,j,b} - \nu_i$. The equation corresponds to the marginal offer surplus or the marginal competitive rent for those who are reducing IC using technologies to control runoff. This equation matches the value of another unit of IC allowance (or an equivalent IC for using BMPs or

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[7] $\sum_{i=1}^{N}\sum_{j=1}^{J} H^{t-u+1}_{i,j,k=z}\, g_{i,j} \le \bar{L}_k \;:\; \eta_{t,z}$, where $H^{t-u+1}_{i,j,k=z}$ is the impact coefficient at the common control point *z* (which may be the outlet of the catchment), and $\bar{L}_k$ is the maximum allowable flow at the common control point *z* in time *t*.

technologies), in terms of extra marginal competitive rent $\mu_{i,j} - P^S_{i,j,b}$ for participant *i*, plus the price ν_i that participant *i* will face for trading their IC, given he/she must keep the same area. Figure 3-3 illustrates demand and supply curves for two IC allowances, when a participant trades both areas, while retaining the same land balance.

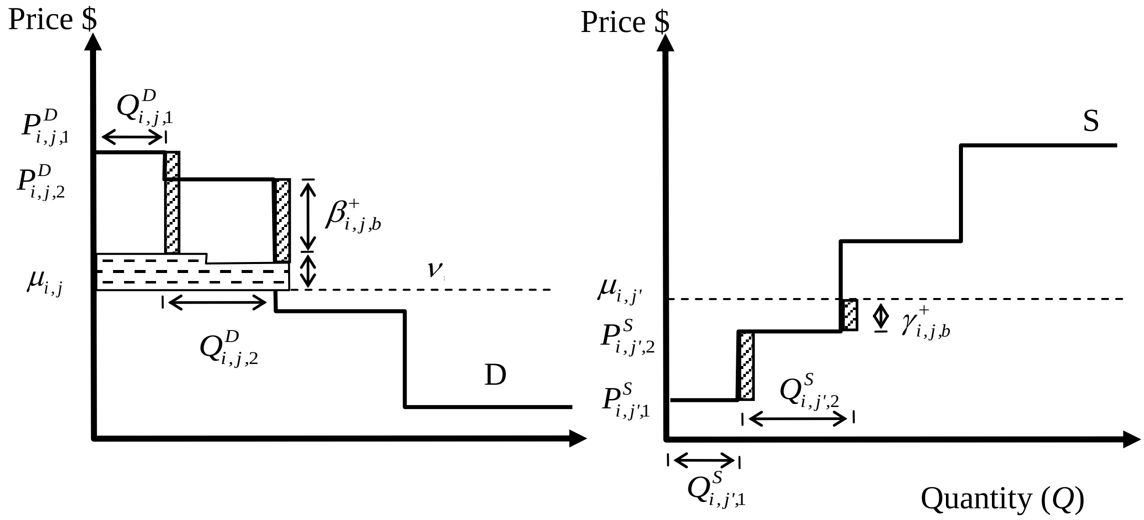

Figure 3-3 Clearing and dual variable values in a trade condition for two impervious levels *j* and *j'*. D and S represent demand and supply. Q^D and Q^S correspond to the demand and the supply area respectively.

The sign of ν_i dependents on whether the SO allows a unit unbalance in the area of IC allowances, although the final balance from constraint [3.5] would be violated. With the [3.5] condition, the SO would ensure that all participants retained the same area, and landholders without IC allowances would not inject runoff into the system (as free riders).

Det_MarketIC1.3 in Appendix A develops alternative primal and dual formulations to analyse prices. The model Det_MarketIC1.3 decomposes ν_i into ν_i^+ and ν_i^- prices, which represent the extra values if the SO allowed participant *i* imbalance in one unit of IC allowance due to extra sale and purchase respectively. Interestingly, the decomposed values depend on the level of allocation in the catchment. This area condition affects the marginal competitive rent and consumer surplus from participants:

$$\mu_{i,j} = P^D_{i,j,b} - \beta^+_{i,j,b} + (\nu_i^+ - \nu_i^-) + \beta^-_{i,j,b} \qquad [3.11]$$

$$\mu_{i,j} = P^S_{i,j,b} + \gamma^+_{i,j,b} + (\nu_i^+ - \nu_i^-) - \gamma^-_{i,j,b} \qquad [3.12]$$

If the catchment were fully allocated, i.e., constraint [3.6] were binding before trading, expected prices would be $\nu_i^+ = 0$ and $\nu_i^- \geq 0$. These dual prices correspond to the cost of keeping participant *i* with the same land area, even though the additional IC allowance may increase the total economic surplus.

On the other hand, in an over-allocated catchment (constraint [3.6] was violated before trading,), market feasibility and effects on individual participants is not at all clear. There would be participants with $\nu_i^+ \geq 0$ and $\nu_i^- = 0$, and others with $\nu_i^+ = 0$ and $\nu_i^- \geq 0$. However, a dominant $\nu_i^+ \geq 0$ and $\nu_i^- = 0$ would be expected for which the total trading surplus would increase.

3.4.4 Trading implication

The model selects the differences in consumer and supply trading surplus by a merit order, because participants are trading their initial and desired final IC allowances. However, those bidding higher do not necessarily buy and those bidding lower do not necessarily sell. The model takes into account the differences in demand and supply bids, adjusted by their impacts at control points, and the land balance. Equations [3.11] and [3.12] show the trade condition, which can be arranged as follows.

$$+\ \mu_{i,j} - \mu_{i,m} + \beta^+_{i,j,b} - \beta^-_{i,j,b} + \gamma^+_{i,m,b} - \gamma^-_{i,m,b} = P^D_{i,j,b} - P^S_{i,m,b} \qquad [3.13]$$

Then, rearranging Equation [3.13]:

$$\beta^+_{i,j,b} + \gamma^+_{i,m,b} = P^D_{i,j,b} - P^S_{i,m,b} - \sum_{k=1}^{K} \sum_{t=u}^{T} \left(H^{t-u+1}_{i,j,k} - H^{t-e+1}_{i,m,k} \right) \lambda_{t,k} + \beta^-_{i,j,b} + \gamma^-_{i,m,b} \qquad [3.14]$$

If $0 < qbuy_{i,j,b} < D^{\max}_{i,j,b}$, so $0 < qsell_{i,j,b} < S^{\max}_{i,j,b}$, this makes its variable "basic" so it sets the price, then the optimal change in IC allowances for participant *i* lies in the middle of step *b*, and this makes *i* “marginal”. This means that neither the upper nor lower bound of that demand and supply curve step will be binding. So, by complementary slackness, $\beta^+_{i,j,b} = \beta^-_{i,j,b} = \gamma^+_{i,m,b} = \gamma^-_{i,m,b} = 0$. Consequently, equation [3.13] implies $\mu_{i,j} - \mu_{i,m} = P^D_{i,j,b} - P^S_{i,m,b}$, for that demand and supply curve step.

$\beta^{+}_{i,j,b}\ +\ \gamma^{+}_{i,m,b}$ represents the total surplus to participant *i* for changing the impervious level from *m* to *j*; the price differential $P^{D}_{i,j,b}\ -\ P^{S}_{i,m,b}$ is the opportunity cost to pay (receive) due to changing the impervious level or using technology; and $-\sum_{k=1}^{K}\sum_{t=u}^{T}\left(H^{t-u+1}_{i,j,k}-H^{t-e+1}_{i,m,k}\right)\lambda_{t,k}$ are the clearing prices at channel points adjusted by individual impact coefficients, which are all positive in the Det_MarketIC1 formulation. A participant may change the IC allowance if the opportunity cost is at least covered by the net clearing value of trade, given the land impervious balance. For instance, if a participant wants to change IC allowances, the model accounts for the net differences in bids, irrespective of the absolute value of either bid. Clearing prices do not change if relative differences do not change. This may look peculiar, if we tried to compare bids between participants, and would make it difficult to monitor abuse of market power. But it could still be mathematically and conceptually correct.

Participants that desire to reduce the level of imperviousness, $n > p$, would have a surplus for trading as in Equation [3.14]. For participants that reduce IC allowance, the model takes into account only the net bid price $\left(P^{D}_{i,p,b}-P^{S}_{i,n,b}\right)$; thus, the total marginal surplus, $\pi_{i,(n,p),b}$, for changes in IC allowances from *n* to *p* becomes:

$$\pi_{i,(n,p),b}=\beta^{+}_{i,p,b}+\gamma^{+}_{i,n,b}=P^{D}_{i,p,b}-P^{S}_{i,n,b}-\sum_{k=1}^{K}\sum_{t=u}^{T}\left(H^{t-u+1}_{i,p,k}-H^{t-e+1}_{i,n,k}\right)\lambda_{t,k}+\beta^{-}_{i,p,b}+\gamma^{-}_{i,n,b} \qquad [3.15]$$

For participants that desire to increase impervious level, $m < p$, the total surplus will be

$$\pi_{i,(m,p),b}=\beta^{+}_{i,p,b}+\gamma^{+}_{i,n,b}=P^{D}_{i,p,b}-P^{S}_{i,m,b}-\sum_{k=1}^{K}\sum_{t=u}^{T}\left(H^{t-u+1}_{i,p,k}-H^{t-e+1}_{i,m,k}\right)\lambda_{t,k}+\beta^{-}_{i,p,b}+\gamma^{-}_{i,m,b} \qquad [3.16]$$

Another implication of the bids is related to the changes in IC allowance and the changes in impacting flows. Thus, lands close to the control point should pay almost nothing if runoffs are quick, and so impacting flow avoids the peak. In this case, bids should be carefully established, because increasing IC allowance would not necessarily imply that they should pay more. In fact, they should actually be paid for covering their land to get quick runoff. Land far from the control point should pay nothing, because its runoff is so slow that it avoids the peak flow at the control point, and they should probably be paid for delaying flows further. However, in most cases, increments in IC allowances increase the runoff-volume and also flows at control points. Appendix B shows an example of changes in IC and flows.

3.4.5 Locational-nodal prices

After clearing the market, participants *i* and *l* with similar impacts in the subcatchment, will most face similar prices when they change IC allowances from *m* to *j*, $\left(\mu_{i,j}-\mu_{i,m}\right)\approx\left(\mu_{l,j}-\mu_{l,m}\right)$ or $\sum_{k=1}^{K}\sum_{t=u}^{T}\left(H_{i,j,k}^{t-u+1}-H_{i,m,k}^{t-e+1}\right)\lambda_{t,k}\approx\sum_{k=1}^{K}\sum_{t=u}^{T}\left(H_{l,j,k}^{t-u+1}-H_{l,m,k}^{t-e+1}\right)\lambda_{t,k}$. The locational and spatial effects correspond to the accumulated impact at different control points (clearing prices $\lambda_{t,k}$ at control point *k* and time *t*). Figure 3-4 illustrates subcatchments with similar clearing prices. For instance, participants located in A with similar impacts $\sum_{t=u}^{T}H_{i,j,k}^{t-u+1}\approx\sum_{t=u}^{T}H_{l,j,k}^{t-u+1}$ would face the price for IC allowance type *j* $\sum_{t=u}^{T}H_{i,j,A_1}^{t-u+1}\ \lambda_{t,A_1}+\sum_{t=u}^{T}H_{i,j,B_1}^{t-u+1}\ \lambda_{t,B_1}+\sum_{t=u}^{T}H_{i,j,C_2}^{t-u+1}\ \lambda_{t,C_2}+\sum_{t=u}^{T}H_{i,j,F}^{t-u+1}\ \lambda_{t,F}$. Alternatively, participants with small properties in the same subcatchment D_1, but with different impervious levels, will probably face similar prices, because $\sum_{t=u}^{T}H_{i,j,k}^{t-u+1}\approx\sum_{t=u}^{T}H_{l,j,k}^{t-u+1}$. This is expected because small differences in imperviousness do not significantly change the impacting flows $\sum_{t=u}^{T}H_{i,j,k}^{t-u+1}$ at control point *k*.

The locational prices mean that participants with similar imperviousness in different subcatchments would face different prices. For instance, participant *i* in subcatchment D with IC allowance type *j* would face a price $\sum_{t=u}^{T}H_{i,j,D_1}^{t-u+1}\ \lambda_{t,D_1}+\sum_{t=u}^{T}H_{i,j,F}^{t-u+1}\ \lambda_{t,F}$, which could be different than the price for the same level of imperviousness *j* in subcatchment A, $\sum_{t=u}^{T}H_{i,j,A_1}^{t-u+1}\ \lambda_{t,A_1}+\sum_{t=u}^{T}H_{i,j,B_1}^{t-u+1}\ \lambda_{t,B_1}+\sum_{t=u}^{T}H_{i,j,C_2}^{t-u+1}\ \lambda_{t,C_2}+\sum_{t=u}^{T}H_{i,j,F}^{t-u+1}\ \lambda_{t,F}$. This observation has also been studied and reported in the electricity and gas literature on locational and spatial spot prices (Bohn et al. 1984; Hogan et al. 1996; Chattopadhyay 2004; Midthun et al. 2009; Nesbitt and Scotcher 2009; Pepper et al. 2012).

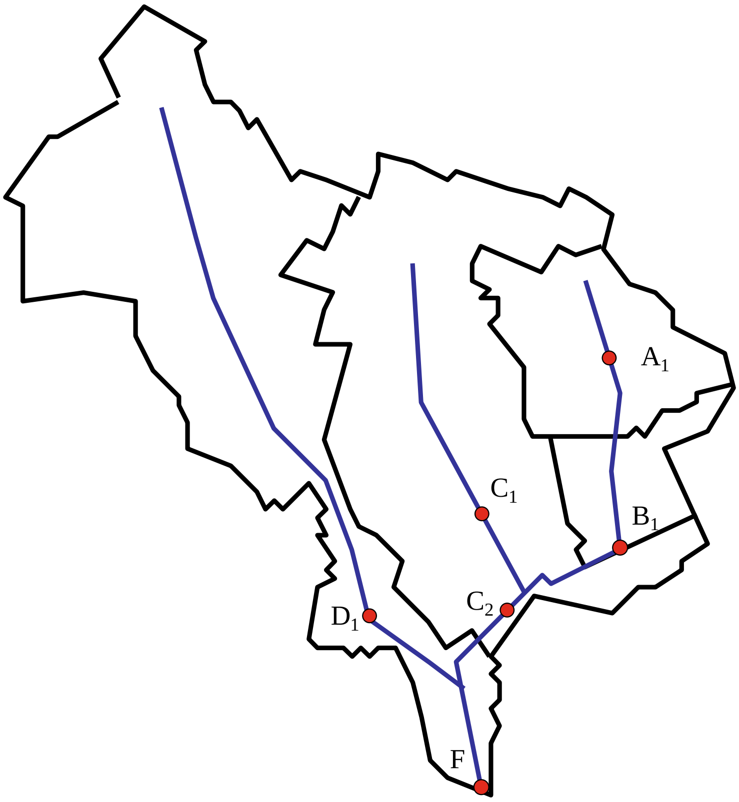

Figure 3-4 Example catchment with sub-catchments A, B, C, and D. A_1, B_1, C_1, C_2, D_1, and F are control points where flooding causes problems. B_1, C_2 and F are common control points between subcatchments.

3.4.6 Operator revenue: Initial rights

Initial IC allowances affect revenue distribution for the SO. After clearing the market, the SO will receive a net payment $NP = \sum_{i=1}^{N} \sum_{j=1}^{J} \mu_{i,j} \left(\sum_{b=1}^{B} qsell_{i,j,b} - \sum_{b=1}^{B} qbuy_{i,j,b} \right)$, which corresponds to a rental of individual valued impacts from participants over time and control points, $\sum_{i=1}^{n} \sum_{m,p \in j}^{J} \sum_{k=1}^{K} \sum_{t=u,e}^{T} \left(H_{i,p,k}^{t-u+1} g_{i,p} - H_{i,m,k}^{t-e+1} g_{i,m} \right) \lambda_{t,k}$.

The auction may not be revenue neutral. In a deterministic formulation, Prabodanie (2010) discusses the initial distribution of loading permits for a nitrate market, noticing that a SO may achieve feasibility with non-negative revenue and infeasible initial conditions with negative revenue. The infeasibility condition is related to the revenue, but an infeasibility solution could also be achieved if total reducing impacts are above the defined thresholds. Additionally, the author above pointed out that the surplus is related to payments among participants derived from receptor capacity. A similar issue will be faced in a market for IC, where initial allowances could produce similar consequences, in revenue and feasibility, according to whether the catchment is under or over-allocated. With an over-allocated catchment, the SO may be a net payer, *NP* < 0, reaching a solution; however, the solution may be infeasible with an over-allocated catchment. Even though all participants reduce allowances, still impacting flows are above the capped control point limits. A scaling factor will be proposed and discussed in Chapter 8.

If the initial set of IC allowances and so the flow rights in the catchment is feasible, i.e., $\sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} A_{i,j} \leq L_k \; \forall \; t,k$, then *NP* could be non-negative. Therefore, if the SO owns the receptor capacities and participants need to change or to increase IC, and hence shift flows without capacity rights, then the SO is a net receiver when selling capacity constraints, $\sum_{k=1}^{K}\left[\sum_{t=u}^{T}\left(L_k - \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1}\right)\lambda_{t,k}\right] \geq 0$ and so the SO reaches revenue adequacy if at least in one control point *k*, the SO is a net seller $\sum_{t=u}^{T}\left(L_k - \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1}\right)\lambda_{t,k} > 0$ for *k*.

If IC allowances from participants and their corresponding initial rights bind all the capacity constraints, $\sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} A_{i,j} = L_k$, $\forall t,k$, or the impacting flows do not change the period of binding capacity, $L_k - \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} A_{i,j} = 0 \; \forall \; t,k$, the SO reaches revenue neutrality $\sum_{k=1}^{K}\left[\sum_{t=u}^{T}\left(L_k - \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1}\right)\lambda_{t,k}\right] = 0$, and then NP≈0. That means, the SO could not sell any changes in flows at control point *k*. On the other hand, the SO could also reach revenue neutrality if in a control point is a net payer and in another a net seller, but both payments are balanced, so NP≈0.

Initial allocations of long-term IC allowances may exceed channel capacities. To solve this problem, the manager could try to buy rights in the market; hence, the expected level of floods and risk could be managed. Alternatively, the SO may try to scale the long-term IC allowances, however, if he/she defined rights for the impervious levels, scaling may be judged in court. Both solutions could be expensive. Scaling will be discussed in Chapter 8.

3.4.7 Auction period

An appropriate auction schedule needs to be determined that suits participants and authorities. For instance, the electricity markets in Australia, New York, and California clear every 5 minutes with partial optimization every 30 seconds in Australia and New York; in contrast, New Zealand has a half hour clearing market. The Victoria gas market is cleared every day with intervals to estimate prices and the schedule (Pepper et al. 2012). To obtain an adequate auction frequency, many factors need to considered, including liquidity, expected demand, risk of flooding, and environmental requirements. These factors may also depend on location, i.e., rural and urban places may require different auction frequencies.

The market may operate with short and long term allowances, and these allowances should be issued with clear expiry times. The time-based allowance would improve liquidity in the market. Raffensperger and Cochrane (2010) noted that a time-based allowance gives incentive to participate and to internalize the cost of the imperviousness in the catchment.

Based on catchment requirements, the SO could choose a schedule which enables the desirable allocations while managing the risk and expected demand. A seasonal right with an annual auction may be more active and self-hedging against flooding. The manager should also consider whether IC allowance periods are short or long.

In the market, a short-term IC allowance may work for those who require temporary land use actions or changes, e.g., land development for construction of real estate, and land cultivation for establishing crops. Thus, participants could link their temporary changes with their required IC allowances. Those participants who could not obtain allowances in previous auctions could alternatively buy short term allowances.

A short auction period (more frequent auctions) is likely to have higher transaction costs. If participants cannot bid in advance for future periods, a longer auction period may delay projects, so the opportunity cost to develop would increase. Participants who desire to change the imperviousness level would have to wait a long time before they could participate in the market, and such delay could change any decision to invest. The IC allowance period should reasonably match the participants' land use project.

In the long-term, sellers and buyers could trade land cover allowances for long-term changes in IC allowances, especially in urban areas. These IC allowances could tie project lifetimes with the time of the rights. Thus, long-term projects could be developed and a constant level of imperviousness could be ensured. However, long-term allowances may counter the desired long-term flood levels for the catchment planned by the SO, whereas short-term allowances allow management of such problems.

While such developers above may provide high bids, there is no certainty of project success; accordingly, higher clearing prices could be an issue associated with such develop. Hence, buyers that need to change the level of imperviousness will try to move their actions to those periods with lower prices. The prices in the risky season would encourage other participants to develop short-term actions. Thus, a market with short-term IC allowances is likely to be more liquid than a market for long-term IC allowances. A long term market could be more liquid only if more are options available to participate. Thus, participants could often trade long term rights in each auctions, which could help investments. The market would be more contestable if there were no entry or exit barriers, no sunk costs, and participants could participate freely in the market (Evenden and Williams 2000). However, if trade is possible only for long term allowances with a long term auction period, the market may be illiquid discouraging investment.

Participants could also trade their requirements for imperviousness in advance, e.g., one year, five years, or by season, which would generate more liquidity in the long-term market. Since participants could bid in advance throughout the planning horizon, BMPs could also be planned in advance. Such planning would ensure participants maintained the minimum level of imperviousness for the required long term.

Having both long-term and short-term allowances traded together is likely to raise many operational issues. We do not address these further in this thesis.

3.4.8 Planning horizon

The model considers the set of extreme storms that could occur in the modelled allowance period. In the ‘deterministic model’ the planning horizon accounts for an established extreme event.

The storm event determines the time component for the lagged effect of impacting flows at control points. Storm flows commonly impact for only short periods after a rainfall event. Usually the storm period lasts hours or days, depending on the hydrological condition of the catchment. The model period should be longer than the lag periods of impacts at control points.

If the market would account for a cumulative effect, as with sediments, the planning horizon must be defined carefully. For instance, Raffensperger et al. (2009) noted that a ground water market has an inter-temporal effect. The authors also noted that a ground water market might be held every week with bids for 52 weeks, being a one-year hydrological cycle. However, the constraints should consider periods beyond the hydrological cycle. The same time components were noted by Prabodanie et al. (2010), who analysed trading nutrients with different planning horizons in a market for nitrate. The authors observed that the market should account for the long-term effects at control points. Both authors agreed that market models should consider long-term constraints to avoid violating omitted constraints after the end of the market planning horizon. However, a market for IC allowances could not face this problem if the SO modelled storm for only one season, for an auction that traded IC allowances rights that had periods of longer than a season.

The second point to be considered is the participants' budget constraints, which depend on the IC allowance period. Participants make decisions about quantities and prices according to their budget constraints, IC allowance, service life of technologies and BMPs, risk aversion, income, etc. With a long IC allowance period, participants must estimate their bid over a long timeframe; however, they may not be able to pay up front for the whole period. Therefore, some participants may wish to bid for a short-allowance period, while others may bid for a long allowance period.

As a consequence of budget constraints, when the allowances and capacity constraints belong to the SO, participants might present strategic bids with lower prices. On the other hand, if IC allowances belong to participants and few participants bid in the long term, there would probably be few transactions.

Liquidity can be improved if participants have access to financial contracts such as contracts for differences, options, etc., as in the electricity market with financial transmission rights (FTR) (Hogan 2002). Long-term contracts could allow them to manage budget constraints, and to reduce the risk related to future prices.

Long term allowance periods may serve to ensure the long-term resource adequacy to control runoff. Thus, participants should tie their investments to the maximum capacities imposed by the SO. A long allowance period would help to deal with lead time for construction. Contracts could be established in the market for future actions or IC allowances. Thus, risks related to prices and quantities could be hedged for participants.

These types of issues are important for a functioning market. However, we do not address them further in this thesis.

3.4.9 Storms to establish short and long term markets

Participants and the SO face different types of risk. Participants face uncertain prices, quantities, investment, and technology. The SO faces risk with security of the system, adjustability under any failures, failure to be revenue neutral, establishment of the storms, and flood damage for extreme rainfall events.

A SO has the sensitive decision of choosing the storms used to design the markets. The decision would depend on the underlying storm probability distribution, the flooding cost, the level of risk that the SO desires, and the actual imperviousness of the catchment. Figure 3-5 shows several examples of storm distributions. This figure illustrates the changes in the storm probability distribution by storm types over seasons (e.g., Plessis 2001). In the figure, E_d is the critical event that corresponds to a storm able to generate floods. Thus, if the SO chooses this storm, the market will cover any storm up to this event with a known probability. Ribaudo et al. (1999) noticed that any goals in terms of runoff had to be established in terms of probabilities of occurrences.

Where auctions cover a single season, choosing the same storm for each season will lead to different probabilities that damage occurs by season. An alternative could be to choose a probability of damage and use a different storm for each season to deliver this level of risk.

The storm decision will affect the impact coefficients at different control points. A market established on a modest storm event of, say, 60 mm rainfall in 24 hours, will have a modest runoff from each property, and the SO would allow relatively more IC in the catchment. However, with a greater storm, flooding would be increased, with larger impacts, so the SO would allow less IC for a given flood stage limitation. This could require reducing the imperviousness in the catchment, in case the catchment were over-allocated; however, this condition may not allow a clearing market solution. The SO might choose an extreme case storm E_d to establish the market. The SO should periodically re-evaluate the cost of extreme flooding, with a view to defining which storm and which risk will define the markets.

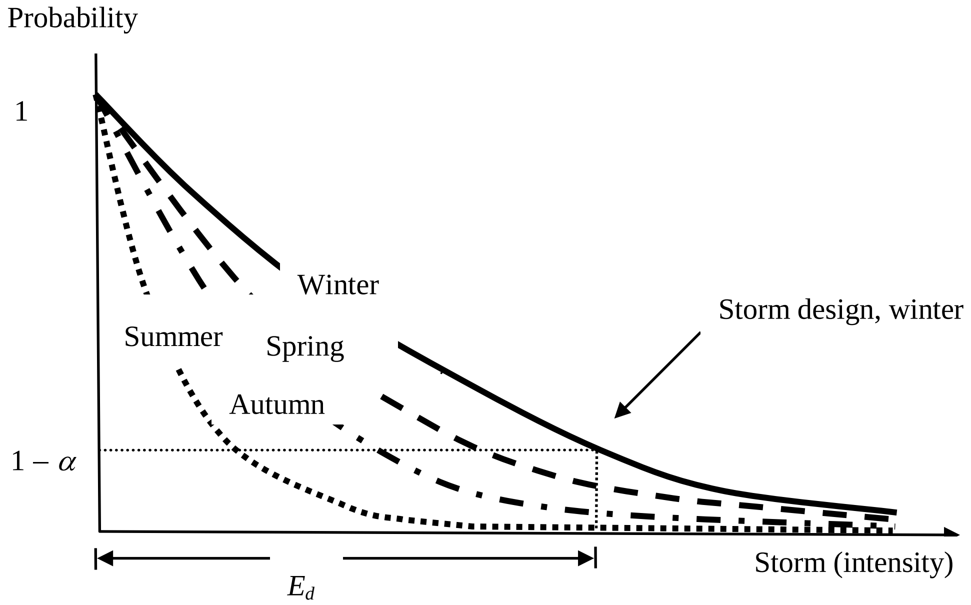

Figure 3-5 Seasonal probabilities at a specific place or catchment. E_d is a specific storm type, e.g., a storm of 100 mm in 24 hours, and α is the probability of a storm greater than the established storm event.

Even though the SO could cover a range of storms, greater events would still affect the catchment. Accordingly, the SO should identify those, and evaluate the risk for flood damage, when establishing the market with a specific storm design.

Establishing a storm design is similar to the risk cost balancing problem in infrastructure design and flood protection (CSIRO 2000; Loucks et al. 2005; Jha et al. 2012). The decision about the infrastructure capacity and so for the "storm design" is conditioned to the expected flood damage that could be faced in the area and the possible protecting benefit for the changes in infrastructure. Additionally, the benefit for improvement in infrastructure capacity could be noticed with the dual price in the market. Thus, an optimum storm design could be defined for the area.

3.5 An alternative LP pricing/impacting model Det_MarketIC2

An alternative market model could be formulated considering trade for changes in IC allowance in the area rather than trading initial and final IC allowances such as Det_MarketIC1. The market formulation “Det_MarketIC2” will account for differences in IC allowances within the area, while adjusted changes in flows are calculated by the LP market model. If participants bid for the differences, it could avoid that the market accounts for the net differences in bids that was observed with the previous market model Det_MarketIC1. The impacting flow coefficients H in Det_MarketIC are positive, and the applied price for participant i accounts for the changes in impacting flows between the initial IC allowance and the final IC allowances, Equation [3.8]. With Det_MarketIC2, the market model considers marginal impacting flows at control points, and the coefficient H accounts for the marginal changes in impacting flows. That means, the H coefficient accounts for the marginal changes in flows between the initial IC allowances and the final IC allowances, and those could be positive or negative at each control point. Det_MarketIC1 and Det_MarketIC2 reach similar clearing prices if the market is competitive.

Participants will trade IC allowance from an initial condition m to a final desired condition j; instead of selling m and buying j. In Det_MarketIC2, participants would sell if the IC allowance type j that they desire to reduce in the properties is lower than the initial IC allowance type n, $n > j$ (or the equivalent IC for implementing the BMPs). This accounts for the changes in impacting flows at control points; in this case, the impacting flows are being increased. Participants would buy if the IC allowance type j that they desire to increase is greater than the initial IC allowance condition m, $m < j$. This assumes that participants have initial IC allowances and the increase in IC allowances increase impacting flows at control points. Participants buy (sell) via bidding in tranches; thus, a participant could demand (offer) more than one IC allowance change for their property. To simplify bidding, participants could demand and offer for the final desired IC allowance in the area. Changing IC allowances is a continuous variable and hence participants will account for their initial IC allowance in their bid decisions. For instance, they could demand as a first tranche 1 ha for changing to B (from A) and as second tranche 0.25 ha for changing to B (from A) in the block area (1.5 ha). Thus, the market could allocate 1.1 ha for changing to B.

The model uses some of the nomenclature of Det_MarketIC1, Det_MarketIC1.1 and Det_MarketIC1.3; for instance, the base flows are kept relatively constant over the period in the channels and streams. However, new parameters and decision variables are incorporated. Additionally the market assumes uniform soil moisture AMC II in the catchment.

Parameters

$A^0_{i,m}$ = Total initial IC allowance type *m* owned by participant *i* who desires to buy IC (ha).

$A^0_{i,n}$ = Total initial IC allowance type *n* owned by participant *i*, who desires to sell IC (ha).

$D^{\max}_{i,j,b}$ = Maximum area in IC allowance to change to allowance type *j* (ha) that participant *i* in bid step *b* is willing to buy at price $P^D_{i,j,b}$.

$S^{\max}_{i,j,b}$ = Maximum area in IC allowance to change to allowance type *j* (ha) that participant *i* in bid step *b* is willing to sell at price $P^S_{i,j,b}$.

$P^D_{i,j,b}$ = Demand price ($/ha) for changing to IC allowance type *j* from participant *i* and bid step *b*. This is the maximum that participant *i* is willing to pay for a change from IC allowance type *m* to allowance type *j* and bid step *b*.

$P^S_{i,j,b}$ = Bid price ($/ha) for changing to area allowance type *j* from participant *i* and bid step *b*. This is the minimum that participant *i* is willing to accept for a change from IC allowance type *n* to allowance type *j* and bid step *b*.

L_k = Maximum allowable flow (volume/time) at channel control point *k*. These capacities depend on the chosen storm scenario at control point *k*.

A^{Tot}_i = Total area of participant *i* (ha). $A^{Tot}_i = \sum_{j=1}^{J} A^0_{i,j}$ for all *i*,*j* that could be checked previously to clear the market model. This area constraint will also be checked in the participants' bidding interface.

$Q^0_{k,t}$ = Initial total flows at control point *k* and time *t*. These flows are related to the initial IC allowance and the chosen storm scenario.

$H_{i,j,k}^{t-u+1}$ = Marginal flow impact at control point *k* of IC allowance type *j* from participant *i*, at the end period of *t* –*u*+ 1. *u* is the lag time between commencement of the storm event and when the flow reaches the control point (volume/time ha). This coefficient relates initial conditions on the participant's property to the marginal impact at control points. The impacts are estimated for a specific storm type. This linear coefficient is likely to depend on the initial IC allowance conditions within the catchment, so it should be updated as IC allowances change to improve the accuracy of the impact coefficients. This marginal impact can be positive or negative. If participant *i* does not impact on control point *k*, then $H_{i,j,k}^{t}$ = 0.

Decision variables

$qbuy_{i,j,b}$ = Area in changing hectare to IC allowance type j and bid steps *b* bought by participant *i*.

$qsell_{i,j,b}$ = Area in changing hectare to IC allowance type j and bid steps *b* sold by participant *i*.

$g_{i,j}^{D}$ = Total hectares bought for changing IC allowance type j for participant *i* (ha).

$g_{i,j}^{S}$ = Total hectares sold for changing IC allowance to type j for participant *i* (ha).

$\mu_{i,j}^{D}$ = Price for buying participant *i* and changing land allowance type *j* ($/ha).

$\mu_{i,j}^{S}$ = Price for selling participant *i* and changing land allowance type *j* ($/ha).

$\lambda_{t,k}$ = Clearing nodal price. Price to discharge at the control point *k*, time *t* ($/ volume/time).

Model: **Det_MarketIC2**

$$\text{Maximize} \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P_{i,j,b}^{D} qbuy_{i,j,b} - \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P_{i,j,b}^{S} qsell_{i,j,b} \qquad [3.17]$$

Subject to

$$0 \le qbuy_{i,j,b} \le D_{i,j,b}^{\max},\ \forall\ i,j,b \qquad : \beta_{i,j,b}^{-}, \beta_{i,j,b}^{+} \qquad [3.18]$$

$$0 \le qsell_{i,j,b} \le S_{i,j,b}^{\max},\ \forall\ i,j,b \qquad : \gamma_{i,j,b}^{-}, \gamma_{i,j,b}^{+} \qquad [3.19]$$

$$\sum_{b=1}^{B} qbuy_{i,j,b} = g_{i,j}^{D}, \forall\ i,j \qquad : \mu_{i,j}^{D} \text{ (free)} \qquad [3.20]$$

$$\sum_{b=1}^{B} qsell_{i,j,b} = g_{i,j}^{S}\ , \forall\ i,j \qquad : \mu_{i,j}^{S} \text{ (free)} \qquad [3.21]$$

$$Q_{k,t}^{0} + \sum_{i=1}^{n} \sum_{j}^{J} H_{i,j,k}^{t-u+1} g_{i,j}^{D} + \sum_{i=1}^{n} \sum_{j}^{J} H_{i,j,k}^{t-u+1} g_{i,j}^{S} \leq L_{k}, \forall\ t,k \quad : \lambda_{t,k} \qquad [3.22]$$

$$g_{i,j}^{D},\ g_{i,j}^{S}, \text{ (free)} \qquad [3.23]$$

Explanation:

[3.17] The objective function maximizes the gains for trading IC allowances across time, for a given rainfall event. The objective function does not measure the absolute welfare, but is indicative of any such changes. It is recognised that participants could try to bid strategically; but that issue is beyond the scope of this thesis.

[3.18] Total changing area bought in each tranche is bounded by demand quantities.

[3.19] Total changing area sold in each tranche is bounded by bid quantities.

[3.20] The final area bought for changing to IC allowance type *j* of participant *i*.

[3.21] The final area sold for changing to IC allowance type *j* of participant *i*.

[3.22] The total flow at control point *k* in time *t* must be less than the maximum flow capacity in the channel. This capacity corresponds to the differences between channel capacity and status quo base flows. The dual price $\lambda_{t,k}$ represents the improvement in economic surplus if the SO permitted another unit of capacity in the channel at control point *k*.

[3.23] Buy and sell quantities ($qbuy_{i,j,b}$ and $qsell_{i,j,b}$) must be non-negative, and will naturally limit the final allocation of IC allowances, $g_{i,j}^{D}$ and $g_{i,j}^{S}$.

The final IC allowance should be limited to the initial IC allowances (ha). In this case, the formulation should use an additional constraint, so participants cannot finish with a negative initial IC allowance or with more area allowance than their property area:

$$\sum_{j \neq m}^{J} \sum_{b=1}^{B} qbuy_{i,j,b} \leq A_{i,m}^{0} \ \ \forall i,m \text{ and } \sum_{j \neq n}^{J} \sum_{b=1}^{B} qsell_{i,j,b} \leq A_{i,n}^{0}, \ \ \forall i,n : \nu_{i,m}^{D},\ \nu_{i,n}^{S} \qquad [3.24]$$

The duals in these constraints, $\nu_{i,j}^{D}$ and $\nu_{i,j}^{S}$, correspond to the marginal value of using a particular piece of land. Thus, it is expected that participants with several high value alternative land uses will have a high $\nu_{i,j}^{D}$ and $\nu_{i,j}^{S}$.

The next dual analysis considers the first model without the condition [3.24]. Participants are bidding for their net changes in IC allowances, and not for a set of possible changes from their initial IC allowance.

This market formulation accounts for the desired and initial conditions. The dual variables $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ account for these changes in impacting flows while the IC allowance balance is maintained. The variables $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ are presented in a canonical way and dual variables are conveyed to be positive for the analyses in this section and in the following chapters.

The dual variable $\mu_{i,j}^{D}$ in constraint [3.20] is the marginal value for another unit of changing impervious area level to *j* bought for participant *i*. The price accounts for the changes in IC condition, for the corresponding changes in impacting flows across time and control points $\mu_{i,j}^{D} = \sum_{k=1}^{K}\sum_{t=u}^{T} H_{i,j,k}^{t-u+1}\ \lambda_{t,k}$, where $\sum_{k=1}^{K}\sum_{t=u}^{T} H_{i,j,k}^{t-u+1}\lambda_{t,j} \geq 0$. This price is equivalent to the final clearing price used to charge participants from Det_MarketIC1.

The dual variable $\mu_{i,j}^{S}$ in constraint [3.21] is the marginal economic surplus for another unit of changing impervious levels *j* sold by participant *i*. This also represents the increasing cost to the system if another unit of changing impervious levels were permitted to user *i*. This is the applied price used to pay participant *i* for the changes in impacting flows. $\mu_{i,j}^{S} = \sum_{k=1}^{K}\sum_{t=u}^{T} H_{i,j,k}^{t-u+1}\ \lambda_{t,k}$, where $\sum_{k=1}^{K}\sum_{t=u}^{T} H_{i,j,k}^{t-u+1}\lambda_{t,j} \leq 0$.

These final prices are also similar[8] to the applied prices for participants from Det_MarketIC1 if the market is competitive, with the following trading conditions:

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[8] The dual price from Equation [3.14] can be arranged to show the relationship with [3.25]. Thus, if a participant is increasing imperviousness, with Det_MarketIC1 could sell their initial IC allowance at $P_{i,m,b}^{S}$ and could buy at $P_{i,p,b}^{D}$, and the applied price for changing imperviousness is $\mu_{i,p} - \mu_{i,m}$. This price accounts for the marginal changes in impacting flows at control points,

$$\mu^D_{i,j} = P^D_{i,j,b} - \beta^+_{i,j,b} + \beta^-_{i,j,b}, \forall i,j \qquad [3.25]$$

$$\mu^{S\ 9}_{i,j} = P^S_{i,j,b} - \gamma^-_{i,j,b} + \gamma^+_{i,j,b}, \forall i,j \qquad [3.26]$$

Participants will trade IC changes if their bids, which represent the opportunity costs of such changes in IC allowances, are greater than the clearing price when they desire to increase impervious, and lower when they are reducing impervious level. The trading conditions [3.25] and [3.26] are similar to those shown in equations [3.15] and [3.16], which account for the differences for buying and selling in Det_MarketIC1 while maintaining a land area balance.

As in Det_MarketIC1, participants would need to establish their bids carefully, because their IC allowances change the flows at control points. Increasing IC allowances does not necessarily mean participants need to pay, as they could get rapid runoff from their property to reduce flows at peak times. Participants should then receive a payment. Participants with land close to the control point should pay almost nothing for increased imperviousness, because the runoff is so rapid that it avoids the peak flow at control point. In this case, a participant's bidding interface could help to see if their decision may reduce, increase or shift peak flows at control points.

3.6 Settlement with market model Det_MarketIC2

The market model allocates with prices $\mu^D_{i,j}$ and $\mu^S_{i,j}$ for each participant. The settlement, r_i, for those participants who demand increased IC allowances is $r_i = \sum_{j=1}^{J} \mu^D_{i,j}\, g^D_{i,j}$, and

---

$\sum_{m,p\in j}^{J} \sum_{k=1}^{K} \sum_{t=u,e}^{T} \left(H^{t-u+1}_{i,p,k} - H^{t-e+1}_{i,m,k} \right) \lambda_{t,k}$. From Equation [3.25], the price to increase imperviousness is $P^D_{i,j,b}$, which accounts for their initial IC allowance implicitly and the applied price $\mu^D_{i,j}$ accounts for the marginal changes $\sum_{m,p\in j}^{J} \sum_{k=1}^{K} \sum_{t=u}^{T} H^{t-u+1}_{i,j,k} \lambda_{t,k}$. With Det_MarketIC2, the coefficient H accounts for the marginal changes in impacting flows from the initial to the final IC allowances, which is the same $\left(H^{t-u+1}_{i,p,k} - H^{t-e+1}_{i,m,k} \right)$ from Det_MarketIC1. Thus, the applied price from Det_MarketIC1 and Det_MarketIC2 are the same, as well as the settlements for participants.

[9] For the purpose of discussion, dual price is represented in a canonical and positive formulation and contains prices with negative values $\mu^S_{i,j} \leq 0$

for those who offer to reduce IC allowance is $r_i = \sum_{j=1}^{J} \mu_{i,j}^S \, g_{i,j}^S$. The settlement considers that participants have rights for the initial IC allowances.

3.7 Comparing trade condition from models

It is possible to choose parameters for the LP market models Det_MarketIC1 and Det_MarketIC2 so that similar primal and dual solutions results. The two models result in the same trading surplus (same total social costs), and further parameters can be chosen such that they result in the same net costs/profits for each market participant. However, participants would trade in conceptually different ways. In model Det_MarketIC1, participants trade their conditions, while in model Det_MarketIC2 participants bid for the final condition, which represents the changes in IC allowance. In Det_MarketIC1, independently of their bids, the model takes into account the marginal effects between price bids. This may confuse participants who may misestimate their bids. This issue could be solved by informing participants of such an inconvenience, or by simply implementing the Det_MarketIC2 model.

3.8 Market competitiveness

Competition concerns government and market designers because there could be significant losses for society when participants exercise market power and alter prices for their benefit away from competitive prices (Ferguson and Gould 1980; Stoft 2002). Market power could be generated due to few participants or concentration in the market. However, a thin market does not necessarily mean a non-competitive market (Varian 1994).

Physical competitiveness in a market for IC allowances can be measured using physical trade conditions which correspond to the impacting flow coefficients, $H_{i,j,k}^{t-u+1}$, and how participants' flows overlap at control points. Unfortunately, different properties with the same IC allowance have flows which reach control points at different times and intensities. These differences may produce market concentration problems and tradability which should be identified by the SO.

Chakrabarti and Goodwin (2008) presented different measurements to analyse competitiveness in a market. Prabodanie and Raffensperger (2009) analysed the problem of tradability in a market for nutrients and used the hydro-geological conditions from participants as a way to test the level of potential physical trade and so the competitiveness in the market. These authors used the Herfindahl-Hirschman Index (*HHI*) to measure market concentration. Day et al. (2002) have noted that markets with competitive *HHI* indices could present prices above competitive levels due, for instance, to transmission constraints. In the same way, non-competitive *HHI* indices could correspond to competitive prices. Hence, the *HHI* could be used only as a first view for market competitiveness.

If *HHI* is used to measure physical competitiveness in a market for IC allowances, the physical measurement of market share $\bar{\beta}_{i,k}^{t}$ can be calculated in terms of physical trade conditions. Thus, the index can be estimated by time t and location k, in particular in places with flooding risk. The index is as follows:

$$HHI^{t} = \sum_{i=1}^{N} \left(\bar{\beta}_{i,k}^{t} \right)^{2} \qquad [3.27]$$

$$\bar{\beta}_{i,k}^{t} = \frac{\sum_{j=1}^{J} H_{i,j,k}^{t-u+1} g_{i,j}}{\sum_{i=1}^{N} \sum_{j=1}^{J} H_{i,j,k}^{t-u+1} g_{i,j}}, \quad \forall i,k,t \qquad [3.28]$$

HHI ranges between 0 and 1; with *HHI* < 0.01, the market is not considered concentrated; with 0.01 < *HHI* < 0.18, the market is moderately concentrated; with *HHI* > 0.18 the market is concentrated (Chakrabarti and Goodwin 2008). Thus, the SO could monitor market concentration for storm water flows at different points in the catchment. This index could also be estimated in terms of storm scenarios, and their relationships with flows and flooding components.

Additionally, the SO could estimate when participants are able to raise the market prices. This requires private cost data from each participant, which the SO does not have. Alternatively, the SO could measure the probable pivotal ability to raise prices using indices such as the Pivotal Supplier Index (*PSI*), the Residue Supply Index (*RSI*), and the Residual Demand Analysis (*RDA*). The Residual Supply Index (*RSI*) measures the capacity to reduce supply in accordance with the total demand. In an IC market, this index could be estimated as the capacity to reduce flows by time at the control point. Note that transmission market operators in the USA often calculate competitiveness of transmission constraints in a similar fashion, especially with RSI (Sheffrin 2002; Sheffrin et al. 2004;

Chang 2007; Lee et al. 2011). Thus, this index could be estimated for the period where flooding reached the peak or throughout the flooding period.

$$RSI_k^t = \frac{TC_k - InC_{i,k}}{TD_k} \quad , \forall k,t \qquad [3.29]$$

TC is the total capacity of a participant to reduce impacting flows, *InC* is the individual capacity to reduce, and *TD* is total demand of impacting flows required by time. *RSI* > 1 indicates the supplier *i* may have little influence on price. With *RSI* < 1, the supplier *i* may exercise market power. Because each participant impacts on specific control points, their market power is specific to those control points but not others. While the participant is not exercising power in these points; however, the index may show she/he is doing so. Therefore, this is only an approach. The indices will not be further discussed nor calculated, and this would be part of future research.

3.9 Final remarks and conclusion

This chapter developed clearing formulations for an IC market under an extreme storm scenario. The models maximise surplus for trading IC allowances, with limits of channel capacity, or maximum flooding levels established under a storm scenario.

In both formulations, duals represent valued changes in impacting flows from an initial and final condition. Additionally, the dual prices account for the requirement that each participant has to keep the same area at the end of the auction as at the start.

The revenue from the SO depends on the catchment conditions. An over-allocated and under allocated catchment, i.e., over and under allocated capacity limits, will result in the SO being a net payer or receiver. Capacity limits are traded and valued to reach the standards that society expects. The limits are established according to a storm scenario. The market formulation accounts for related changes in physical storm water flows in the channel, streams and flooding conditions in the catchment.

The market models Det_MarketIC1 and Det_MarketIC2 were established with capped control points, where flow capacities cannot be violated. Thus, participants implicitly trade as if a specific storm scenario occurred. This market condition could generate inefficient allocations, because the marginal value of increasing impervious levels may be higher or lower than the marginal cost to the system if the SO allowed an extra unit of flood capacity at a specific place. Therefore, the nodal clearing price could be higher or lower than the true marginal incremental damage $\left(\lambda_k > \frac{\partial C_k^f\left(f_k \right)}{\partial f_k}, or \ \lambda_k < \frac{\partial C_k^f\left(f_k \right)}{\partial f_k} \right)$. Consequently, the price signals may promote inefficient IC in the catchment. The models in this chapter assumed a strict non-violation of the control points. That means participants are trading, but may not incorporate the cost of flooding in their decisions. To establish which flooding level could be obtained in the catchment within the market, the market design should consider which storm and flooding level should be chosen in the catchment; the damage cost and the convexity in the flooding cost; the damage cost approximations, how the economic implications affect the market decision; and the revenue adequacy that would be observed related with the damage approximation.

The market should account for the rainfall uncertainty as well as any possible change in flood distribution derived from changes in IC allowances. Marginal changes in flood distribution would then be priced. Thus, participants would face a price when their actions shift the flood damage distribution.

Charter 4 will present a market formulation that accounts for the storm distribution and changes in flooding distribution. Chapter 5 will present a formulation that constrains for hastening peak flood (hastening peaks) and flood durations associated with particular scenarios and hazard areas.

Chapter 4

4 A MARKET FOR IC ALLOWANCES UNDER RAINFALL UNCERTAINTY

4.1 Motivation for stochastic programming

The previous chapter presented a deterministic market proposal with the models Det_MarketIC1 and Det_MarketIC2, where the SO had to define a single maximum rainfall event to hedge against flood in the catchment. A stochastic version of Det_MarketIC1 and Det_MarketIC2 models, which had hard constraints [3.6] and [3.21] respectively, are not required, because it is likely that just one scenario would have the tightest constraint in both models, and none of the others would be binding in the optimal solution. These markets do not fully consider the stochastic nature of rainfall and flood; thus, participants do not incorporate the cost of flooding from storms greater than that established under the storm design criteria. The market should ideally be affected by the rainfall distribution and the resulting private and social costs of flooding during extreme events.

Thus, the stochastic model Sto_MarketIC is instead an attempt to clear the market by creating a demand curve for flood reductions (indirectly, imperviousness reductions) that will be cleared against the implicit supply (based on the bids of participants). The demand (supply) curve is based on the marginal expected changes resulting from increases (decreases) in imperviousness levels (accounting for the transfer functions, flows and stage-damage relationships ((Krzysztofowicz and Davis 1983c; Loucks et al. 2005)).

The resulting IC allowances take advantage of specific relationships in the single ‘worst case’ storm, relying, for example, on the timing of rainfall to mitigate the impact. There will always be bigger storms than those modelled.

The catchment's IC allowances and storm events would affect storm water, surface runoff, and hence flows and flooding (Chapter 3) at control points. Thus, any change in IC allowances, BMPs and control runoff technologies can affect the patterns of runoff from the property, and also the impacting flows at control points. These changes are not instantaneous due to inflexibilities of demand and supply; furthermore, under any extreme event, demand and supply do not respond quickly enough to modify IC allowances. Thus, a proposed market should incorporate ex-ante participant preferences for IC allowances, and the model should account for a range of rainfall events and possible changes in flood distribution. The proposed market takes into account a flood cost only for maximum peak flow at the control point.

In the market model, the stochastic nature of rainfall is incorporated by assigning probabilities of rainfall events. The effect of a rainfall event depends upon the impact coefficients derived from the IC allowances, BMPs and technologies. Those coefficients correspond to the flows per unit of time at different points across the channel under different rainfall scenarios. Figure 4-1 illustrates the stochastic nature of rainfall events and storm water flows observed from different storms and impervious levels.

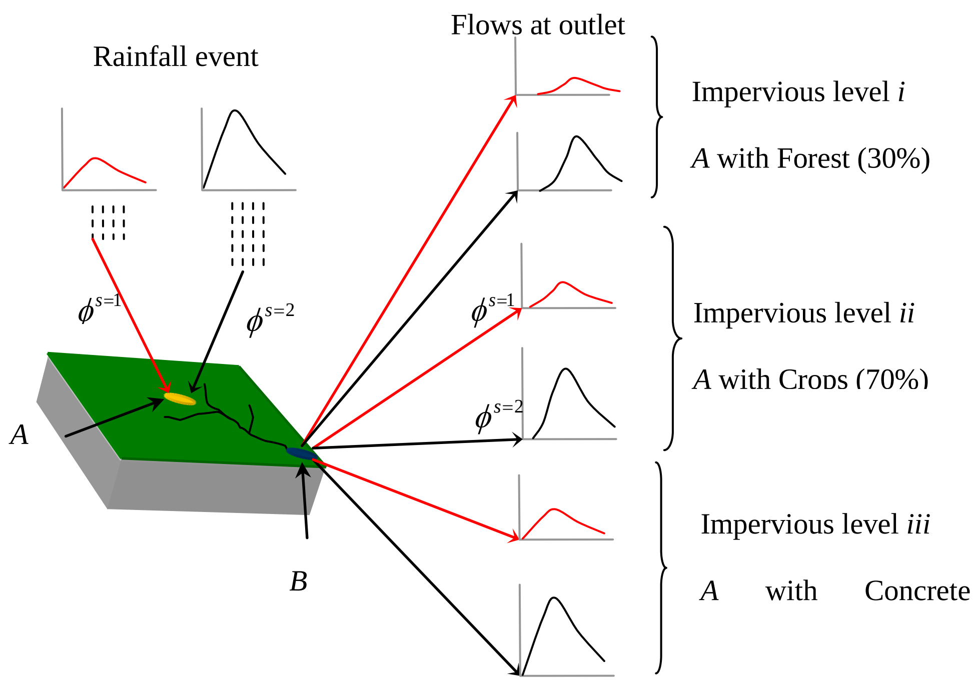


Figure 4-1 Rainfall events on land with different impervious levels ($i < ii < iii$). $\phi^{s=1}$ and $\phi^{s=2}$ are probabilities of rainfall events. A is a property in the catchment, and B is the outlet control point.

Different models have been proposed and applied to deal with uncertainty, in particular with inflow uncertainty. Loucks et al. (2005) presented different methods to manage stochastic water resources for river basin, urban water, reservoirs, lakes, and flooding problems. The authors pointed out that for any water planning, which involves uncertainty (rainfall, stream flows and flood flows), a probability distribution function is required to manage the resource.

Pereira and Pinto (1985) used stochastic programming for managing reservoirs in a hydro dominated electric system; 37 reservoirs in Brazil were used to illustrate the proposal which included an extension of Benders decomposition. Yang and Read (1999) used constructive dynamic programming (CDP) to optimise reservoir releases for power under uncertain inflows, and Carrion et al. (2007) proposed stochastic programming with recourse, for solving the electricity supply problem of a large consumer. Pritchard et al. (2010) proposed a stochastic programming model for scheduling electric power generation under uncertainty, inflexibilities, and uncertain demand. They assumed that scenarios were finite; thus, they solved the problem by using standard techniques.

Tilmant et al. (2008) presented a stochastic programming model to obtain the marginal water value in an integrated economic hydrologic model for a multipurpose multi-reservoir system, where agriculture and hydroelectric power compete for water.

Hollinshead and Lund (2006) reported a stochastic model, in three stages with recourse, to minimize the expected cost of long-term spot and option water purchases as a way to meet environmental demands. As a result, they optimized seasonal water purchases for a manager, taking into consideration uncertainties derived from the hydrological, operational, and biological issues.

Calatrava and Garrido (2005) analysed water market systems under uncertainty of water supply in Spain. Barquin et al. (2004) presented a market model under uncertainty for medium-term inflow uncertainty, where the equilibrium took into account uncertain fuel prices, demand, hydro inflows and generator failures, based on a scenario tree representation.

Tilmant et al. (2008) pointed out that stochastic programming such as a two stage stochastic programming (TSSP) is suitable for addressing hydrologic uncertainty with a hedging strategy against extreme events, such as floods and droughts. Moreover, stochastic formulations are able to take into account the uncertainty, violation of constraints (flood cost), and the complex spatial and temporal trade-offs, which would allow determining marginal water values in multipurpose multi-reservoir systems.

Concerning flooding, Piantadosi et al. (2008) presented a stochastic model to choose a policy to manage urban runoff, and Liu et al. (2009) proposed a two stage programming to manage flood diversion under uncertainty. In the latter, the authors noticed that many impacting flood components were uncertain due to flow variability, and that two-stage (or multi-stage) stochastic programming (TSSP) is particularly useful for working with infeasibilities. Accordingly, the TSSP is a plausible approach to model the randomness from different rainfall events and flooding; thus, this approach will be used for the IC market clearing formulation.

A hypothetical illustration of flood control options using a TSSP model is presented by Lund (2002), whose goal was to minimise the expected cost of flood damage from different options to manage flooding. The author reported that an additive non-convex flood damage function could increase the intractability of a solution; accordingly, the author proposed a piecewise linear approximation to deal with this limitation.

With this market framework, the proposed stochastic IC model uses linear piecewise approximations to avoid convexity difficulties of the flood damage function.

Piecewise linear function and penalties were also proposed by Dupacová et al. (1991) who approximated a convex function and used penalty functions for floods, irrigation and recreation. Van der Vlerk (2002) considered a lower and an upper bound for the total supply with piecewise linear penalties for violating those bound. The author optimised the electricity distribution system in the Netherlands, with uncertainty of the future energy supply. The proposed IC market formulation follows this penalty condition and piecewise approximation by convexifying the flood damage cost.

Finally, the SO would have revenue adequacy in expectation, although in some scenarios the SO may be a net payer or net receiver. Pritchard et al. (2010) noticed that prices and allocations could vary from the deterministic solution. Prabodanie (2010) discussed revenue for the SO from the view of a deterministic condition, Raffensperger (2011) presented alternatives to match available water with a revenue sufficiency for the

SO. Pinto et al. (2012) dealt with revenue neutrality when the catchment is over and under allocated. The revenue issue will be faced in the proposed Sto_MarketIC of this chapter with initial allocations, and extra rent with the convexification.

The proposed TSSP market model will maximise the expected economic surplus for trading impervious land cover, while accounting for flood cost due to flow violations in channels, streams and floodplain areas. Thus, flood cost, associated with the flows and constraint violations, will account for the flood damage in extreme storm events. The model assumes that damage functions would not change during the planning horizon. The recourse in the TSSP does not represent recourse decisions of participants, who cannot adjust imperviousness or BMPs under different scenarios; rather, this allows calculation of the flood damage function $C(f)$, where f is a vector of stage flood from changes in demand $D_{i,,j}$ and supply $S_{i,j}$ from participant i and IC allowance j.

The SO will be in charge of protecting areas against flooding and maintaining an acceptable level of risk for the community. The SO does not participate in the market directly, but the SO defines the flood damage cost in the market model. Price signals will incentivise reduction of storm water flows and damage due to flooding. Consequently, the market could be hedged against a range of storms, and flooding in the catchment. Figure 4.2 illustrates a hypothetical rainfall distribution and flood damage that could be observed with different IC allowances in the catchment. The dashed line indicates the shifted flood damage under different IC allowances and storms; these changes correspond to changes in flood distribution and consequently changes in the hedged range of storms.

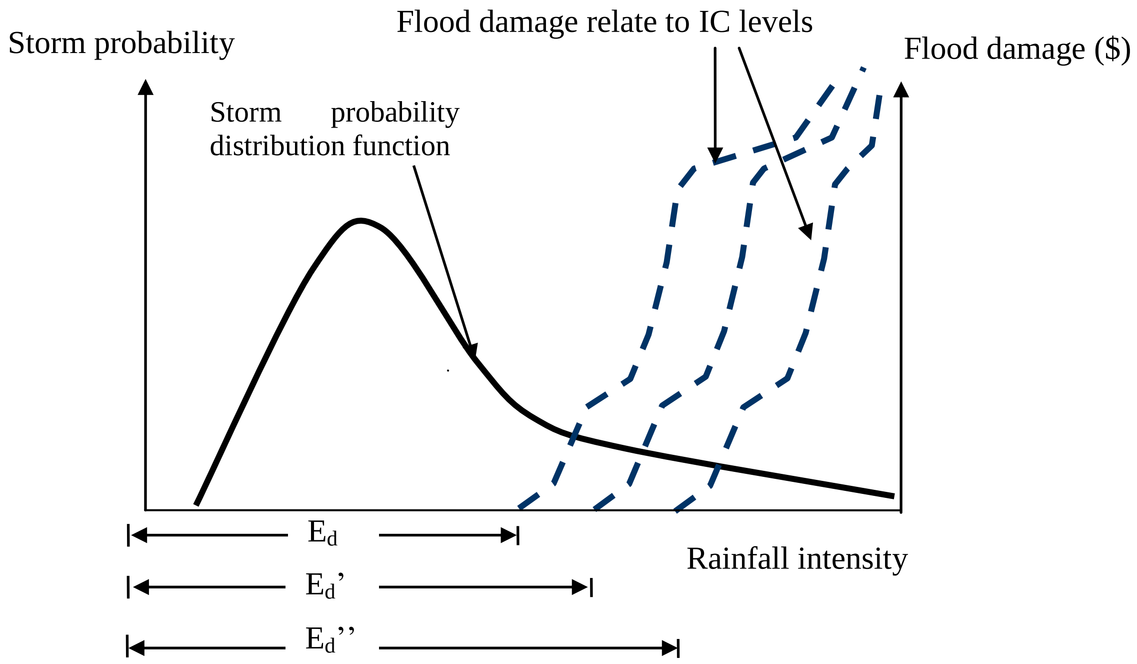


Figure 4-2 Probability distribution of rainfall events and flood damage for extreme events. The axis on the left represents storm probability and the axis on the right flood damage ($). Ed, Ed', and Ed'' represent storms that the market would hedge against.

4.2 The general two-stage stochastic model

This section describes the general formulation for the two stage stochastic program with recourse, also known as a stochastic model with penalties (Dupacová et al. 1991; Kall and Wallace 1994; Birge and Louveaux 1997). A first stage considers decision variables *x* under uncertainty. A second stage, begins with a known value of *x*, and the variables *y* work to evaluate consequent recourse flood cost penalties.

The formulation for an equivalent deterministic model is as follows (Kall and Wallace 1994; Birge and Louveaux 1997):

$$\max_x \mathrm{E}[\, f(x,\xi)\,]: c^T x - E_\xi\left[\min\ q(\xi)^T\ y(\xi)\right] \qquad [4.1]$$

$$\text{s.t. } Ax = b$$

$$T(\xi)x - W(\xi)y(\xi) = h(\xi),\ \xi \in \Lambda$$

$$x, y \in R^n$$

In the market model for IC, $c^T x$ represents the expected objective function for trading IC, where the objective is to maximise trading value. $q(\xi)^T y(\xi)$ is the recourse flood cost function which accounts for flood damage, where ξ is a random (unknown) variable for rainfall events, $\xi \in \Xi \subset \Re^N$. The decision variable x needs to be selected before the random variables are known; ϕ^s is the rainfall probability for $\xi = \xi^s$, defined for a discrete set of possible storms s=1,…,S, $\phi^s \geq 0$ and $\sum_s^S \phi^s = 1$; $W(\xi) y(\xi)$ corresponds to the violation of the constrained capacities constraints; $h(\xi)$=h and $T(\xi)$ are the participants' impact coefficients. Thus, the formulation takes into account the chance of future flood damage which will depend on IC trading x, the final impervious levels in the catchment, and the probability of rainfall events. The stochastic model [4.1] is linear and the objective accounts for a linearised damage cost function of flooding.

4.3 Choosing scenarios

Dye (1994) pointed out that suitable scenarios must be chosen to obtain efficient outcomes. In this case, suitable storm scenarios are selected to represent the storm distribution and hence the flood damage distribution. From this discretised distribution, storm probability scenarios could be calculated (Loucks et al. 2005). These representative scenarios should be used to clear the market; however, they should give insight into all the storms and in particular for the extreme storm scenarios and their probabilities Scenarios should include details about the flooding and the expected flood damage. For instance, the chosen storms could simply be as 100 and 200 mm rainfall in 24 hours, or as a series of storms such as 100, 30 and 110 mm rainfall in 76 hours. Accordingly, the SO should select from storms scenarios that represent flood damage conditions, while maintaining a robust solution.

Different methods could be used to select scenarios. The method could be simply an arbitrary selection of storms and their related probabilities by the SO, which should account for the distribution focusing on extreme events and keeping the condition that the scenario probability selections must sum one $\sum_{s=1}^S \phi^s = 1$; for further detail see Loucks et al. (2005). More complex methods include scenario reduction to define the scenarios that match with the distribution. Dupačová et al. (2003) proposed a scenario reduction technique based on a different probability metric for scenario reduction in stochastic programming. Growe-Kuska et al. (2003) proposed an algorithm to reduce scenarios to determine scenario subsets and adjust probabilities. The scenarios and probabilities are modified and bundled into similar scenarios to reach a smaller number of scenarios. In this research, scenario selection and their probabilities are obtained based on a discretised distribution based on HIRDS (NIWA 2002; 2008).

As was noted in section 2.8, in Chapter 2, any possible flood scenario may have non-linear and non-convex flood damage functions. Figure 4-3 shows a situation where, according to the changes in IC allowances and storm severity, different stage-floods and costs are reached; the width of the bar represents the minimum and maximum stage-flood and damage derived from the storm. A, B and C represent sections of the damage cost function which are non-linear and convex, non-linear-non-convex, and non-convex respectively. In the market design, possible non-convexity will be addressed by convexification of the damage function. If this is not fixed, the non-convexity issue may raise externalities. Therefore, some participant will not correctly internalise the marginal changes in flooding for the changes in IC allowances, even though they contribute to the stage-flood; consequently, there will be differences between the final payment and the marginal flood damage contribution from the participant as well as between the expected flood damage and the total payments from participants. These differences could be internalised for the SO or society. This point is discussed throughout this chapter.

Additionally, marginal flood damage could be almost zero during extreme storm scenarios. Because the clearing price represents the marginal change in flood damage, it may change little with the extreme storms, even though IC allowances change (see C in Figure 4-3). Thus, a low event probability, such as for a 400 mm or 500 mm storm in 24 hours, could reduce the clearing price nearly to zero in these scenarios. For this reason, these scenarios may be omitted in the chosen storm scenarios for the market formulation. This effect is emphasised when the damage function is linearised but the non-convex condition remains unchanging. This effect is illustrated in Appendix B.

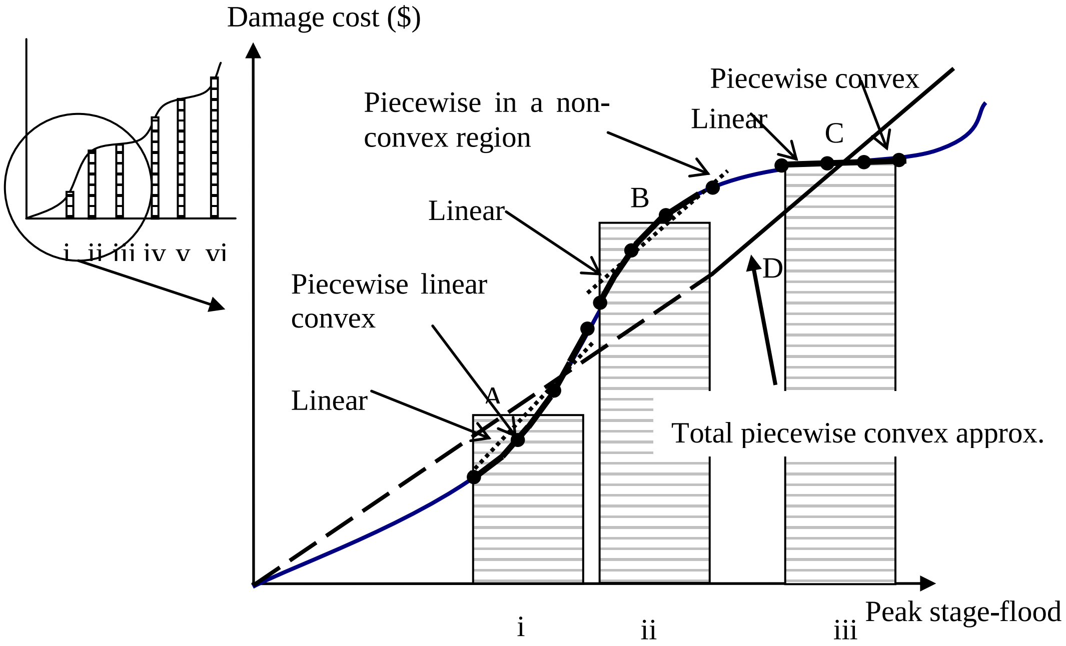


Figure 4-3 Flood damage curve and chosen storm scenarios *i*, *ii* and *iii* with their equivalent stage-flood levels. A, B and C are the different costs related to impervious level combinations in scenarios: linear (dots), piecewise linear approximation (dashed). D is the total piecewise convex approximation.

4.4 Overview of the Sto_MarketIC market

The proposed market for IC works as an auction system, accepting selling offers and purchasing bids, to trade IC allowances (continuous variable). Participants would not trade pair-wise, but buy from and sell to a central auction manager, through common pools which would have control points and areas where flooding occurs.

The SO should evaluate which policy he/she desires to implement through the market, given the SO must include the flooding mitigation in the market model. The SO may limit the value of contents and properties to discourage new investments in the floodplain area. Accordingly, any change in the flood damage cost should be internalised for people in the flooding area, i.e., they should face changes in the expected flood damage under changes in imperviousness in their properties; otherwise, the SO could periodically update the damage valuation. Thus, any change in flood distribution would be progressively more expensive, and participants that change IC would face higher prices. Although participants in the high areas have rights to ICs, increasing flood costs may discourage imperviousness in these properties. This also implies that they have the rights only for a specific expected flood damage and flood distribution. The SO should evaluate this strategy as a way to discourage imperviousness. As will be seen in next sections, prices depend on changes in flood distribution and the SO could finish as a net payer if participants have sufficient initial capacity rights.

The demand and supply curves are expressed in steps of price and quantity pairs for the right to trade IC allowances; thus, each participant provides a piecewise linear approximation of a value function for changing IC allowances. Basically, the price and quantity pairs represent the characterization of participants in terms of location, IC allowances, control technologies, BMPs, and development projects on their properties.

As with the proposed market described in Chapter 3, the Sto_MarketIC model is not formulated to select which technology each participant should use, and so the clearing formulation does not have integer decision variables. Consequently, it avoids corresponding non-convexity issues and the related problems with prices.

The Sto_MarketIC allows participants to bid in advance according to an auction schedule which would depend on the hydrological season, flooding period, and the main economic activities. For instance, a catchment comprised of farms would run with regard to the agricultural season, e.g., two or three times per year, but in an urban catchment, the market may run monthly or annually. As the optimal timeframe depends on the catchment land uses, this should be evaluated regularly by the authority. Outside the market, participants may evaluate the opportunity costs of different options to control their own runoff, whilst satisfying the SO, and considering that they are not going to change their initial runoffs. To simplify the IC calculation, the land cover could be estimated online by implementing a web site administrated by the SO.

Boundaries in each control point would be set in terms of capacity at channels, pipes and streams, as well as flooding components such as depth and duration. Based on those boundaries, the authority will approximate mitigation costs and flood damage for extreme events. The resulting damage depends on the storm scenario, described in the next section.

Additionally, if the market is cleared every year and allowances account for a year, the flood damage corresponds to the expected flood damage in the year. However, in a market with longer timeframes and allowances, the flood damage cost may account for the present value of the damage for these events. The scenarios will account for the combination of these events. The following section will focus on a year timeframe and leave for future research the market that accounts for longer IC allowance periods.

4.5 A piecewise convex approximation to flooding cost

In modelling a system with hydraulic flow movements, flooding and flood damage can be non-linear and non-convex with respect to decision variables which would require convexifying the function. In approximating a non-linear function, the piecewise linear approximation approach can be used.

Beale et al (1970) introduced an ordered set of variables and constraints that force a consecutive set to be positive; accordingly, a global optimal solution over a non-convex piecewise linear function could be found. That approximation requires integer variables. An NLP with linear constraints and convex piecewise approximation in the objective function can be solved efficiently as a LP. Similarly, Hogan (2002) pointed out that an economic market solution relies on local linearisation in an electricity dispatch. Midthun et al. (2009) and De Wolf and Smeers (2000) noticed that linearisation allows feasible solutions in large scale networks, such as a gas market with linearised gas flows. Recently, Pepper et al (2012) showed a clearing market engine in the Victoria gas market which uses successive multi-dimensional piecewise linearisation in the constraints. Dupacová et al (1991) reported that piecewise linear costs, based on simple algorithms, can be used to solve water management problems.

Van der Vlerk (2002; 2004) proposed a convex approximation of the expected recourse cost function. The author divided the penalty cost function into a one-sided or two-sided function to obtain a function for shortage and surplus costs. This approach would be equivalent to estimating the flood damage using a one-sided cost penalty approach.

In using a piecewise linear approximation, the expected trading combination of IC allowances will raise or lower the total peak flows, and the flood damage will move along the damage curve. The piecewise convex approximation may model this movement as illustrated in Figure 4-3.

The piecewise approximation[10] could handle the nonlinearity and could be approximated in a convex way when having non-convexity in the range. A similar implication was noted by van der Vlerk (2002) and by Pepper et al. (2012), who used successive linearisation to deal with non-convexities. However, even though a convex hull could be obtained, the real problem is still non-convex. In the IC market, a piecewise approximation implies forcing a convex feasible region into a particular scenario, such as B shown in Figure 4-3.

For a particular rainfall scenario, the final flooding is defined by the market. Cost would be between a minimum and maximum flood damage that could be reached with the lowest and highest IC allowances respectively, for the trading combination and their flows. Thus, the market would be cleared between the peak flows $f_{k,1}$ and $f_{k,5}$, and linearised flood damage $C\left(f_{k,1}\right)$ and $C\left(f_{k,5}\right)$ (see Figure 4-4). Thus, $f_{k,r}^{s}$ is the flow in the break point in the range *r*, and $C\left(f_{k,r}^{s}\right)$ is the damage evaluated at these break points. The approximation accounts for a number of ranges *R*, and $\frac{C\left(f_{k,r}^{s}\right)-C\left(f_{k,r-1}^{s}\right)}{f_{k,r}^{s}-f_{k,r-1}^{s}}$ is the slope that represents the marginal damage in this range. The illustration also shows a rough linear approximation (dashed line) which may be generated by the Taylor expansion, and by splines choosing knot points (e.g., Fox 2008).

[10] A piecewise linear approximation of non-linear continuous function *g* in $\Re$ is linear in each interval $\left[c_r, c_{r+1}\right]$. Thus, for real numbers $c_1 < c_2 < \ldots < c_R$, and $r = 1,\ldots,R$, there is a linear slope in the interval for which c_r is a break point, and from this variable the slope changes in the next range interval $r + 1$. For a convex function *g*, the piecewise approximation is also convex.

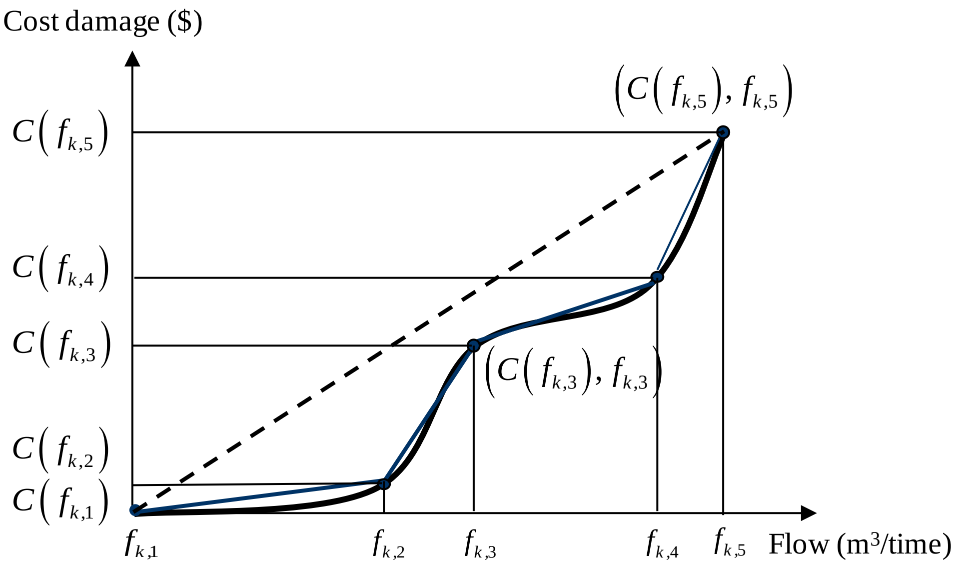


Figure 4-4 Piecewise linear cost approximation of a convex function for flow vs. damage cost.

4.6 Market model for IC, Sto_MarketIC

The Sto_MarketIC model approximates the flood damage by using a flood cost which relates flood parameters to damage. This market assumes that i) no flow losses and no non-source flows are injected in the system during a storm event; however participants have initial allocations; ii) storms in the chosen scenarios are the most representative in the catchment and they affect the area as a whole; iii) the flood damage cost function does not change during the planning horizon, which implies that changes such as new investments in infrastructure and real estate projects in the flooding area are ignored in the flood cost function estimation; iv) the market is perfectly competitive; v) the base flows remain unchanged in channels and streams, and the soil moisture is constant over the period.

4.6.1 Damage and flood cost

The market-clearing model will take into account violations at the control points due to flows at channel, stream, and floodplain areas above threshold capacities. In that case, the catchment could have flood damage as shown in Figure 2-13 A and Figure 2-16 B (Chapter 2), and the flood damage will be estimated by the maximum peak flow. The peak flow is related to the maximum flood depth (stage flood) in the flooded location; this implies the maximum damage. This assumption could be debatable since hydraulic processes at the channel control point may not necessarily be related to the flood in the surrounding area; however, this is an acceptable approximation due to the relationship between peak flow and stage flood (Loucks et al. 2005). Damage is also a function of duration of inundation. However, this could be considered as a referential flood cost related to peak flow.

In places such as streams and rivers, the maximum peak is closely related to the maximum depth, so it is possible to assume a direct relationship between peak flow and maximum flood depth. Several studies have a similar approach relating maximum depth and damage estimates (Hannan and Goulter 1985; Smith 1994; Dutta et al. 2000; 2003; Herath 2003). Figure 4-5 shows the peak flow at a control point. The maximum peak flow at a control point can also be linked to the spatial distribution of flooding in the area, which would include damage of the whole affected area (see Figure 4-4).

The market-clearing formulation considers penalties based on peak flows from established thresholds at each control point. In an electricity market, ancillary services (electricity reserves) are used in response to contingency events (Chattopadhyay et al. 2003; Chen et al. 2005; Read 2010). Chen et al. (2005) minimises the expected cost of energy and reserve cost for such contingency events; thus, the market operator estimates an optimum spinning reserve schedule to cover contingencies. The authors defined probabilities for each contingency event, which may affect prices as was also pointed out by Read (2010). The Sto_MarketIC will penalise such contingency events, rather than mobilising reserves to cover unexpected power needs in electricity markets. Thus, the Sto_MarketIC1 will penalise using a flood cost for the maximum stage-flood.

A flood damage cost function $C(f)$ accounts for a peak flow (maximum flood depth) at control points across scenarios. This flood cost will be used in the TSSP market-clearing model (Sto_MarketIC1). The following section presents additional market assumptions as well as the Sto_MarketIC1 model.

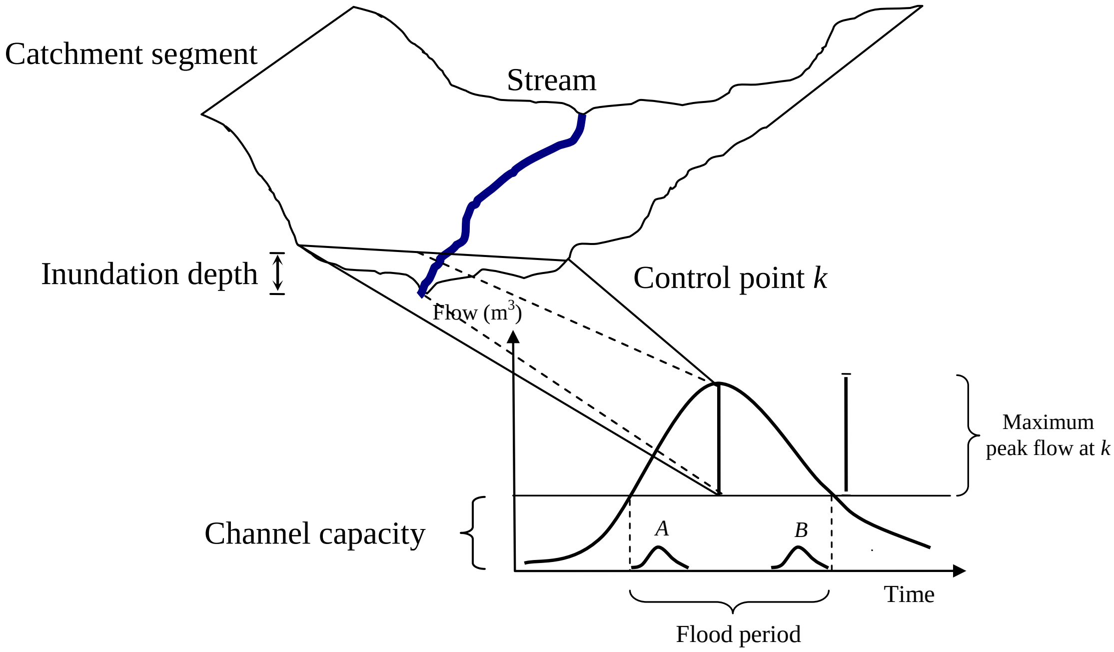


Figure 4-5 Total flows and flooding. Flood flows are above channel capacity. *A* and *B* are individual impacting flows from contributing properties to control point *k*.

4.6.2 Market-clearing model Sto_MarketIC1

This section presents the primal formulation of the clearing market model with the nomenclature previously introduced with the Det_MarketIC2 models in Chapter 3. In addition, it is assumed that flood damage is estimated for each storm scenario by the violations to capacities, and that any technology used to control runoff is perfectly divisible. The main source of stochasticity are the storm scenarios. It is recognised that other sources of uncertainty exist, but the focus is on the storm uncertainty. The expected flood damage is convex under changes in imperviousness, as outlined previously in Chapter 2 and Section 4.5 of this chapter. The base flows are kept relatively constant over the period in channels and streams. Participants have perfect knowledge regarding outcomes of their decisions and agree to the outcomes from simulations. Participants are demanding (offering) changes to their IC allowances on their properties based on their initial positions. The initial positions correspond to a status quo of initial IC allowances on their property. The market clearing model accounts for these differences. Participants cannot bid for changes more than their initial IC allowance within the area. If participants bid for a set of possible changes in allowances within their properties, the market formulation should include conditions similar to [3.24] in Chapter 3. Nodal prices vary over time, across network locations and storm scenarios at places where flooding is an issue (channels, streams, and floodplain areas).

The market-clearing model can be formulated by its equivalent deterministic formulation, which accounts for a finite set of realizations of the random vector of storms s=1,…,S, and their related probabilities ϕ^s. Thus, the market clearing can be as follows.

Indices

i = Participant, 1,...,N.

j,m,n,p= Land type (CN or imperviousness), j= 1,...,J.

b = Bid step, b=1,...,B.

k,l = Control point (node), k=1,...,K.

t,u,r = Storm time period, t=1,…,T.

s = Scenarios of rainfall (storms), s= 1,…,S.

Parameters

$A^0_{i,m}$ = Total initial IC allowance type m owned by participant i who desires to buy IC (ha).

$A^0_{i,n}$ = Total initial IC allowance type n owned by participant i, who desires to sell IC (ha).

$D^{\max}_{i,j,b}$ = Maximum area in IC allowance to change to allowance type j (ha) that participant i in bid step b is willing to buy at price $P^D_{i,j,b}$.

$S^{\max}_{i,j,b}$ = Maximum area in IC allowance to change to allowance type j (ha) that participant i in bid step b is willing to sell at price $P^S_{i,j,b}$.

$P^D_{i,j,b}$ = Demand price (\$/ha) for changing to IC allowance type j from participant i and bid step b. This is the maximum that participant i is willing to pay for a change from IC allowance type m to allowance type j and bid step b.

$P^S_{i,j,b}$ = Bid price (\$/ha) for changing to area allowance type j from participant i and bid step b. This is the minimum that participant i is willing to accept for a change from IC allowance type n to allowance type j and bid step b.

M_k = Flow capacity (volume/time) at channel control point *k*. The flow capacities depend on the channel sectional shape and the chosen base-flow at control point *k.*

$C_k^f\left(f_k^s\right)$= Flood cost at control point *k* in scenario *s*. This cost represents the flood damage under peak flow f_k^s in volume (m^3) at control point *k*. This damage will be incorporated as a cost ($ volume/time) and could be linearly approximated as was outlined in section 4.5.

ϕ^s = Probability of storm in scenario *s*. This parameter satisfies the following properties: $0 \le \phi^s \le 1$, and $\sum_{s=1}^{S} \phi^s = 1$.

$Q_{k,t}^{0,s}$ = Initial total flows at control point *k* across time *t* and scenario *s*. These flows are related to the initial IC allowance.

$H_{i,j,k}^{t-u+1,s}$ = Marginal impact at control point *k* of IC allowance type *j* from participant *i* and scenario *s,* at the end of the time *t* –*u*+1. *u* is the lag time between the storm time and the flow which reaches the control point (volume/time ha) and scenario *s*, as well. This coefficient relates conditions of the participant's property with the impact at control points across rainfall scenarios, e.g., volume/time/ha. This linear coefficient is likely to depend on initial IC allowance conditions from the property within the catchment, so it should be updated as IC allowances change. This marginal impact can be positive or negative. If participant *i* does not impact control point *k*, then $H_{i,j,k}^{t,s} = 0$.

Decision variables

$qbuy_{i,j,b}$ = Area in changing hectare to IC allowance type j and bid steps *b* bought by participant *i*.

$qsell_{i,j,b}$ = Area in changing hectare to IC allowance type j and bid steps *b* sold by participant *i*.

$g_{i,j}^D$ = Total hectares bought for changing IC allowance type j for participant *i* (ha).

$g_{i,j}^S$ = Total hectares sold for changing IC allowance to type j for participant *i* (ha).

$\mu^{D}_{i,j}$ = Price for buying participant *i* and changing to IC allowance type *j* (\$/ha).

$\mu^{S}_{i,j}$ = Price for selling participant *i* and changing to IC allowance type *j* (\$/ha).

$\phi^{s}\lambda^{s}_{t,k}$ = Price to discharge at control point *k*, time *t* and scenario *s* (\$ volume/time) only if flooding occurs.

f^{s}_{k} = Peak flow above maximum capacity under scenario *s* at control point *k* (volume/time). The recourse flood cost represents the flood damage for each scenario *s*.

Primal: **Sto_MarketIC1**

Maximize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{D}_{i,j,b} qbuy_{i,j,b} - \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{S}_{i,j,b} qsell_{i,j,b}$

$$-\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K} C^{f}_{k}\left(f^{s}_{k}\right) \qquad [4.2]$$

Subject to:

$$0 \le qbuy_{i,j,b} \le D^{\max}_{i,j,b},\ \forall\ i,j,b \qquad : \beta^{-}_{i,j,b}, \beta^{+}_{i,j,b} \qquad [4.3]$$

$$0 \le qsell_{i,j,b} \le S^{\max}_{i,j,b},\ \forall\ i,j,b \qquad : \gamma^{-}_{i,j,b}, \gamma^{+}_{i,j,b} \qquad [4.4]$$

$$\sum_{b=1}^{B} qbuy_{i,j,b} = g^{D}_{i,j},\ \forall\ i,j \qquad : \mu^{D}_{i,j}\ \text{(free)} \qquad [4.5]$$

$$\sum_{b=1}^{B} qsell_{i,j,b} = g^{S}_{i,j},\ \forall\ i,j \qquad : \mu^{S}_{i,j}\ \text{(free)} \qquad [4.6]$$

$$Q^{0,s}_{k,t} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^{D}_{i,j} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^{S}_{i,j} \le M_{k} + f^{s}_{k},$$

$$\forall\ t,s,k \qquad : \phi^{s}\lambda^{s}_{t,k} \qquad [4.7]$$

$$f^{s}_{k} \ge 0, \qquad : \theta^{s}_{k} \qquad [4.8]$$

$$x^{D}_{i,j},\ x^{S}_{i,j}\ \text{(free)} \qquad [4.9]$$

Explanation

[4.2] The objective function maximizes the expected gains for trading IC, less the costs for flood damage under different rainfall events. This formulation considers a discrete probability distribution of rainfall events, and recourse cost for flood damage at different control points. The objective function does not measure the absolute welfare, but changes in the objective are the appropriate measure of changes in welfare (assuming the market is sufficiently competitive that offer/bids reflect marginal opportunity costs). It is recognised that participants could try to bid strategically; but this issue is beyond the scope of this thesis.

[4.3] Total changes of IC allowances that are bought in each tranche are bounded by demand quantities.

[4.4] Total change of IC allowances sold in each tranche is also bounded by bid quantities.

[4.5] The final area bought of changing to IC allowance type j of participant *i*.

[4.6] The final area sold of changing to IC allowance type j of participant *i*.

[4.7] For each scenario *s*, the total flows at control point *k* in time *t* should be lower than the flow capacity M_k. However, flows may violate the capacity M_k and f_k^s which estimates the peak flow above capacity in the channel area across scenarios. In several scenarios $Q_{k,t}^{0,s} \geq M_k$, this means there is no chance of avoiding a flood in these storms, just an ability to reduce the damage caused for the flows. This constraint deals with possible upstream participants that are prepared to pay for the flood damage cost.

This general representation corresponds to the flows from participant *i* ($Q_{i,k}^{0,t,s}$) related to their initial IC allowances plus a flow gradient $\left(\Delta_{i,j,k}^{t,s}\right)$ for the changes in IC allowances, with the gradient being positive or negative across scenarios. Figure 4-6 illustrates this condition and two linear approximations across scenarios. This is just one dimension of a multi-dimensional surface; $G_{i,j}$ corresponds to the initial IC allowances, and $Q_{0,k}^{t,s=1}$ and $Q_{0,k}^{t,s=2}$ are flow levels related to the scenario *s* =1,2; $h_{0,k}^{t,s}$ and $\overline{h}_{0,k}^{t,s}$ are the calculated intercept flows of the linearisations. Any change in IC allowances would increase or reduce the flows from the initial flow conditions by $H_{i,j,k}^{t-u+1,s}\, g_{i,j}$.

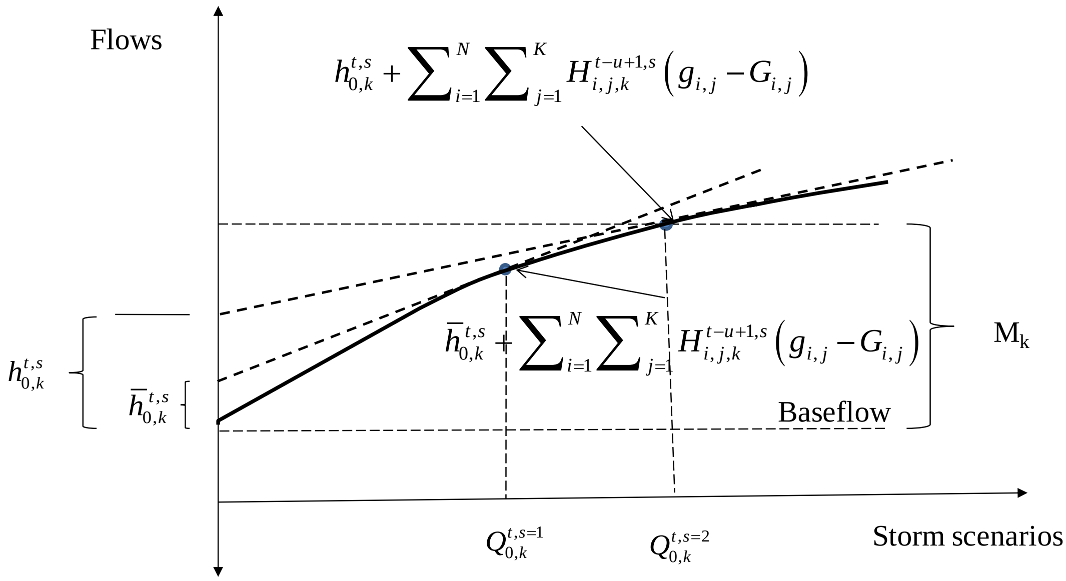


Figure 4-6 Impact of flow linearisation for two storm scenarios

[4.8] Condition of non-negative flows above capacities.

[4.9] Quantities traded in the market as well as the initial allowance (area) with the initial IC allowance must be non-negative. This will limit the final allowance allocations, $g_{i,j}^{D}$ and $g_{i,j}^{S}$.

The recourse flood can have different piecewise linear approximations. Firstly, $\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}\sum_{r=1}^{R}\alpha_r^s\ f_{k,r}^{s}$ where α_r^s is the marginal damage in range *r* and scenario *s*, with a constraint for exceeding peak flows $0 \le f_{k,r}^{s} \le f_{k,r}^{\max,s}$ $\forall$ *k,s,t*. Secondly, with the Lambda method (Beale and Tomlin 1970; Lee and Wilson 2001), where the damage function becomes $\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}\sum_{r=1}^{R}D_{k,r}^{s}\, l_{k,r}^{s}$, and the variable f_k^s from [4.7] is $\sum_{r=1}^{R}F_{k,r}^{s}\ l_{k,r}^{s}$; where $D_{k,r}^{s}$ is the damage at grid point *r*, $l_{k,r}^{s}$ is a variable introduced for each grid point *r*, $F_{k,r}^{s}$ is the flow (flood level). Additionally, a convexity constraint on $l_{k,r}^{s}$ should be introduced in the formulation. However, this formulation may have non-convexity problems (Beale and Tomlin 1970). Fortunately, a smooth flood damage cost can be found by convexification and piecewise approximation.

$$\sum_{r=1}^{R} l_{k,r}^{s} = 1, \qquad \forall\ k,s \qquad\qquad : \phi^{s}\varpi_{k}^{s} \qquad [4.10]$$

$$\sum_{r=1}^{R} F_{k,r}^{s}\, l_{k,r}^{s} = f_{k}^{s},\ \forall\ k,s \qquad\qquad : \phi^{s}\overline{\varepsilon}_{k}^{s} \qquad [4.11]$$

$$l_{k,r}^{s} \geq 0,\ \forall\ k,s,t \qquad\qquad : \tilde{\omega}_{k,r}^{s} \qquad [4.12]$$

4.6.3 Price analysis

A price analysis can be derived from the dual of Sto_MarketIC1 (the dual in Appendix A is presented in terms of the flood damage cost and its derivative).

The objective function [4.2] provides a net benefit measurement, which is fixed given the primal solution. The duals specify how this wealth is distributed among participants. The dual variables' definitions and implications are given in the following analysis and discussion.

The clearing prices associated with constraint [4.7] correspond to a set of flows received at control points, by time and by storm scenario, and will be non-zero for those flows at or above threshold capacities. The prices $\phi^{s}\ \lambda_{t,k}^{s}$ are the changes in the expected flood damage that may occur in the area.

The shadow price $\mu_{i,j}^{D}$ for constraint [4.5] is the marginal expected value for another unit *j* due to the changes in IC allowances *j* to participant *i*. This shadow price accounts for the expected marginal flood damage due to increasing IC allowance, and also for changing flow patterns at the control points and scenarios. In other words, the dual variable $\mu_{i,j}^{D}$ corresponds to a weighted set of flow impacts across all scenarios and period peaks *t*. Thus, the applied price that will be used to charge participant *i* will be:

$$\mu_{i,j}^{D} = \sum_{s=1}^{S} \phi^{s} \sum_{k=1}^{K} \sum_{t=u}^{T} H_{i,j,k}^{t-u+1,s}\,\lambda_{t,k}^{s},\ \forall\ i,j \qquad\qquad : g_{i,j}^{D} \qquad [4.13]$$

The shadow price $\mu_{i,j}^{S}$ in constraint [4.6] represents the cost to the system, in terms of marginal surplus or the marginal change in the expected flood damage, for an extra unit of changes in IC allowance from *n* to j in the area. The dual variable is $\mu_{i,j}^{S}$ is the applied price to pay participant *j*:

$$\mu^{S}_{i,j} = \sum_{s=1}^{S} \phi^{s} \sum_{k=1}^{K} \sum_{t=u}^{T} H^{t-u+1,s}_{i,j,k} \lambda^{s}_{t,k} \text{ , } \forall \; i,j \qquad : \; g^{S}_{i,j} \qquad [4.14]$$

The condition [4.14] corresponds to the extra expected return for marginal changes in flows across time *t* at control point *k*. (In this case, this is expected to be negative, but a canonical expression is conveyed to be positive.) This value will be recalled in the next section when discussing its relationship with the flood costs.

Equations [4.13] and [4.14] account for all impacts at the subcatchment (nodal price) and common points for potential flooding as well as flooding peaks by period across scenarios. These prices are influenced by changes in flow impacts, the probabilities for extreme storm events and their relationships with the clearing prices, which in turn are linked to the flood damage. The last point will be discussed in the next section.

As similar to Det_MarketICs, participants need to establish their bids carefully, because their IC allowances change the flows and the peak flow times at control points across scenarios. In most cases, increased IC allowance means increasing total runoff, flows at peak times at control points and so participants pay for the imperviousness. But in particular situations participants might be paid to increase imperviousness, e.g., the increment in IC allowances avoids peak flows and so reduces flow at peak times; thus, the condition is $\sum_{s=1}^{S} \phi^{s} \sum_{k=1}^{K} \sum_{t=u}^{T} H^{t-u+1,s}_{i,j,k} \lambda^{s}_{t,k} < 0$ at the new peak time. This could be observed in properties close to the control points whereby participants could pay or receive almost nothing.

The trading condition could also be analysed from the next dual conditions.

$$\mu^{D}_{i,j} = P^{D}_{i,j,b} - \beta^{+}_{i,j,b} + \beta^{-}_{i,j,b} \text{ , } \forall \; i,j,b \qquad [4.15]$$

$$\mu^{S}_{i,j} = P^{S}_{i,j,b} + \gamma^{+}_{i,j,b} - \gamma^{-}_{i,j,b} \text{ , } \forall \; i,j,b \qquad [4.16]$$

If $0 < qbuy_{i,j,b} < D^{\max}_{i,j,b}$ and $0 < qsell_{i,j,b} < S^{\max}_{i,j,b}$, the optimal IC allowance changes for participants in the demand and supply side lie in step *b* from each, resulting in both being marginal. This means that neither the upper or lower bound for the demand and supply steps will at their bound; hence, by complementary slackness, $\beta^{+}_{i,j,b} = \beta^{-}_{i,j,b} = \gamma^{+}_{i,j,b} = \gamma^{-}_{i,j,b} =$ 0. The marginal conditions are $\mu^{D}_{i,j} = P^{D}_{i,j,b}$ and $\mu^{S}_{i,j} = P^{S}_{i,j,b}$, and correspond to the marginal clearing points between adjusted demand and supply curve steps in terms of impacting flows along control points and storms scenarios. Thus, a set of these marginal changes will be clearing, one for each bounded limit at each control point and scenario, and prices will value violations as will be seen next.

All other demand/offer steps will be either infra-marginal or supra-marginal, and these shadow prices will balance the equations [4.15] and [4.16], because participants' marginal values (demand and supply bid prices) have exceeded the costs imposed on the rest of the system for increased flooding. Thus, when $qbuy_{i,j,b} > 0$ and $qsell_{i,j,b} > 0$, so by complementary slackness $\beta^{-}_{i,j,b} = \gamma^{-}_{i,j,b} = 0$, the surpluses in the demand and supply for participants in step *b* are $\mu^{D}_{i,j} = P^{D}_{i,j,b} - \beta^{+}_{i,j,b}$ and $\mu^{S}_{i,j} = P^{S}_{i,j,b} + \gamma^{+}_{i,j,b}$.

If the market model accounts for an equivalent constraint [3.24], i.e., the participant could bid in a set of possible changes in IC allowances, the dual variables related to this constraint will affect the final trade. Thus, the dual equations [4.15] and [4.16] are as follows:

$$\mu^{D}_{i,j} = P^{D}_{i,j,b} - \beta^{+}_{i,j,b} + \beta^{-}_{i,j,b} - \nu^{D}_{i,j=m}, \forall\ i,j,b \qquad [4.17]$$

$$\mu^{S}_{i,j} = P^{S}_{i,j,b} + \gamma^{+}_{i,j,b} - \gamma^{-}_{i,j,b} + \nu^{S}_{i,j=n}, \forall\ i,j,b \qquad [4.18]$$

The new dual conditions could be seen as $\mu^{D}_{i,j} = P^{D}_{i,j,b} - \nu^{D}_{i,j=m}$ and $\mu^{S}_{i,j} = P^{S}_{i,j,b} + \nu^{S}_{i,j=n}$ with the opposite implication from $\nu^{D}_{i,j=m}$ and $\nu^{S}_{i,j=n}$. Thus participants who want to increase imperviousness will face an additional cost for their preferred IC allowance conditions, with the dual prices as the marginal value of IC allowance. This internal condition may increase the opportunity cost of increasing IC allowances, especially in areas with high opportunity cost such as city centres. Thus, they could not easily increase their IC allowances, and they could be encouraged to adopt BMPs.

4.6.4 Prices relating to flood costs

Dual prices can be decomposed into the marginal expected flood damage across control points and scenarios. These prices are linked to the flooding costs $C^{f}_{k}\left(f^{s}_{k}\right)$ at control point *k*, as well as with the rainfall and its probability of occurrence ϕ^{s} in scenario *s*. In this situation, the dual accounts for the marginal expected changes in the flooding distribution.

From the dual Equations [4.13] and [4.14], as well as from the next dual Equation [4.19], new price conditions are obtained for $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$.

$$-\phi^{s}\sum_{t}^{T}\lambda_{t,k}^{s}-\theta_{k}^{s}=-\phi^{s}\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}\quad,\forall\ k,s\qquad:\ f_{k}^{s}\qquad[4.19]$$

Notice that if $\lambda_{t,k}^{s}$ > 0, the new condition will be $\sum_{t=1}^{T}\phi^{s}\lambda_{t,k}^{s}=\phi^{s}\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}$ due to complementary slackness where θ_{k}^{s} = 0. In addition, given that the flood cost takes into account the maximum flow, the condition $\sum_{t=1}^{T}\phi^{s}\lambda_{t,k}^{s}$ would be bounded in time $t^*=t^*(k,s)$. Assuming that the peak is reached for each scenario and control point, the condition becomes $\phi^{s}\lambda_{t^*,k}^{s}$ in the scenario *s*. The clearing price $\lambda_{t,k}^{s}$ accounts for the marginal flood damage cost for an additional unit of peak flow at a control point. Thus, as the value of $\phi^{s}\lambda_{t^*,k}^{s}>0$, the dual condition will be $\lambda_{t^*,k}^{s}=\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}$ and the dual prices will be as follows:

$$\mu_{i,j}^{D}=\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}H_{i,j,k}^{t^*,s}\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}\ ,\forall\ i,j\qquad[4.20]$$

$$\mu_{i,j}^{S}=\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}H_{i,j,k}^{t^*,s}\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}\ ,\forall\ i,j\qquad[4.21]$$

The prices generated from equations [4.20] and [4.21] correspond to the expected marginal flood damage increment (or reduction), at peak time *t**, across the rainfall scenarios. Since participants are trading IC allowances, these prices represent the marginal changes in flows at the peak of flooding, at control points and the resulting marginal flood costs for extreme storms.

In the dual conditions of Det_MarketIC models described in Chapter 3, clearing prices were related to established storms and thresholds, and the dual prices were related to the cost to the system under an extreme storm condition. However, in this case, the dual variables $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ depend on the expected market clearing values along discrete rainfall scenarios.

4.7 Settlement

The participant's settlement depends on their initial IC allowance and the applied prices $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ account for the marginal changes in the expected flood damage. This non-arbitrage- price condition does not depend on what flow occurs in a given year nor for the flood realisation. The condition actually accounts for the changes in the expected flood damage at control points in the catchment, and the market may be revenue inadequate. This condition will be discussed in Section 4.9 in Chapter 4, and an adjusted method to reach revenue neutrality will be proposed in Section 8.9 in Chapter 8.

The market model allocates with prices $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ for each participant. The participant's settlement, r_i; for those who are demanding to increase IC allowances, is $r_i = \sum_{j=1}^{J} \mu_{i,j}^{D}\, g_{i,j}^{D}$. For those participants who are offering to reduce IC allowances, $r_i = \sum_{j=1}^{J} \mu_{i,j}^{S}\, g_{i,j}^{S}$. The settlement considers that participants have rights for the initial IC allowances. The settlement is $r_i > 0$ for those who are increasing IC allowances, and $r_i < 0$ for those who are reducing IC allowances.

4.8 Hedging against storms

Participants and the SO will observe that the catchment is hedged against a range of storms defined by the rainfall scenarios. In this research, a hedging level corresponds to a range of storm scenarios that would not produce any flooding damage. Figure 4-2 shows the effect of hedging: in the initial market condition Ed, participants are not internalising flood damage, nor controlling runoff. This can be interpreted as the participants having the rights to damage in a specific flood distribution. During a market, prices will be the signal to change impervious levels, and to allocate impervious levels until the marginal surplus of trading matches the marginal expected flood damage. In essence, this is a classic competitive market equilibrium in which the marginal benefit of consumption (the demand curve) equals the marginal cost of supply. In Figure 4-2, the hedging effect corresponds to

Ed → Ed’ → Ed’’; accordingly, the damage will start from an extreme storm, producing changes in the probability distribution of flooding in the catchment.

Figure 4-7 illustrates hypothetical changes in flood probability distributions with different final impervious levels in the catchment. Commonly, an increase in the final impervious levels would increase the probability of damage. For example, an increased IC would result in 100 mm of rainfall within a 24 hour storm reaching similar damage level to of 150 mm rainfall in a 24 hour storm prior to the increased impervious levels.

The hedging will produce different effects for participants and the SO, depending on trade conditions. Next, cases are presented that summarise different trading situations, price conditions, and final hedging.

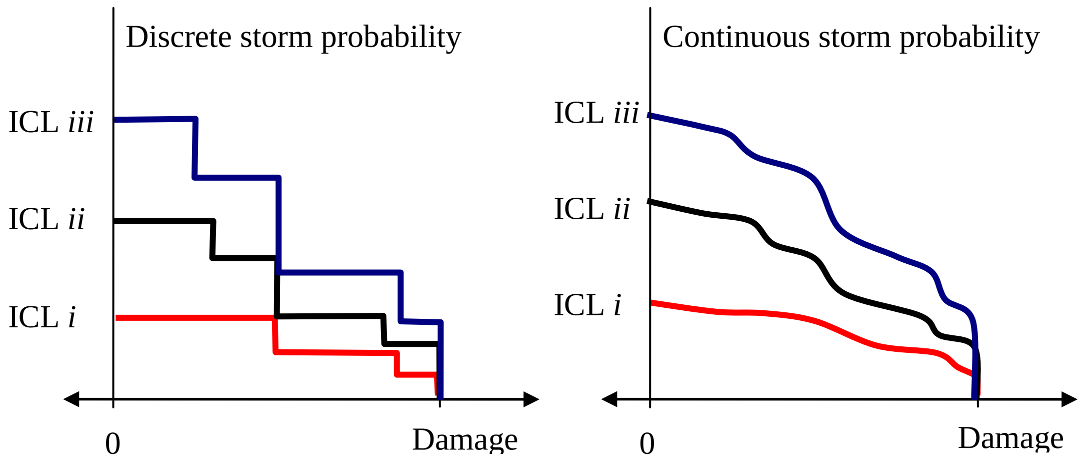


Figure 4-7 Hypothetical flood damage probability density functions with different IC allowances (ICL) in the catchment. *i*, *ii* and *iii* are incremental final IC allowances which raise flood distribution in the catchment.

4.8.1 Trading cases

Case 1: If a participant desires to increase their imperviousness, increased runoff will result and peak flows will shift at control points in each storm scenario. The price that a participant pays should take into account the expected marginal increment in flood damage for both shifting effects. These effects would increase the clearing price at control points. If other participants are not trading, the marginal cost for increasing floods at each control point should be paid by the developer. As a result, the developer would pay for increasing flooding damage within the flood areas. The price will correspond to the changes in area between ICL *iii* or ii and ICL *i* in Figure 4-7. The hedging changes will be paid by the participant who is increasing the IC allowance. Additionally, given the SO is overseeing the flood damage areas, the new flood condition will be priced and the anticipated incremental damage will be paid to the SO.

Case 2: A developer desires to increase the impervious levels which consequently will increase flows and flood damage across scenarios, but other participants desire to trade with the objective of reducing stage-flood times. Previous auctions had similar clearing prices, but with the new developer, participants may face higher clearing prices. Consequently, participants whose bids were previously not accepted are now able to trade. Developers may face higher clearing prices, and hence incur greater costs for their increasing flows. The price signals may encourage other participants to reduce their impervious levels within the catchment at the next auction.

Case 3: A developer desires to buy more IC allowances, which increases the peak flow, but previously, the capacity constraints were loose in some scenarios, and the catchment was hedged against a range of storm scenarios and flooding. To date, flooding was not an issue and trading participants paid almost zero. However, in changing the impervious condition, the participant would face rising clearing prices due to violations of capacity constraints. The expected flood damage would rise, and the hedging level would fall. As a result, the clearing price would rise, and participants who were used to trading IC allowances almost for free, now face higher prices.

Case 4: The current impervious levels within the catchment are high with serious flooding problems, and participants desire to trade IC allowances. In this case, different trading situations may occur. The SO may accept bids to reduce the expected cost of flooding for the purpose of reducing the peak flows. Thus, accepted bids would correspond to those allocations which reduce the expected flood damage more than the bid prices. Additionally, accepted bids may raise imperviousness, shifting peak time t^* towards time t^{**} where flooding is not a problem, as illustrated in Figure 4-7. In some trading scenarios, the shifting effect in peak period (from t^* to t^{**}) may reduce the final clearing prices $\phi^s \lambda_{t,k}^s > \phi^s \lambda_{t,k}^{s*}$ due to the reducing effect of flood damage, via hypothetical changes in flood patterns between A and B shown in Figure 4-8. In both situations, the market may work as a one-sided auction, and the SO would be a net payer for reducing impervious levels and hedging against a range of storm events.

Participants' marginal surplus for trading is greater than the marginal damage. Participants who increase IC would trade to the extent that their demand prices are greater than the expected marginal flood damage. In the same way, participants who reduce impervious levels would trade until their bid prices were lower than the expected marginal flood damage. Therefore, the final hedging condition will depend on the trading scenarios. A scenario that reduces flood damage will be hedged against storms with a corresponding movement in the probability function as illustrated in Figure 4-7, from ICL *iii* or *ii* to ICL i. Other trading scenarios could increase the flood damage and the movement in the probability density function would be the opposite (from ICL *i* to ICL *iii*), and the hedging level would be reduced. Any increments in the expected damage would be internalised for those participants who increase the damage, or change the flood damage distribution.

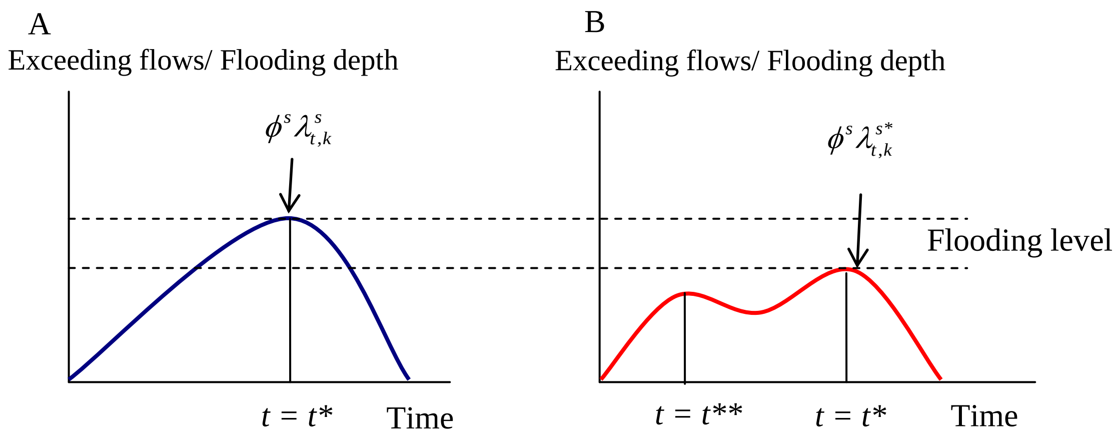


Figure 4-8 Flooding patterns with different exceeding flows and peak flow with a storm scenario; $\phi^s \lambda^s_{t,k}$ and $\phi^s \lambda^{s*}_{t,k}$ represents clearing prices with impervious levels A and B respectively; t is time.

In summary, different trading scenarios could occur in the catchment and consequently different hedging levels could be reached as well. Any changes in the flood probability distribution and consequently in the hedging level will be internalised for those participants who change their IC allowances. Thus, the changes in flooding would be priced and those prices would encourage participants to use more efficient BMPs and technologies, or to change IC allowances to avoid flood damage.

For each case presented above, hypothetical situations of trading IC allowances, property size, storm scenarios, control points and flood damage will be presented or analysed. The market timeframe is a year, so the recourse flood cost of the TSSP model accounts for the expected damage during the year. Piecewise convex functions will represent flood damage. The main flooding component will be the maximum flooding depth.

Ten participants are bidding in the market, with different impervious areas. Each participant has 20 ha and is able to change their IC allowances in 10 ha.

Table 4-1 presents initial IC allowances from participants. Figure 4-9 shows the catchment where participants inject their runoff to the system, as well as flooding control points CP1, CP2 and CP3. Lag periods between control points are assumed.

Table 4-1 Hypothetical IC conditions for ten participant properties

Participant	Initial area (ha)	Non trade area		Area to trade IC	
		IC condition	ha	IC condition	ha
1	20	Forest	10	Meadow	10
2	20	Forest	10	Crop	10
3	20	Crop	10	Crop	10
4	20	Concrete	10	Meadow	10
5	20	Concrete	10	Crop	10
6	20	Crop	10	Meadow	10
7	20	Concrete	10	Concrete	10
8	20	Concrete	10	Concrete	10
9	20	Crop	10	Crop	10
10	20	Meadow	10	Crop	10

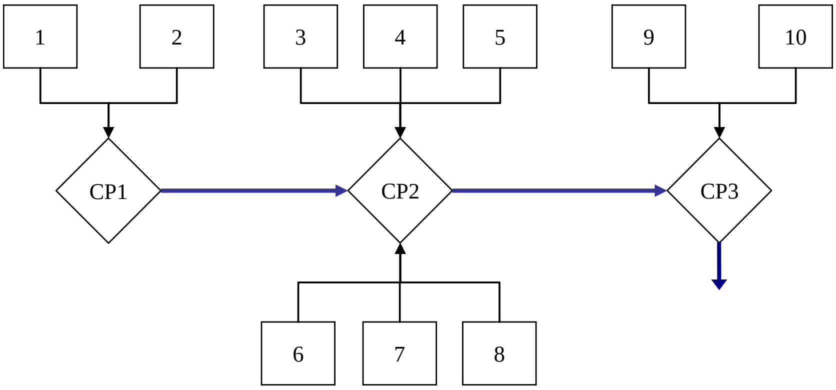
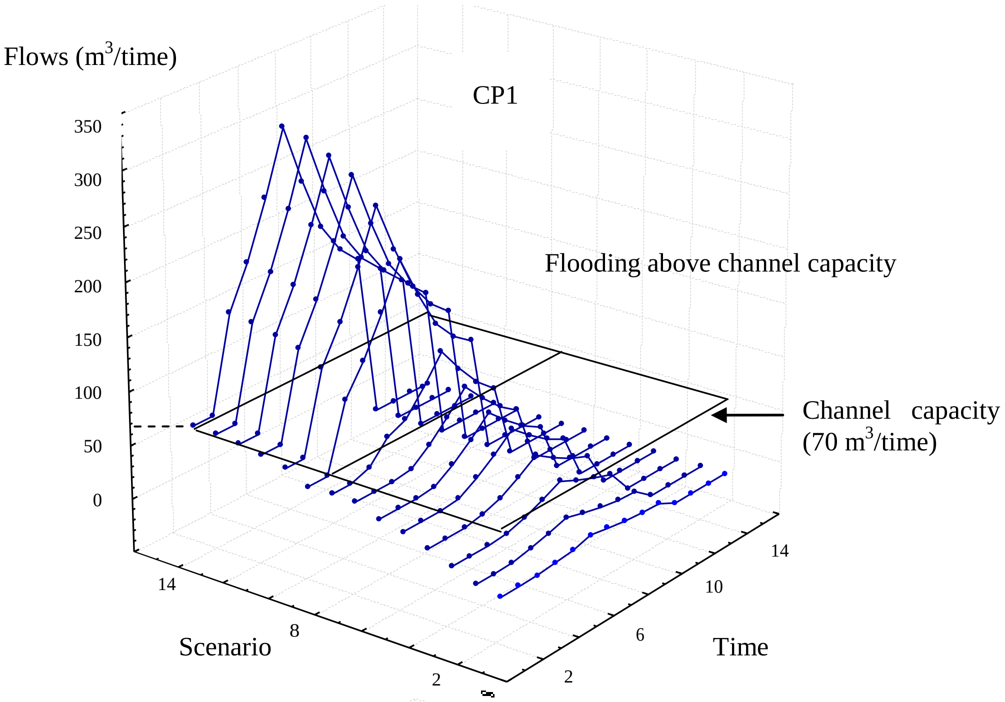


Figure 4-9 Hypothetical catchment showing properties (1-10), control points, and flow routing.

The catchment has flooding problems at CP1, CP2 and CP3. Different maximum flows can be reached with current IC allowances, across scenarios. Additionally, 14 storms are used to establish the market and their probabilities (in order of increasing storm intensity) are 0.399, 0.2, 0.15, 0.1, 0.07, 0.04, 0.02, 0.01, 0.005, 0.003, 0.0012, 0.0008, 0.0005, and 0.0005. Figure 4-10 illustrates the maximum flows reached across the 14 storm scenarios at control points CP1 and CP3. This assumes that storms affect different control points in the same way.

The flood damage cost functions relating to maximum peak flows are $Dm_1 = 0.22 f^2$, $Dm_2 = 0.00011 f^3$, and $Dm_3 = 0.000055 f^3$ at CP1, CP2 and CP3, respectively. Grid points for the piecewise flood damage cost function at CP1 are estimated as $f_i = \left(\frac{(F - I_0)}{0.22} \frac{n_i}{n_T} \right)^{0.5}$, where I_0 is the initial and F final damage values in the damage interval (I_0, F), n_i is the number of the grid points between $0,..,n_T$, and n_T is the number of ranges in the interval. The flood damage cost functions are piecewise linear approximated and the threshold capacities M_k at control points are 70, 230 and 250 m^3/time at CP1, CP2 and CP3 respectively. These threshold capacities correspond to the difference between channel capacity and the base flows.

Participants have different IC preferences, and we assumed that they have evaluated the opportunity costs of their changes in IC allowances. Those preferences are in Table 4-2



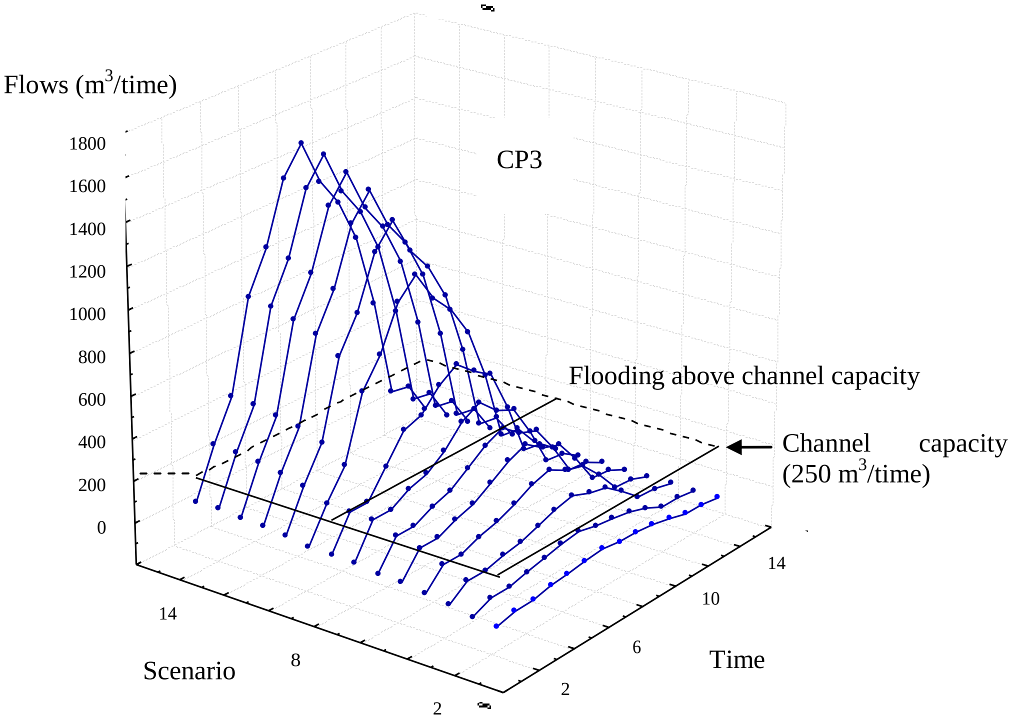


Figure 4-10 Initial maximum flow levels by time, at control points (CP) 1 and 3, across 14 storm scenarios.

Table 4-2 Participants' preferences for changing IC allowances

Particip.	Initial IC	Initial area (ha)	Impact at control point	Option 1			Option 2			Option 3		
				ha	IC	$/ha	ha	IC	$/ha	ha	IC	$/ha
1	Meadow (F)	10	1,2,3	5	Cr	$8	3	Cr	$7	2	Cr	$6
2	Crop (Cr)	10	1,2,3	5	Cn	$2	3	Cn	$1	2	Cn	$1
3	Crop (Cr)	10	2,3	5	Cn	$9	3	Cn	$7	2	Cn	$5
4	Meadow (M)	10	2,3	5	Cn	$10	3	Cn	$8	2	Cn	$6
5	Crop (Cr)	10	2,3	5	Cn	$11	3	Cn	$8	2	Cn	$7
6	Meadow (M)	10	2,3	6	F	$2	3	F	$8	1	F	$9
7	Concrete (Cn)	10	2,3	6	M	$4	3	M	$7	1	M	$10
8	Concrete (Cn)	10	3	6	Cr	$5	3	Cr	$8	1	Cr	$9
9	Crop (Cr)	10	3	6	F	$7	3	F	$10	1	F	$12
10	Crop (Cr)	10	3	6	F	$5	3	F	$10	1	F	$15

Figure 4-11 illustrates the exceeding maximum flows at control points and the damage across storm scenarios, where CP1 has a lower probability of flooding than CP3 as well as lower damage. The expected flood damage in the catchment is $1,303.50, divided into $94.49, $445.06 and $763.98 at CP1, CP2 and CP3 respectively. In current conditions, CP1 and CP2 are hedged up to storm type 4, and CP3 up to storm type 3.

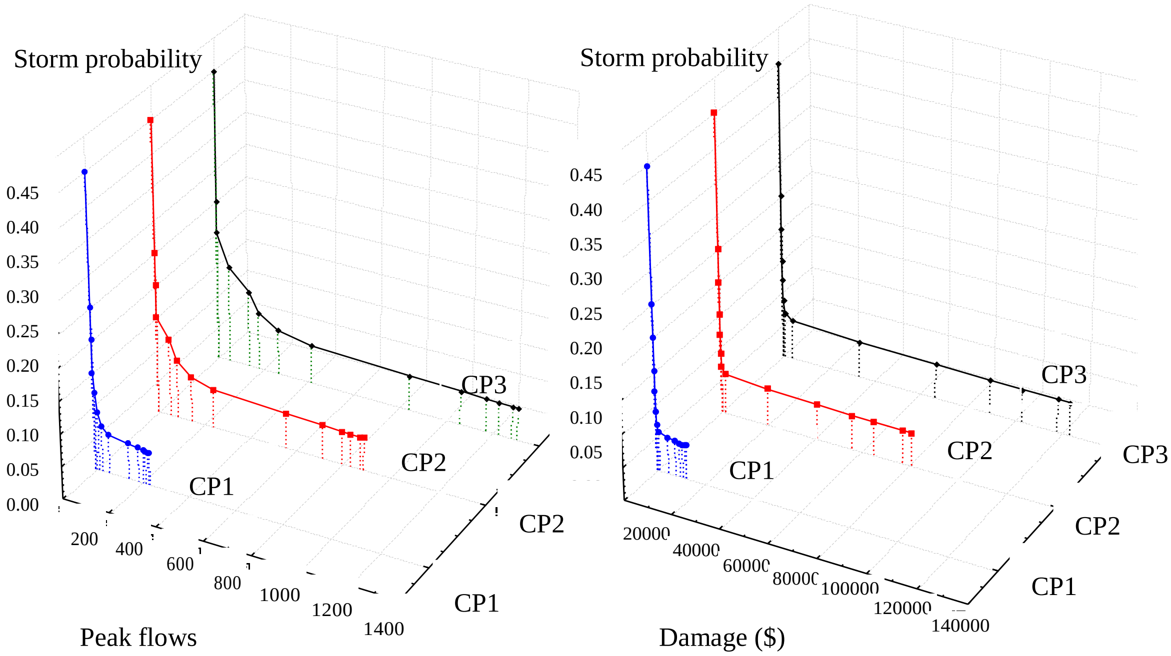


Figure 4-11 Hypothetical initial conditions of peak flows and flood damage at CP1, CP2, and CP3. On the left, storm probability vs. peak flows, and on the right, storm probability and flood damage costs.

Case 1

In this case, participant 2 desires to change IC allowances from the current IC allowance "crops" to "concrete". The expected flood damage rises to $1,350.40. The participant has a maximum willingness to pay $20, $16 and $14/ha for increasing 5, 3 and 2 ha to "concrete". Because the participant bids high enough, he/she could change IC allowance in the 10 ha to a final payment of $48.54. The expected flood damage is increased by $46.90, which is a little lower than the total payment of participant 2. This difference is due to the piecewise approximation of flood damage, and in this case the SO would obtain extra revenue.

On the other hand, if the participant bids low prices such as $6/ha, $3/ha and $2/ha, the market would clear with the participant only being able to change 5 ha, paying $23.64. The expected damage in the catchment would be $1,326.50 (rising by $23).

Case 2

This case considers an established market where small changes in flood damage have been noticed and traded through the market. Participant 2 desires to develop a project which would change the flooding in the catchment. If participant 2 was not in the market or bid low prices, initially participants 1, 3, and 5 increase IC allowances by 10 ha to "crop", 5 ha to "concrete", and 8 ha to "concrete" respectively. In contrast, participants 6 and 7 reduce IC allowances by 6 and 10 ha respectively. The final expected damage costs are $1,344.60. Let us see changes in prices, allocations and flooding if participant 2 bids higher by some x to increase IC allowances. Assume bid prices \$1.50×x, \$1.20×x and \$$x$/ha, for respective quantities 5, 3 and 2 ha, and assuming x between 1 and 6, the following will be observed:

- Participant 1 who already changed 10 hectares to "crop" could increase only 8 ha, since the clearing prices were raised by participant 2's bid.
- Previously, participant 2 did not change IC; but now with a high demand price, the participant could change 10 ha to "concrete". These changes shift the peak flows and the damage across scenarios and at control points CP1, CP2 and CP3. Figure 4-12 illustrates these effects in the peak flows and flood damage at CP1. Thus, with incremental IC the expected damage increases from $102.60 to $125.90 at CP1, and from $1,344.60 to $1,380.76 at the catchment level.

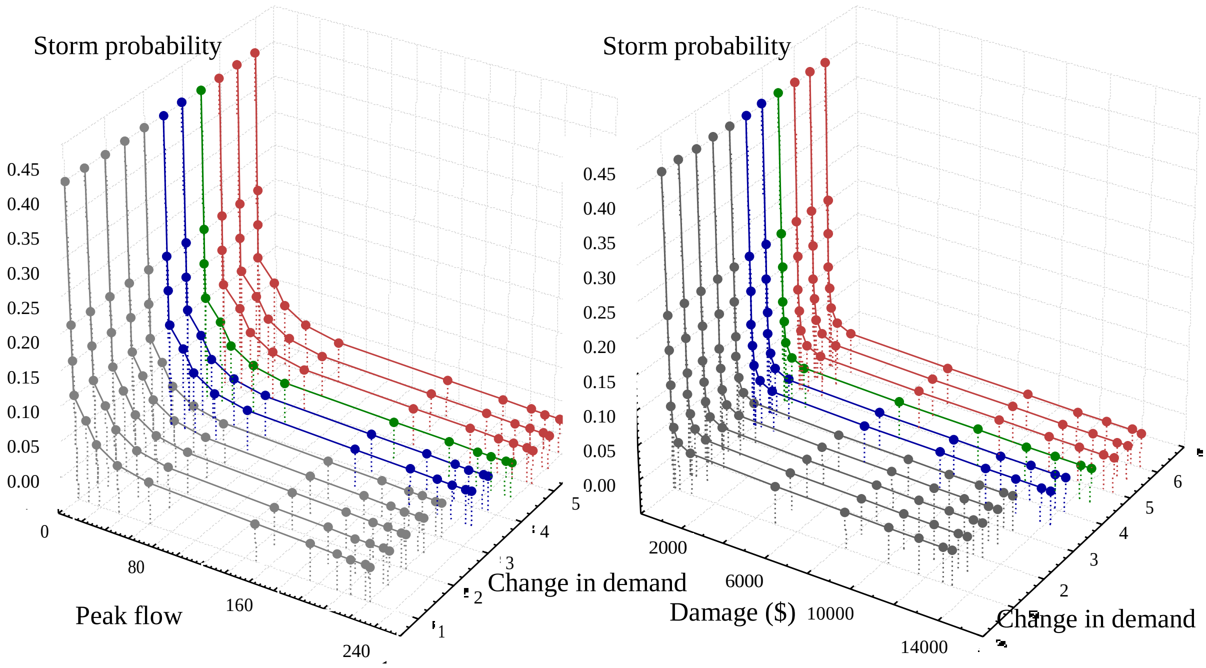


Figure 4-12 Changes in peak flow and flood damage at CP1 under changes in demand from participant 2.

- Changing demand changes clearing prices across storm scenarios. Figure 4-13 illustrates the changes in clearing prices at control points 1 and 3. Participant 2 is at the CP1 area, so changes in clearing prices are stressed at CP1, due to the changes in the incremental flood damage and that no participants reduce IC allowances at CP1. At control point CP3, prices changed smoothly due to the trade-off between the marginal flood increments and the incremental surplus for changing impervious levels, as well as the piecewise linear approximation to the damage function. The solid circle in Figure 4-13 shows the change in the clearing price at the CP3.

- This section describes the scenario selection, the marginal effects for the extreme storm events and the minimum marginal contribution to the final price (see Section 4.3). The last two storm scenarios (scenarios s13 and s14), which correspond to the dashed circles in Figure 4-13, show the reducing effect of the marginal increment of flood damage. In these scenarios, the duals correspond to \$0.052 and \$0.062 at CP1; \$0.118 and \$0.149 at CP2; and, \$0.133 and \$0.133 at CP3 (\$/peak flow) (see Table 4-3). The marginal contribution is reduced by the storm probability; however, in this example the effect is not emphasised and could easily be observed when the marginal flood damage is almost zero. Additionally, at CP1 the observed final total damage and the expected damage are \$14,639.90 and \$15,137.80, and \$7.31 and \$7.46.

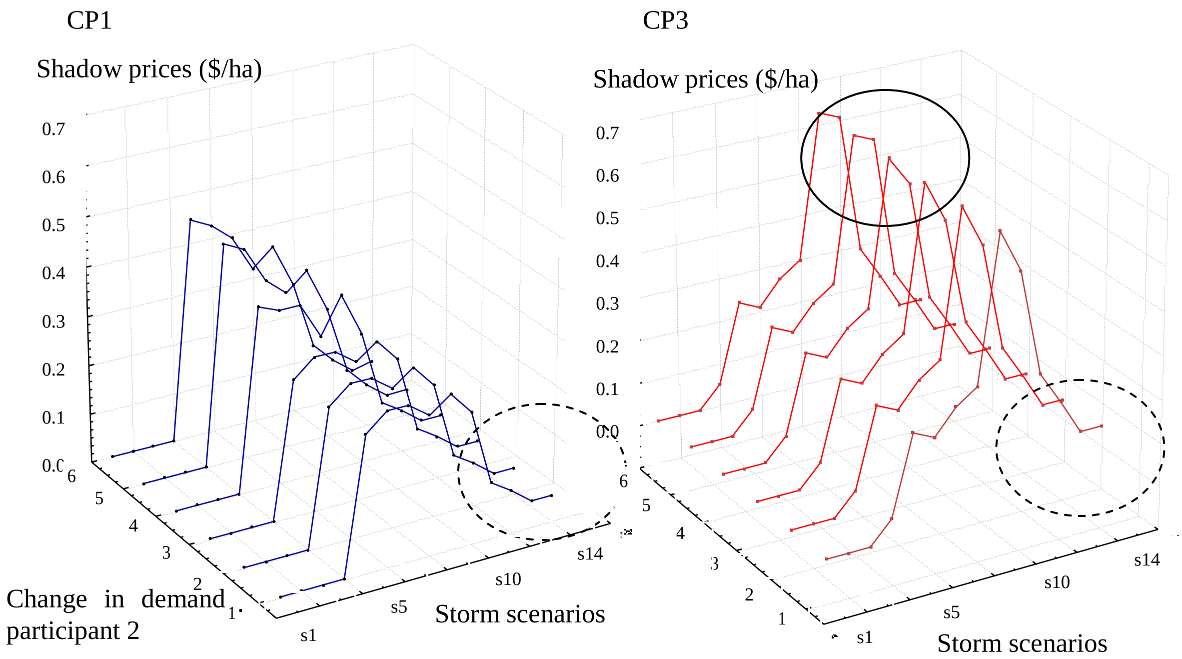


Figure 4-13 Changes in clearing prices (shadow prices) across storm scenarios and control points.

Table 4-3 Total and expected damage, and dual in scenario s13 and s14

Control point	Total damage		Expected damage		Duals	
	Storm scenario s13	Storm scenario s14	Storm scenario s13	Storm scenario s14	Storm scenario s13	Storm scenario s14
CP1	\$14,639.90	\$15,137.80	\$7.31	\$7.46	\$0.052	\$0.062
CP2	\$77,797.30	\$81,626.90	\$38.89	\$40.81	\$0.118	\$0.149
CP3	\$121,2270	\$127,127.00	\$30.61	\$63.56	\$0.133	\$0.133

Case 3

In this case, the capacities are changed to emphasise the effects in flood conditions across scenarios. At CP1, CP2 and CP3, the threshold capacities M_k are 78, 284.5 and 250 m^3/time hedged against flood conditions up to storm scenario 5, 5, and 3 respectively. However, participants desire to change IC, and consequently the flood distribution may change at each control point. Participants have the same preferences as previously illustrated, but participant 2 has two options (F1 and F2): F1 bid a low price, and F2 bid high enough to increase IC allowances. Other participants may have strategic decisions, but this issue is outside the scope of this thesis.

The market is cleared under the F1 condition, and participants 1, 3, 4 and 5 increased IC allowances, but participants 6 and 7 reduced IC allowances. In this case, 10, 5, 5, 8, 6 and 9 ha of IC allowances would be changed. Under F2 the final land allocation would remain similar; however, participant 2 could increase IC allowance by 10 ha. Figure 4-14 illustrates the dual prices under both situations.

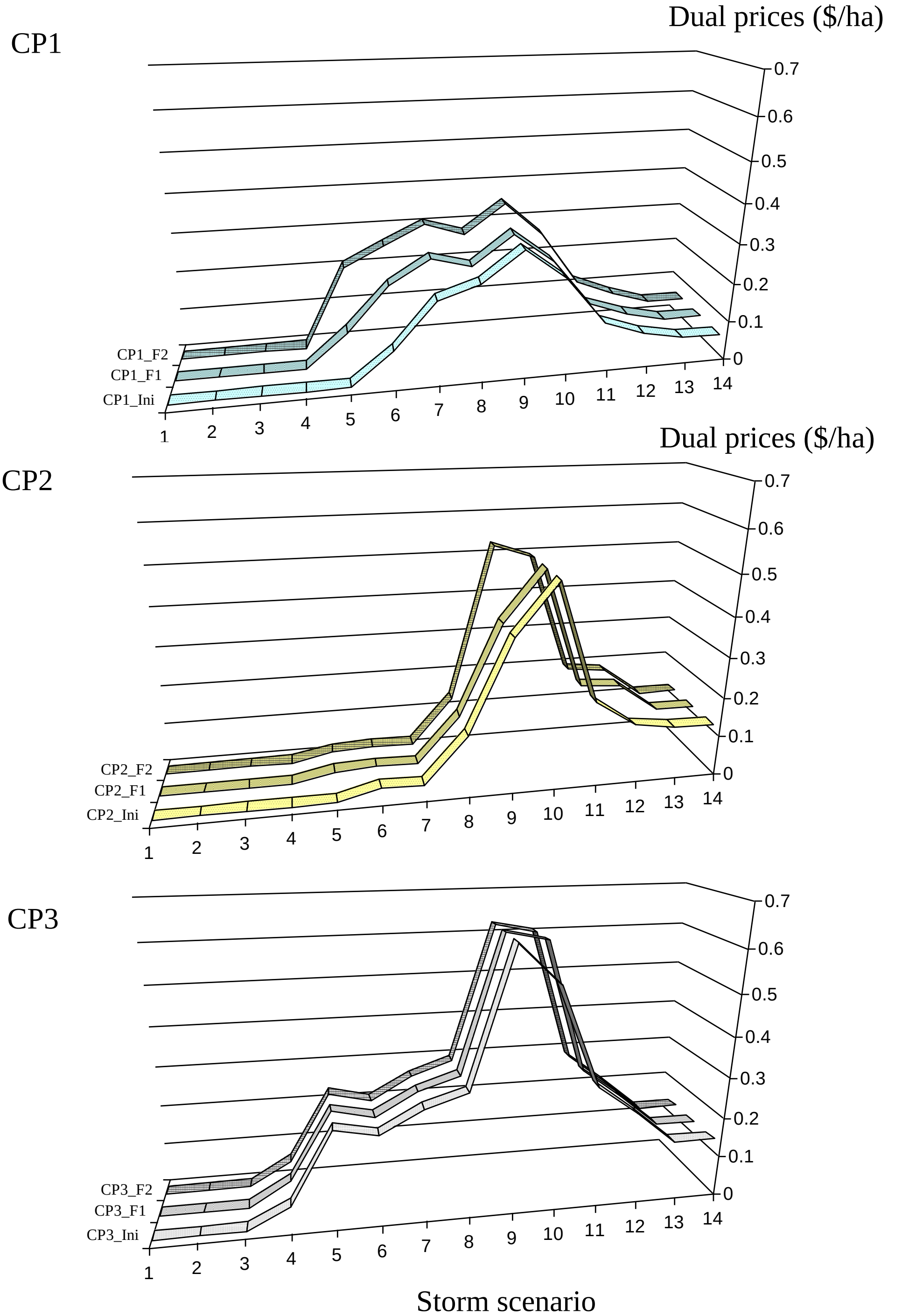


Figure 4-14 Dual prices across scenarios at control points CP1, CP2 and CP3. CP_Ini, CP_F1 and CP_F2 are initial, F1 and F2 conditions at control points respectively.

With options F1 and F2, CP1 and CP2 are hedged up to storm type 4, and CP3 remained with the same hedged storm type 3, but flooding increased its intensity across scenarios (see Figure 4-14). Prices at CP1 rise because insufficient participants reduce their IC allowances at CP1, and participants demand sufficiently high to compensate the incremental expected flood damage along control points. At CP2 and CP3, participants 6 and 7 reduced IC allowances, smoothing the flooding levels as well as the dual prices reached at the control points under F1 and F2.

Case 4

In this case, control points CP1, CP2 and CP3 have threshold capacities M_k of 60, 280 and 300 m^3/time with respective damage functions of $Dm_1 = 15f^2$, $Dm_2 = 0.001f^3$, and $Dm_3 = 0.0005f^3$. Fourteen storm scenarios with different intensities are used in the market. Under the current IC, the expected flood damage at CP1, CP2 and CP3 are \$7,982.10, \$28,003.10 and \$5,001.28 respectively, and \$40,986.50. Flooding occurs from storm scenarios 4, 5 and 6 at CP1, CP2 and CP3. Participants are assumed to have the previous demand and supply preferences.

Flooding occurs often with significant damage. Thus, if the market were cleared, the SO would be a net payer. However, the imperviousness would be reduced, so the catchment would be hedged against a wide range of storms. In addition, the flood distribution would be shifted, reducing expected damage.

In this case, any participants who desire to reduce IC can do so, e.g., participants 5-10 would reduce IC by 10 ha each, but participants who desire to increase IC would not do so. Consequently, the catchment is now hedged up to storm types 5 and 4, and flows are reduced across scenarios at CP2 and CP3, see Figure 4-15..

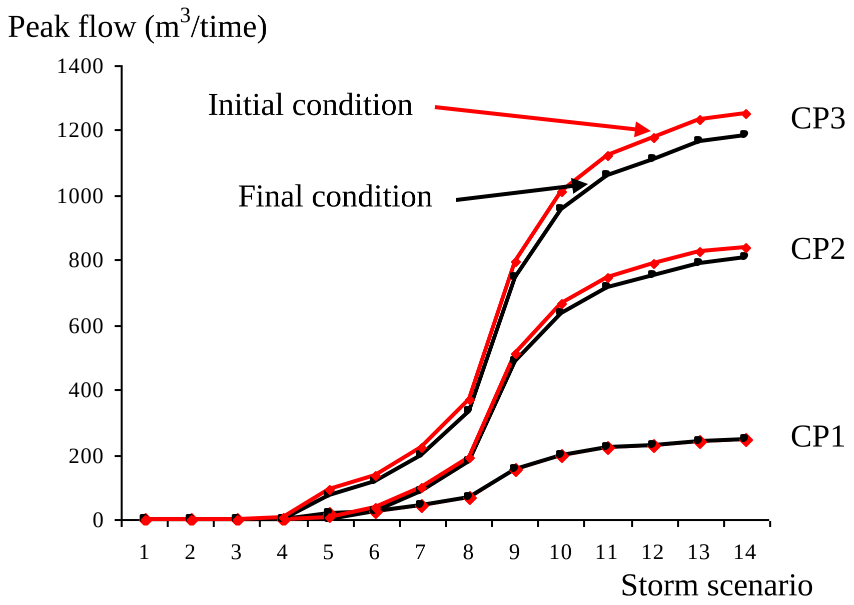


Figure 4-15 Changes in peak flows at control point CP1, CP2 and CP3 before and after the market is cleared.

4.9 System operator revenue

After clearing the market, the SO's revenue (*RE*) is obtained as the difference between the payment from participants with increasing demands and reducing supplies, which is represented as follows:

$$RE = \sum_i^n \sum_j^J \sum_s^S \sum_k^K \sum_t^T \phi^S \lambda_{t,k}^s H_{i,j,k}^{t*,s} g_{i,j}^D + \sum_i^n \sum_j^J \sum_s^S \sum_k^K \sum_t^T \phi^S \lambda_{t,k}^s H_{i,j,k}^{t*,s} g_{i,j}^S \qquad [4.22]$$

In the electricity market, revenue adequacy is defined as payouts to financial transmission rights being less than congestion revenues from dispatch. With a dispatch clearing model under uncertainty, Pritchard et al. (2010) noticed that revenue is reached only in expectancy, and that in some scenarios the SO could be a net payer and in others a net receiver. However, in the proposed Sto_MarketIC clearing model, the SO can reach different revenue in expectancy That means the SO is exposed to a weighted payment or receipt related to the changes in flood damage and the probability of the storm scenario, ϕ^s ,. Thus, the SO could be a net receiver or net payer depending on the marginal changes in flood distribution. The SO could be a net receiver if the SO would sell expected flood damage in the flood area; contrarily, the SO could be a net payer if the expected flood damage is reduced. More detail is discussed in next Section. In addition, the market is not necessarily revenue neutral in expectancy, but depends on initial IC allowances and the flood cost function approximation.

4.9.1 Initial IC allowances and revenues

Section 3.6 of Chapter 3 pointed out the SO may reach different revenue positions according to initial IC allowances, and over and under-allocated capacity constraints in a market established with one storm scenario. In the Sto_MarketIC, the IC allowance is a set of impacting flows $G_{i,j,k}^{t-u+1,s} A_{i,j}$ along period and scenarios at control points, where $G_{i,j,k}^{t-u+1,s}$ is the impact flow related to the initial IC allowance and $A_{i,j}$ is the initial land area. The IC allowance's price represents the value of violating capacity at different places in the catchment.

Initial loose constraints which allow increasing flows in some scenarios, which implies that the SO could sell these rights, and receive revenue for the equivalent increment in IC allowances, if the channel capacity will become binding due to flooding in the area. Additionally, high prices could increase IC allowances and the SO could receive payments for those increments. Contrarily, the SO could be a net payer when marginal flood expected damage is greater than the offer prices to reduce IC allowances. Marginal prices for changing flood and IC allowances could be different than the marginal flood damage, and the SO may pay or receive the difference.

Thus, if $P_{i,j}$ is the bid price to increase IC allowance, and because a non arbitrage condition is faced in the market, the clearing price will be $\sum_{s=1}^{S} \phi^s \frac{\partial C_k^f \left(f_k^s \right)^*}{\partial f_k^s}$ at control point k. This prices represents the marginal change in the expected flood damage at the new flood distribution. So if the new condition $\sum_{s=1}^{S} \phi^s \frac{\partial C_k^f \left(f_k^s \right)^*}{\partial f_k^s} \geq \sum_{s=1}^{S} \phi^s \frac{\partial C_k^f \left(f_k^s \right)}{\partial f_k^s}$ to previous flood distribution, the SO could face revenue adequacy $P_{i,j} - \sum_{s=1}^{S} \phi^s \frac{\partial C_k^f \left(f_k^s \right)}{\partial f_k^s} \geq 0$. This means, the SO is exposed to an outward shift to the flooding distribution and would sell this change. Contrarily, the SO could be a net payer, if the demand curve is shifted and the new bid prices were reduced. In this case, the SO could buy expected flood reduction $\sum_{s=1}^{S} \phi^s \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s} - P_{i,j} \geq 0$.The revenue for the SO depends on the total initial IC allowances position and participants' bids. The total initial IC allowances (owned by participants and the SO) is the set of total flows $\sum_{i=1}^{N} \sum_{j=1}^{J} G_{i,j,k}^{t-u+1,s} A_{i,j} = \hat{f}_{k,t}^s$, $\forall$ s,k,t, which is equivalent to the set of peak flows $\hat{f}_k^s$ at control points and storm scenarios, i.e., the flood distribution. The effects of these allowances on the expected flood damage is $\sum_{s=1}^{S} \sum_{k=1}^{K} \phi^s C_k^f\left(\hat{f}_k^s\right)$. Landholders' rights have a set of physical impacts at receptor constraints $\sum_{j=1}^{J} G_{i,j,k}^{t-u+1,s} A_{i,j} = \hat{h}_{i,k,t}^s$, $\forall$ s,k,t. The effect of these peaks on the expected flood damage is $\sum_{s=1}^{S} \sum_{k=1}^{K} \phi^s \hat{C}_k^f\left(\hat{h}_{i,k}^s\right)$.

The SO may be a net receiver in expectancy if participants shift flow distributions along scenarios, particularly, toward periods and scenarios when the peaks are below the threshold capacities, M_k. Thus, the SO may sell violated capacity $M_k - \sum_{i=1}^{N} \sum_{j=1}^{J} G_{i,j,k}^{t*} A_{i,j}$ at the peak flow time $t*$, priced by the marginal flood damage $\phi^s \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s}$. This sale allows floods in scenarios that were free of flooding. Additionally, the SO may be a net receiver if participants increase IC allowances, and pay high enough prices for shifting flood damage distribution at some control points, $P_{i,j,b}^D > \sum_{k=1}^{K} \sum_{s=1}^{S} \phi^s \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s}$. Participants are changing capacity for their violating flows across scenarios.

The SO may be a net payer, if the opportunity cost for changing IC allowances is lower than the marginal expected damage, $P_{i,j,b}^S < \sum_{k=1}^{K} \sum_{s=1}^{S} \phi^s \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s}$.

The SO may reach revenue neutrality by being a net payer at some control points and net receiver in others, or when the opportunity cost for IC matches the marginal damage of the flood distribution which remains constant at the end.

4.9.2 Operator revenue due to flood damage cost approximation

Different approximations of the flood damage cost can generate different rents for the SO. This has been observed when modelling transmission losses, financial transmission rights and flow pressure in electricity and gas markets (Hogan et al. 1996; Hogan 2002; Philpott and Pritchard 2004; Lesieutre and Hiskens 2005; Pepper et al. 2012). In this case, the extra rent corresponds to the possible surplus or deficit for the SO, which is termed supra and infra revenue. (Note that if the flood damage cost is linear, this extra revenue is almost zero.) A net surplus is reached based on the differences between the changes in the expected flood damage and the price paid/received for the SO for allowing trading. This effect will be illustrated and discussed in section 4.9.3.

Figure 4-4 and Figure 4-16 illustrate linear approximations of flood damage costs, and a linear approximation for particular scenarios. Convexification of the function may produce this additional rent (supra and infra revenue) to the SO. The SO's extra rent will be analysed in one scenario which corresponds to a simple flooding scenario model, and under several scenarios in the Sto_MarketIC in expectation.

If A or B were chosen, the nodal clearing prices would be $\lambda_A = P_{A1} + P_{A2}$ or $\lambda_B = P_{B1} + P_{B2}$, and P_{A1} and P_{B1} which correspond to the marginal flood damage at specific flooding levels A and B. Accordingly, P_{A2} is the supra revenue that the SO may receive (pay) for the linear approximation. The extra rent corresponds to $P_{A2}\times(Qsell - Qbuy)$ in A and this should be minimised by doing a more accurate approximation.

On the other hand, the SO could be a net extra payer if point C were chosen and IC allowances were increased. In this case, the price signal in the market would be $\lambda_c < (\lambda_A \; or \; \lambda_B)$. However, the flood damage is more expensive than the linearised marginal prices P_{C1}, and the SO would be finally paying P_{C2} (infra revenue), which is the difference between the linearised and the real marginal damage. Under this situation a generalised condition such as *C* could produce an infeasible market condition, and hence the SO should evaluate the market design carefully.

In summary, if rainfall scenarios and flooding levels like A, B, or C were chosen, the market will account for the marginal flood damage in those scenarios. In A, participants may face high marginal cost at equilibrium, and the SO becomes a net payer, and most of the supply offers would be accepted. Thus, the price signal may incentivise flood reduction. In C, the marginal cost would be low at equilibrium, and participants will observe the opposite situation to A or B. In this case, the price signal would be almost zero, and most of the demand for increasing IC would be accepted, raising flood damage, and participants would not pay for contributing to the damage. However, this market design could fail, and the SO would need to convexify the damage function. As was previously stated, the expected flood damage is convex, and so the market would not likely face this situation.

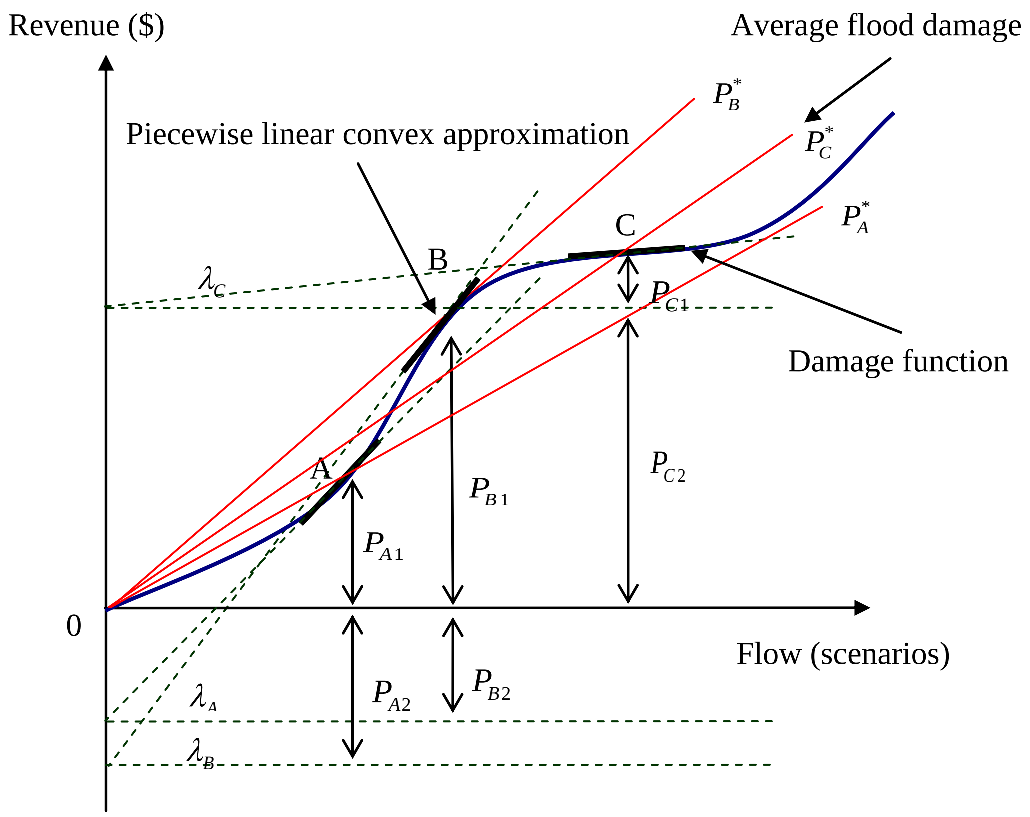


Figure 4-16 Cost approximations and flood scenarios. A, B and C are scenarios; λ_A, λ_B, and λ_C are the corresponding clearing prices (linearised marginal cost). P_{i1} and P_{i2} decompose the clearing price; in A and B, P_{i1} is the marginal damage and P_{i2} is the supra revenue for the non-linear function; in C, P_{i1} is the clearing price and P_{i2} is the infra revenue (cost).

In a market established with scenarios A, B and C, the estimated expected marginal damage estimated (EEMD) would be $\phi^A \lambda_A + \phi^B \lambda_B + \phi^C \lambda_C$ or $\phi^A \left(P_{A1} + P_{A2}\right) + \phi^B \left(P_{B1} + P_{B2}\right) + \phi^C P_{C1}$. The real expected marginal damage (REMD) would be $\phi^A P_{A1} +$

$\phi^B P_{B1} + \phi^C (P_{C1} + P_{C2})$. The difference EEMD–REMD equals $\phi^A P_{A2} + \phi^B P_{B2} - \phi^C P_{C2}$. When $\phi^A P_{A2} + \phi^B P_{B2} \geq \phi^C P_{C2}$ the EEMD would be close to the REMD, whereas damage could be underestimated in other scenarios. On the other hand, if $\phi^A P_{A2} + \phi^B P_{B2} \leq \phi^C P_{C2}$ participants do not cover 100% of the expected marginal cost, and price may encourage them to increase IC allowances. This will also be observed if convexification is applied to the flood damage cost rather than for each scenario, as will be used to illustrate trading cases using convexification in Appendix B and then in Chapters 6 and 7.

4.9.3 Operator revenue surplus

As stated, the SO is exposed to revenue conditions according to the initial IC allowances, the flood damage approximation and the net demand for changing IC allowances. The exposure from the SO corresponds to changes in the expected flood damage due to changes in the demand curves. Because the market is designed as a net market, the demand can correspond to the marginal net income for trading IC allowances. The Sto_MarketIC clearing model assumes that the flood cost functions across scenarios do not change, and changes in clearing prices are only due to changes in demand, as shown in Figure 4-17. Therefore, the SO is exposed only to changes in demand curves.

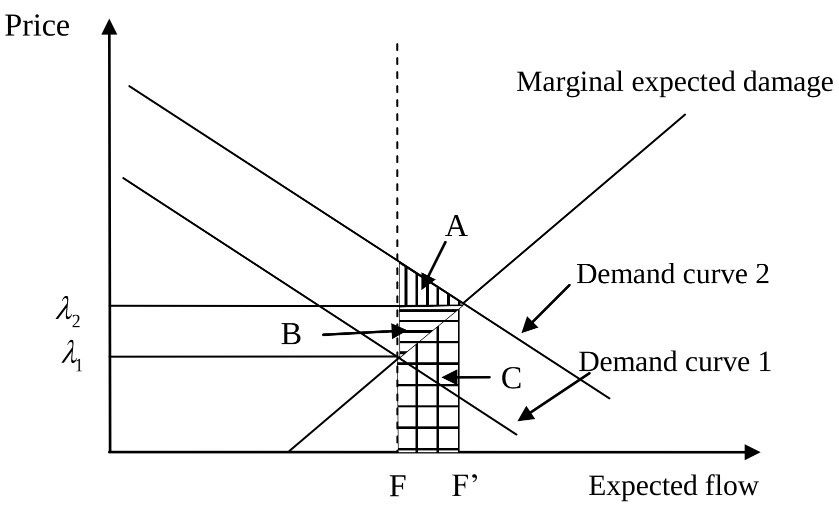


Figure 4-17 Changes in demand curves given a marginal expected damage supply curve. λ_k is the clearing price. A, B, C are net surplus areas. F and F' are the expected flow levels.

Figure 4-17 illustrates the changes in market equilibrium for changes in the demand curves. Initially, the market is cleared at λ_1 with expected flows F (demand curve 1); however, demand preferences could change to demand curve 2 and reach a new equilibrium at λ_2 and F’. Thus, the SO is exposed to a new expected flood damage condition. The SO receives (F’−F)*λ_2, allowing increments in expected damage C and having a surplus B. The participants pay a net (F’−F)*λ_2 and have a net surplus A.

Figure 4-18 illustrates the clearing condition in the next trading period. Net demand is reduced and the final flood condition is F’. The SO is a net payer for reducing flood and pays D = ((F+Δ−F’)*λ_k^3), but has a net surplus area G. However, the initial flood condition F could not be achieved because the net demand was not reduced sufficiently to reach an equivalent flood level condition.

Because changes in flood distribution could be achieved via IC allowances, the SO is exposed to revenue conditions after clearing the market. This revenue could be used to mitigate flood damage, to insure against floods, etc. Also, the extra revenues such as B and G in Figure 4-18 (i) and (ii) could be scaled between market participants. The decision regarding this revenue as well as scaling will be determined by the SO and the authority.

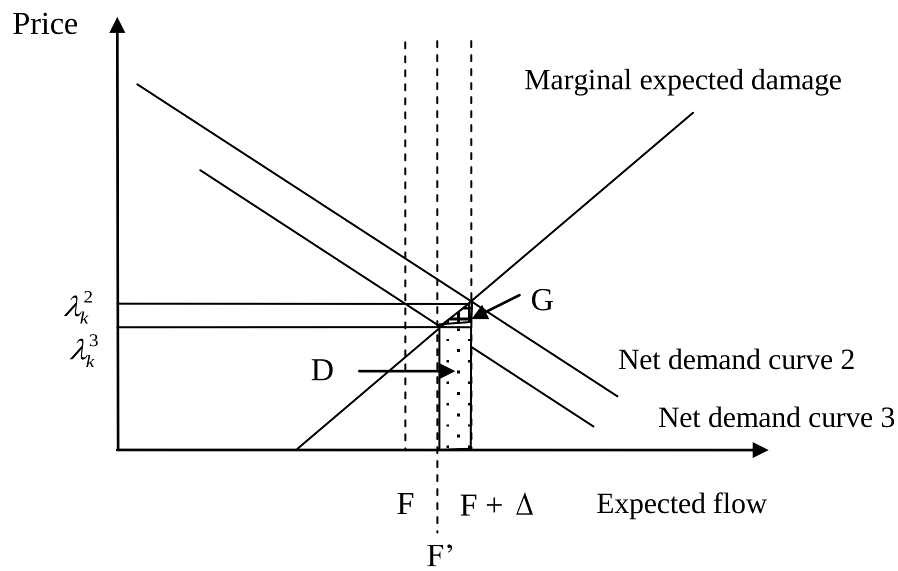


Figure 4-18 Net demand curve and changes in the expected flood damage. λ_k is the clearing price, and D and G are net surplus areas. F, F’ and F+Δ are flood levels.

On the other hand, after several auctions with the expected flow level F and clearing price λ_2, the SO may observed that the damage function in the flood area has significantly changed. The new damage function is damage 2 (see Figure 4-19). Thus, under this new market clearing condition, the clearing price could be λ_3 and expected flows F'. The SO needs to pay (F–F')* λ_3 to reach the new condition F'. Participants reach an extra surplus A, since the payment is higher than changes in demand B. Interestingly, the SO may reach a surplus C due to not compensating all the changes in the new expected damage curve.

The previous figure showed the SO as a net receiver in the first trading period and net payer in the second period, but the SO was actually a net receiver after both trades. Indeed, the flood damage in the catchment was changed. The SO could use this revenue to improve infrastructure, to mitigate flood or to use as insurance in the flood location.

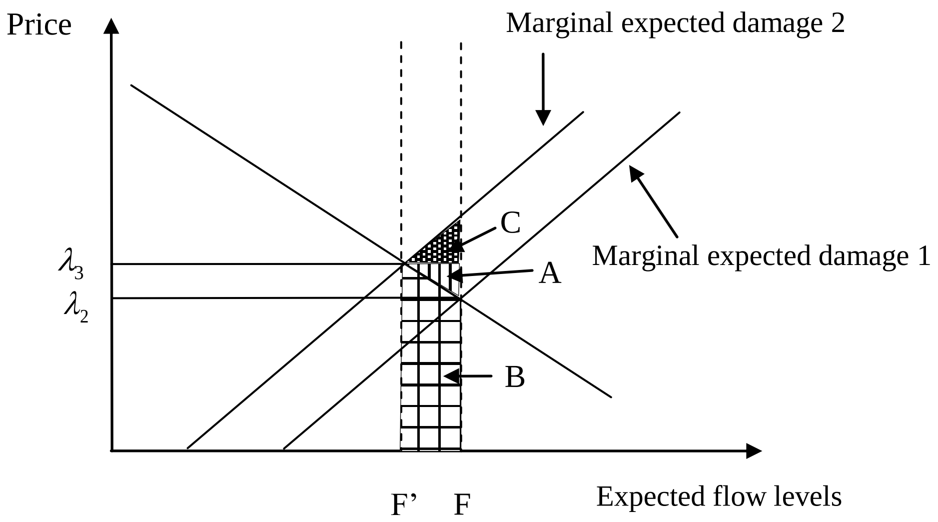


Figure 4-19 Demand curve and changes in the expected flood damage curve. λ_k is the clearing price, and A, B, and C are net surplus areas. F and F' are flow levels.

Those changes in flood damage could also mean that the SO is required to carefully evaluate the established thresholds to avoid over constraining the market, which could cause an extreme net payment condition at the end. The payment could be more than the SO's budget; however, it is expected that the SO could accommodate small changes.

Alternatively, to avoid extreme changes at the flood distribution tail, the SO could try to hedge against extreme changes and could use a risk measure for changing the extreme flood distribution. This proposed risk condition in a possible market formulation will be analysed in Chapter 6.

4.10 Dealing with a non-convex flood damage cost in the market formulation

A piecewise linear approximation may fail when the approximated function is non-convex (Beale and Tomlin 1970). However, a non-convex condition can be handled by adding binary variables (Keha et al. 2004). The method known as SOS2 forces a weighted combination between adjacent grid points (Beale 1963; Lawler and Wood 1966; Beale and Tomlin 1970; Lee and Wilson 2001; Keha et al. 2004; Loucks et al. 2005).

Pepper et al. (2012) pointed out that the Victorian gas market in Australia deals with non-convexities in the physical equations. They noticed that SOS2 allows acceptable solution and clearing prices, accordingly, the gas market in Victoria has being working efficiently for the past 10 years. The Sto_MarketIC could deal with the issue of non-convexity in some scenarios. Currently, other approximations can deal with non-convexity issues, particularly with prices (O'Neill et al. 2005; Bjørndal and Jörnsten 2008), but these non-convexity issues are outside the scope of this thesis. Loucks et al. (2005) presented a non-convex stage-damage function which will be noticed if peak flows exceed channel capacities, banks, levees, dykes, etc. This flood damage function could be addressed in the Sto_MarketIC model via discretisation, linearisation and using SOS2 method.

The SO can estimate flood damage with a local solution from adjacent points with SOS2, calculated from a convex combination between grid points of the linearised and non-convex flood damage cost for each storm scenario. The SOS2 formulation includes the following conditions:

$$l^s_{k,r} = \delta^s_{k,r-1} + \delta^s_{k,r}, \quad \forall\ k,s,t \text{ where } r \in (2,R\text{-}1) \qquad [4.23]$$

$$l^s_{k,1} \leq \delta^s_{k,1},\ \forall\ k,s \qquad [4.24]$$

$$l^s_{k,R} \leq \delta^s_{k,R-1},\ \forall\ k,s \qquad [4.25]$$

$$\sum_{r=1}^{R-1} \delta^s_{k,r} = 1,\ \forall\ k,s \qquad [4.26]$$

$\delta^{s}_{k,r}$ forces a solution in the range (r–1, r). The market formulation could include conditions [4.23]-[4.26] to deal with non-convexity. However, this could create externalities in the market as can be seen with a numerical example in the first section of Appendix B.

Using SOS2 and convexification for the flood damage produces different revenue for the SO. The approximations also change prices and allocations along with the final IC allowances in the catchment as was previously discussed. These affect the flood distribution and the hedging flood condition that could be reached in the catchment. Implications of flood damage approximations and also using SOS2 are extended in Appendix B along with several numerical examples.

4.11 Final remarks and conclusion

The Sto_MarketIC is not a stochastic version of Det_MarketIC, which had a hard constraint [3.21], because with Det_MarketIC model just one constraint would be binding in the optimal solution. Rather, the Sto_MarketIC includes a flood damage function which accounts for the transfer functions, flow-stage, flood-stage and stage-damage relationships in the flooding area (Loucks et al. 2005).

The market model Sto_MarketIC1 considers only the peak flow period, not flows by period along the flood periods. The cost is internalised only for participants who contribute to the stage-flood over the modelled scenarios. Sto_MarketIC is constructed by first creating a demand curve for flood reductions (increase) and so indirectly for imperviousness. The curve is based on marginal changes in the expected flood damage, and this marginal damage will be faced for those participants by the no-arbitrage condition.

Duality shows the no-arbitrage condition of prices; the price represents the marginal changes in the expected flood damage at the equilibrium. This price is accounted for at settlement for each participant. The settlement also depends on the initial IC allowances [reference usage (land use)] that each participant owns in their property. Consequently, the SO could be exposed to different revenue conditions, being a net payer or a net receiver according to the net changes in the expected flood damage at each control point and the status quo of the initial land uses in the catchment. This revenue also depends on the flood damage approximation.

When applying the cost approximation to the flood damage, the SO may not have revenue adequacy; however, the SO may reduce the stage-flood in some parts, or move stage-floods among areas especially in sensitive zones. Thus, the price signal may incentivise participants to move out the peak period as long as they receive a net payment. The expenses would be paid by the SO and the catchment would be hedged against a wide range of storms events as well as flood scenarios. On the other hand, the price may also be a signal to increase flooding.

In convexifying the flood damage cost, the Sto_MarketIC will allocate IC allowances in different ways as was stated in previous paragraphs. Now the question is who should internalise those differences when the damage function is convexified? Society may accept to be a net payer if the conditions allow a net social benefit, or if those who are increasing the damage or changing risk position for society internalise the extra damage. However, who should pay for these differences for the approximation as well as possible distributional effect for scaling are beyond the scope of this research.

To deal with non-convexity of the flood damage function and possible failure when the approximated function is non-convex, a SOS2 method is proposed. The method allows getting prices and allocations, and the price accounts for the expected flood damage.

Finally, in a small catchment where most flows arrive at control points with similar impacting flows by time, each participant’s flow will be impacting at the peak period, resulting in such flows being priced correctly. However, in a larger catchment, participants whose flows arrive before or after the peaks may not pay. Accordingly, when participants inject flows that hasten flood and lengthen flood duration, they are being *free riders*. In both situations, participants are free riders because the market considers only the peak flow to charge participants for their flood contribution. Even if the damage was well estimated, participants contributing to floods may not be correctly charged. This situation can be seen as an externality that could be addressed by market model design. To prevent these free riders, the next chapter will propose ways to penalise non-peak flows.

The SO may observe that in some flood areas, participants could change IC allowances, and their flows would arrive before the stage-flood. These flows do not increase damage, and so this condition could be a desired pattern for the SO. The same could be noticed with those IC allowances that delay flows, but lengthen flood. The SO could also desire this condition if peak flows are reduced. In both cases, participants are not contributing to damage and so are not free riders.

Insurance companies (private and public) could be free riders if the IC market were established in the catchment. Changes in flood distribution and hedging for flood condition will change the risk premiums. A less impervious catchment for a long term would reduce risk premiums resulting in reducing the adverse selection to participants in the flood area. However, companies could also face extra losses if the catchment became impervious and the flood distribution were shifted with potentially more frequent events. These implications are outside the scope of this thesis, but an important point for future research.

Chapter 5

5 EXTENDING PENALTIES FOR FLOOD DAMAGE

5.1 Introduction

The Sto_marketIC1 market model was designed considering that the peak flow was an approximation of the total damage. Those peaks flows then resulted in flood costs which were estimated based on the recourse flood cost in the TSSP model, and for a range of scenarios based on location, and the catchment. The damage depends on peak flow or stage flood, but also flood rate (flash flood) and flood duration (Krzysztofowicz and Davis 1983b; c; d; Georgakakos 1986; Smith 1994; Penning-Rowsell and Green 2000; Loucks et al. 2005; Thieken et al. 2005; Kreibich et al. 2009; Merz et al. 2010). Empirical studies demonstrating these effects are reviewed in Sections 2.8 and 2.9 in Chapter 2, and Section 5.2 later.

Krzysztofowicz and Davis (1983a; b), and de Moel and Aerts (2011) pointed out that most studies on flood damage simplify the damage estimation by using the maximum depth “stage-flood”. This approach is also suitable to Sto_MarketIC1; however, this simplification may carry inefficiencies by having free riders, who are injecting runoffs and their flows contribute to the flood damage because those flows are hastening the peak time (flash flood) or lengthening flood duration (Krzysztofowicz and Davis 1983a; c; Embrechts and Schmidli 1994; Dutta et al. 2003). Additionally, the stage-flood approach does not capture interactions from flood components such as flash flood or flood duration. A market focused only on penalties based on maximum flow depths may result in biased price signals. Accordingly, this chapter will explore alternative penalty costs which will account for flood components such as flash flood or hastening peak flood measured as increment in the rising limb slope (hastening stage-flood), and flood duration as increment in the reducing slope flow, enabling physical changes to floods, and for the cost of flood damages. Penalties will be approximated to the cost of flooding. Therefore, the TSSP market model will estimate the flood cost taking into account additional flood components.

This chapter proposes alternative cost penalties to decompose the flood damage. The Sto_MarketIC2 formulation decomposes flood damage into the components of depth, duration (lengthening flood), and hastening peak flood (hastening peak period). Thus, any changes in flow due to changes on IC allowances, BMPs and technologies could be penalised by the flood component and its related costs. Penalties related to flood cost would be incurred by those participants who contribute to the flood damage.

In Sto_MarketIC2, flows reduce capacities, for which a recourse flood cost of flood damage is raised for a given scenario. The market model can penalise for ramping up or down around established thresholds; however, in this case the penalties cost will be related to violate an established rising limb slope and so hastening peak time of flood (flash flood) and lengthening flood duration related to violate established flow reducing slope.

In the Sto_MarketIC1 model, flood damage was linked to peak flows (maximum depth flooding), and the market model accounted for peak flows over a finite set of storm scenarios. This flood cost could be adjusted according to the storm’s type, i.e., different cost estimates for storms that hasten or lengthen flood duration. Such an approach could be cumbersome due to the need of having as many cost estimates as storms used to establish the market. However, with this approach, any change in flood patterns outside of the stage-flood may not be penalised. Thus, rather than establishing only a flood cost for each peak flood, a more general cost scheme for flood patterns could be established, which is able to cost any change that could modify the physical flood distribution and the flood damage across storm scenarios.

As stated in previous chapters, two similar flooding depths could result in different damage costs, depending on the period to reach the maximum depth or the duration after reaching a given flood peak. The proposed penalties would reflect the changes in flood physical distribution and associated cost. Penalties will not account for all real and possible combinations of flood damage; but provide a plausible and referential measure of such damage.

Section 5.2 presents the flood damage components and the related penalties for each flood component. Section 5.2.1 discusses the depth cost related to stage-flood damage.

Section 5.2.2 introduces the slopes dependences related to warning and flood duration. Section 5.2.2.1 presents penalties constraints which specifically relate the rising limb of the hydrograph curve with a penalty for violating this slope with warning flood (flash flood), and the lengthening flood with flood duration. Section 5.2.2.2 presents an alternative way to relate these penalties with flood components. Section 5.3 describes the market model. Section 5.4 presents the price analysis with the proposed penalties. Section 5.5 is final remarks and conclusions.

5.2 Relating cost to flood damage

Flood components such as hastening peak flood and lengthening flood should be linked to penalties which in turn should account for violations of physical constraints. For instance, large increases in flows may increase the damage for changes in the slope of the rising limb of the hydrograph curve, which reduces the time to reach a peak level of flooding (flash floods). Slow decreases in flows may increase flood duration (discussed in section 5.2.2.1). Hastening of floods and their duration could be approximated via establishing flow thresholds during the flood warning period (further discussed in section 5.2.2.2). Implementing penalties enables the market model to price the flood components that influence flooding. As a result, floods can be managed with clearer and long term price signals.

Flooding components from governing hydraulic equations at control points and scenarios could be added into the flood cost function. Thus, instead of approximating piecewise hydraulic components, a piecewise approximation could be determined directly from the damage cost function, e.g., the corresponding peak flow could be related to the flood damage condition.

The SO could also use the proposed penalties to obtain the desired flood conditions. In such cases, the SO could establish thresholds that actually achieve the desired flood levels. For any increase in the established thresholds, penalties will be raised; thus, participants prepared to pay such penalties, which correspond to flood costs, can change IC allowances and also the physical flood distribution. The SO could penalise for hastening peak flooding in a specific period based on a desired depth increase, with the aim of achieving a smooth flood. Also the SO could desire a particular depth over a given time period, and penalties would be raised for increases in flood depth.

5.2.1 Depth cost in floodplain areas

Depth of flooding contributes to damage. In the Sto_MarketIC2 model, the cost depends on the maximum flow that is reached at each control point, which represents the maximum depth in the channels and floodplain. Damage $C_k^f\left(f_k^s\right)$ is related to maximum flow f_k^s by control point *k* and storm scenario *s*.

5.2.2 Rapid flooding and lengthening duration

Flood damage is raised by sudden and large increases in flows or depth levels, and also by slow declinations (Thieken et al. 2005; Kreibich et al. 2009). Flash floods may produce considerable damage cost (Georgakakos 1986); in fact, regional authorities monitor changes in flows to avoid such extreme damages, and if necessary evacuating people in anticipation of such floods. Penalty costs could account for the anticipation of peak flows.

These penalty costs could be related to warning costs (Krzysztofowicz and Davis 1983b; d; c; Penning-Rowsell and Green 2000). Smith (1994) noted that in urban areas, reductions in the warning period, i.e., notification that the floods would start in a short period, may increase the damage by up to 25%. In addition, the author showed that in a less prepared area, the relationship of actual/potential damage could increase from 0.6 to 0.85 when warning time is reduced from 5 hours to 1 hour. Similarly, Penning-Rowsell and Green (2000) noted that with advanced warning, potential flood damage can be reduced by about 13%. Other empirical studies that estimate flash flood damage were addressed by Barrera et al. (2006), and by Kreibich et al. (2009).

Longer flood periods can also result in increased damage and such damage cost could be calculated by the flow declination penalties for flood duration.

Dutta et al. (2003) presented stage-damage curves, based on flood depth and duration in the agricultural sector in Japan. From those curves, it was possible to see increased damage associated with flood duration; e.g., after 4 days of flooding, damage ranged between 10% and 15%, and after 7 days of flooding, increased to 45%. Thieken et al. (2005) showed that duration increases losses in buildings from 10% with 2 to 7 duration days to 20% of losses with 21 days of flooding. Other studies of flood duration damage were pointed out by Scott (1989), Smith (1994), and Penning-Rowsell et al. (2005).

The SO could establish thresholds (illustrated by the series *i* in graphs *A* and *B* in Figure 5-1), for a possible maximum flood level, storm scenario and the status quo of IC allowances in the catchment. These figures show an upper inclination and a lower declination from established flows or depth levels. The market model could raise penalties according to levels illustrated by α and β angles in Figure 5-2, which would account for the resulting damage. This threshold inclination (declination) may correspond to a cost of flooding in a particular scenario. The electricity market uses similar constraints for generator ramp-rates (Alvey et al. 1998); in fact, such constraints ensure electricity injections in the systems. In our case, for instance, if the current flow situations were illustrated by *iii*, the manager would try to move flows toward situations such as *ii* and *i*.

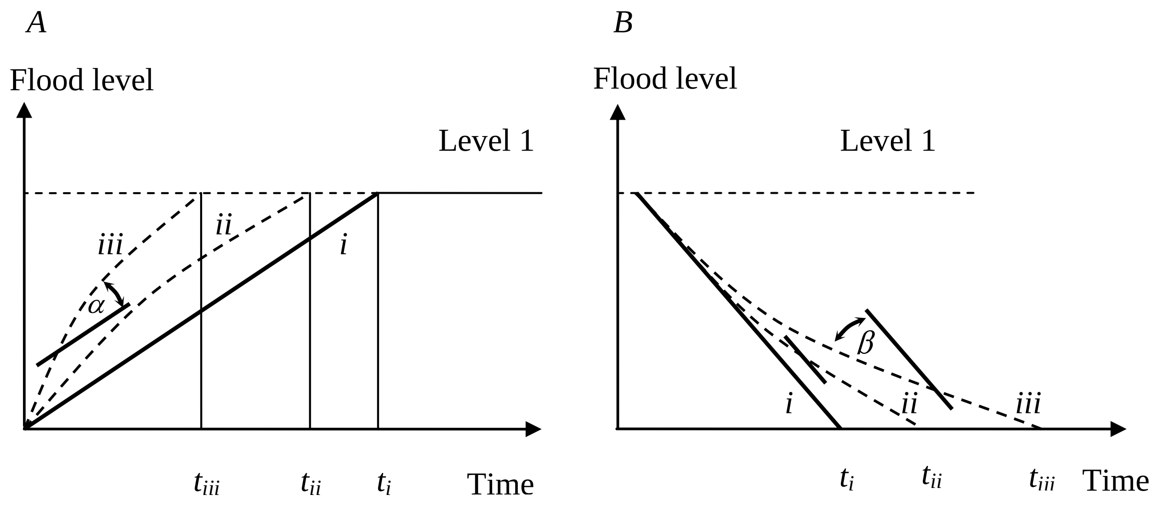


Figure 5-1 (*A*) inclination to reach a flood level and (*B*) declination to reach a desired condition; indices *i*, *ii* and *iii* represent flood inclination (declination) to reach level 1 (being a more preferred flood level), and time $t_i > t_{ii} > t_{iii}$ in *A* and $t_i < t_{ii} < t_{iii}$ in *B*.

Figure 5-3 shows the hastening peak by time for flooding levels L_1 and L_2. If the threshold angle conditions were continuously violated over a period, a flash flood event would more likely be noticed in scenario *s*, and the period to reach this flood level would be reduced by $t-t'$ periods ($\Delta t'$). Flows above threshold capacity increase flood damage, which is represented by changing the angle from α to $\alpha+\gamma$ as in Figure 5-3 *B*.

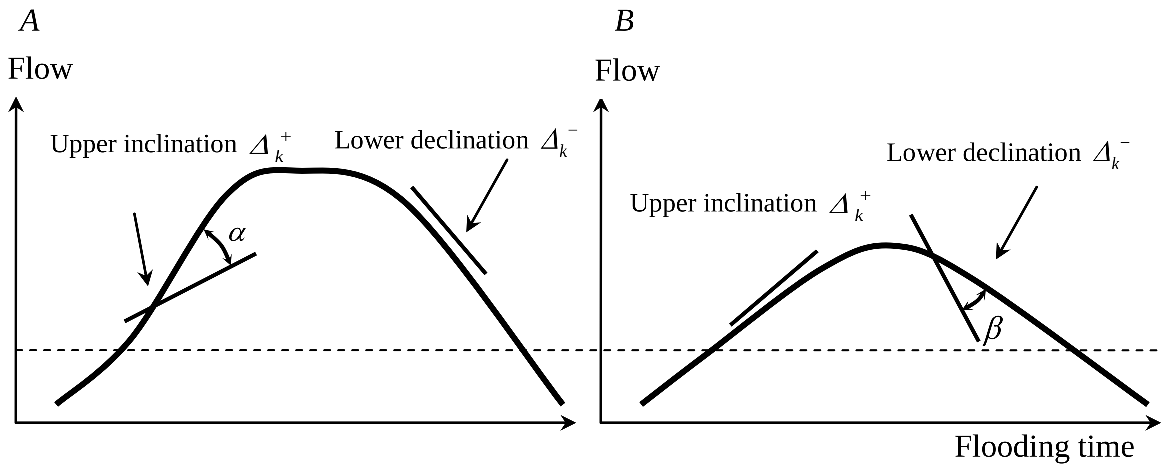


Figure 5-2 Upper inclination and lower declination penalties with hypothetical total flow patterns *A* and *B* at a control point. Angles α and β represent the severity of the upper inclination and lower declination violations.

If a participant increases their impervious level, it will probably increase the threshold depth inclination, and he/she will most likely pay for increased flood costs. The change in the angle as well as its effect on flash flooding is represented in Equation [5.1]. This equation approximates the reduction in the period to incremental damage which is observed when the changes in flow thresholds are significantly violated-

$$\Delta t' = \Delta t \frac{\Delta z_{k,t+1}^{\gamma}}{\Delta z_{k,t+1}^{\alpha} + \Delta z_{k,t+1}^{\gamma}} = \Delta t \left[1 - \frac{\Delta z_{k,t+1}^{\alpha}}{\Delta z_{k,t+1}^{\alpha} + \Delta z_{k,t+1}^{\gamma}} \right] \qquad [5.1]$$

$\Delta z_{k,t+1}^{\alpha}$ is the marginal change in depth (flow) at in control point *k* and time *t*, $\Delta t'$ is the reduction in period to reach level L_1, and $\Delta z_{k,t+1}^{\alpha+\gamma} = \Delta z_{k,t+1}^{\alpha} + \Delta z_{k,t+1}^{\gamma}$. Figure 5-3 shows that at time *t'*, the flood reaches depth L_1; thus, if $\frac{\Delta z_{k,t+1}^{\alpha}}{\Delta z_{k,t+1}^{\alpha} + \Delta z_{k,t+1}^{\gamma}} > 0$, an upper flood bound could be reached in a shorter period, given a constant period $\Delta time$. A similar approach could be used in case of violating declination flow changes as well as any violation able to lengthen the flooding duration.

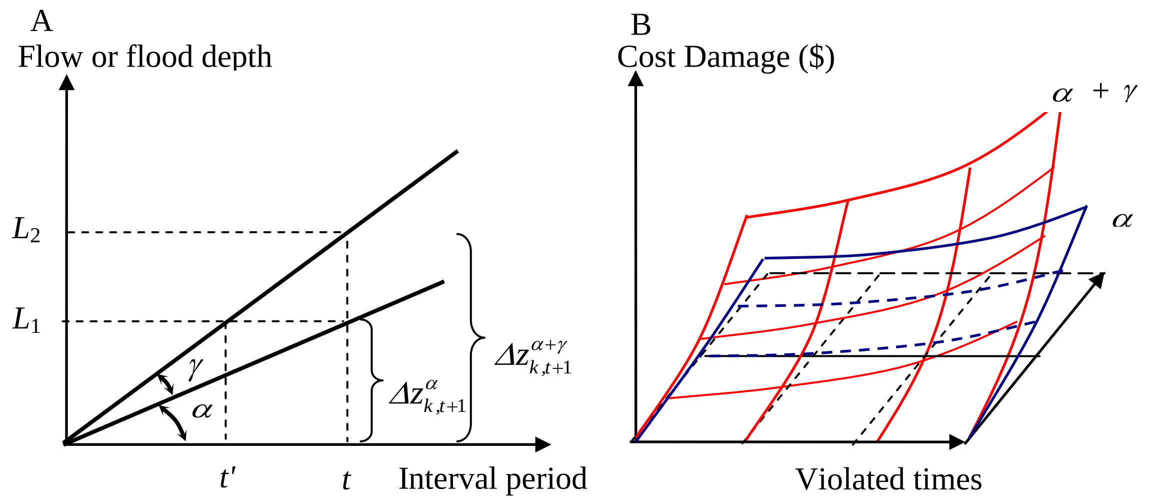


Figure 5-3 (A) Flash flood inclinations and relationships between flood costs and angles α and $(\alpha+\gamma)$. The difference between angles corresponds to the increment of flash flood damage. (B) Changes in damage for the times that threshold is violated.

5.2.2.1 Changes in flow levels and penalty cost

The market formulation could include flood physical conditions such as those indicated with *A* and *B* in Figure 5-1 and Figure 5-2. Equation [5.2] links changes in flows over time for hastening peak flood (flash flood). Equation [5.3] links changes in flows with lengthening flooding duration.

$$z^s_{k,t+1} - z^s_{k,t} \leq \Delta^{s+}_{k,t+1} + Vz^{s+}_{k,t+1} \qquad \forall\ s,k \text{ and } t' \leq t \leq t'' \qquad [5.2]$$

$$z^s_{k,t-1} - z^s_{k,t} \geq \Delta^{s-}_{k,t} - Vz^{s-}_{k,t} \qquad \forall\ s,k \text{ and } t^* \leq t \leq t^{**} \qquad [5.3]$$

Equations [5.2] and [5.3] represent the changes in flows $z^s_{k,t+1} - z^s_{k,t}$ at an inundation area *k*, time *t*, and scenario *s*. Flow changes also represent changes in flow velocity, and so the period to reach a particular flood condition. Consequently, $\Delta^{s+}_{k,t}$ and $\Delta^{s-}_{k,t}$ are the enlarging and reducing thresholds defined by the SO at control point *k*, storm scenario *s* and time *t*. Accordingly, $Vh^{s+}_{k,t+1}$ and $Vh^{s-}_{k,t}$ are the flow changes that violate thresholds at control point *k*, time *t* and scenario *s*. In period $t' \leq t \leq t''$ and $t^* \leq t \leq t^{**}$, changing flows hasten the flood period and lengthen flood duration respectively.

The violated boundaries in [5.2] and [5.3] can be linked to flood damage or flood cost $C_k^{v+}\left(Vz_{k,t}^{s+}\right)$ and $C_k^{v-}\left(Vz_{k,t}^{s-}\right)$. For any violation $Vz_{k,t}^{s+} > 0$, the flood damage would have a marginal increment due to a possible flash flood. Thus, participants who increase this damage will face the corresponding cost. Figure 5-3-B illustrates the violations from the threshold α. Recalling Figure 2-14 in Chapter 2, the stage-flood was shown to be reached at an earlier time, increasing the damage from $C(t^*)$ to $C(t^{**})$. The market model in this chapter will account for this category of violation.

When penalties for declinations are established, and thresholds are violated, i.e., a long-lasting flood, $Vz_{k,t}^{s-} > 0$, the marginal increment in flooding could be estimated by $C_k^{v-}\left(Vz_{k,t}^{s-}\right)$, and the resulting damage could be greater, due to the extended flood duration.

The SO may be more concerned about maximum depth and flash floods than flood duration. In this case, the market clearing formulation would have penalties only for stage-flood and enlarging slope. An alternative flood cost for increasing flows at the first or final flood stages is proposed in this chapter.

5.2.2.2 Alternative penalty cost for large flow increments

Alternative constraints and thresholds to represent hastening peak flood, and lengthening flood duration could be as follows:

$$z_{k,t}^{s} \leq \Delta_{k,t}^{s+} + Vz_{k,t}^{s+} \qquad \text{for } s,k,t \text{ and } t \leq t^* \qquad [5.4]$$

$$z_{k,t}^{s} \leq \Delta_{k,t}^{s-} + Vz_{k,t}^{s-} \qquad \text{for } s,k,t \text{ and } t \geq t^{**} \qquad [5.5]$$

The flood costs that account for the flows above thresholds represent a flood level or a condition that the SO determines across scenarios within the catchment. Figure 5-4 illustrates the thresholds $\Delta_{k,t}^{s+}$ and $\Delta_{k,t}^{s-}$, and times t^* and t^{**} established by the SO. The thresholds also represent tolerance flow levels, below which the SO would not apply flood cost penalties. Participants who change total flows above these thresholds would be penalised by the cost $C_k^{v+}\left(Vz_{k,t}^{s+}\right)$ and $C_k^{v-}\left(Vz_{k,t}^{s-}\right)$.

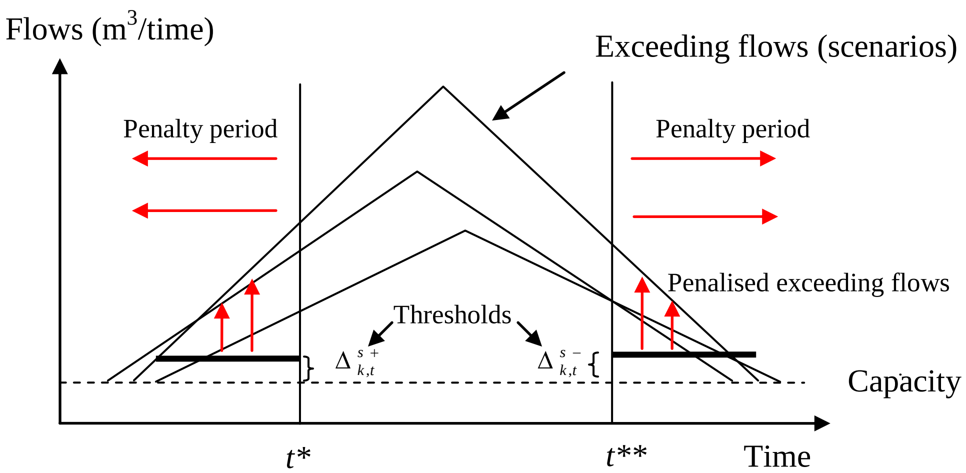


Figure 5-4 Flows above thresholds ($\Delta^{s+}_{k,t}$ and $\Delta^{s-}_{k,t}$) leading to flood penalty cost before and after times t^* and t^{**}.

This chapter focuses on constraints and thresholds associated with flood cost described in section 5.2.2.1). Chapter 7 extends examples for both types of proposed constraints.

5.3 Sto_MarketIC2 model

The Sto_MarketIC2 model extends the Sto_MarketIC1 model, by incorporating new parameters and variables to represent flooding in channels. Concerning channels and streams, model Sto_MarketIC2 will consider the maximum depth as well as maximum and minimum flow changes to approximate flood damage for hastened floods and duration respectively. Violated boundaries have flood costs which represent the changes in flood damage. The market model formulation will be as follows.

Parameters

$\Delta^{s+}_{k,t}$ = Maximum inclination of changing flows at control point k and scenario s (channels or streams) between time t and t–1. The SO establishes these differential thresholds, which represent manageable levels that do not increase damage (volume/time).

$\Delta^{s-}_{k,t}$ = Minimum declination of changing flows at control point k and scenario s (channels or streams) between time t and t+1. The SO establishes these differential flood thresholds as manageable flood levels that do not increase damage (volume/time).

ϕ^s = Probability of storm scenario *s*. This parameter satisfies the following properties: 0 ≤ $\phi^s \leq 1$, and $\sum_s^S \phi^s = 1$.

$C_k^f\left(f_k^s\right)$ = Flood damage for peak flow f_k^s at control point *k* and scenario *s*. This flood cost represents the flood damage in channels and streams related to the maximum depth ($/ volume/time).

$C_k^{v+}\left(Vz_{k,t}^{s+}\right)$ = Flood damage for a large change in the flow inclination between time *t* and *t*+1 at floodplain area *k*, and scenario *s*. This flood cost represents the increase flood damage for accelerating flooding (flash flood) in scenario *s* ($/ volume/time).

$C_k^{v-}\left(Vz_{k,t}^{s-}\right)$ = Flood damage for a small change in the flow declination between time *t* and *t*+1 at floodplain area *k*, and scenario *s*. This flood cost represents the incremental flood damage for lengthening flooding duration in scenario *s* ($/ volume/time).

Decision variables

f_k^s = Maximum peak flow above threshold capacity M_k in scenario *s* at control point *k* (volume/time).

$z_{k,t}^s$ = Flow above capacity M_k at control point *k*, by time *t* and scenario *s* (volume/time)

$Vz_{k,t}^{s+}$ = Exceeding changes in flow at control point *k*, time *t* and scenario *s*. These flows are those above the established maximum flow inclination, Δ_k^+ (volume/time).

$Vz_{k,t}^{s-}$ = Small changes in flows which are below the established minimum changes in flow declination, Δ_k^-, at control point *k*, time *t* and scenario *s* (volume/time).

$\phi^s \varphi_{k,t}^s$ = Clearing price for hastening peak flood and lengthening flood at control point *k*, time *t* and scenario *s* ($/ volume/time).

$\phi^s\ \lambda_{t,k}^s$ = Clearing price, at control point *k*, time *t* and scenario *s* ($/ volume/time).

$\phi^s \chi_{k,t}^{s+}$ = Price for reducing the time to reach a flood level (resulting in flash flood damage or lengthening of flooding) at control point *k*, scenario *s* and time *t* ($/ volume/time).

$\phi^s \chi^{s-}_{k,t}$ = Price for lengthening flooding at control point *k*, scenario *s* and time *t* ($/ volume/time).

Market clearing model**: Sto_MarketIC2**

Maximize

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{D}_{i,j,b} qbuy_{i,j,b} - \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P^{S}_{i,j,b} qsell_{i,j,b} - \sum_{s=1}^{S}\phi^s \left[\sum_{k=1}^{K} C^f_k \left(f^s_k\right) + \sum_{k=1}^{K}\sum_{t=1}^{T} C^{v+}_k \left(Vz^{s+}_{k,t}\right) + \sum_{k=1}^{k}\sum_{t=1}^{T} C^{v-}_k \left(Vz^{s-}_{k,t}\right)\right] \quad [5.6]$$

Subject to:

$$0 \le qbuy_{i,j,b} \le D^{\max}_{i,j,b}, \forall\ i,j,b \qquad : \beta^-_{i,j,b}, \beta^+_{i,j,b} \qquad [5.7]$$

$$0 \le qsell_{i,j,b} \le S^{\max}_{i,j,b}, \forall\ i,j,b \qquad : \gamma^-_{i,j,b}, \gamma^+_{i,j,b} \qquad [5.8]$$

$$\sum_{b=1}^{B} qsell_{i,j,b} = g^S_{i,j}\ , \forall\ i,j \qquad : \mu^S_{i,j}\ \text{(free)} \qquad [5.9]$$

$$\sum_{b=1}^{B} qsell_{i,j,b} = g^S_{i,j}\ , \forall\ i,j \qquad : \mu^S_{i,j}\ \text{(free)} \qquad [5.10]$$

At channel

Peak flow

$$Q^{0,s}_{k,t} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^D_{i,j} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^S_{i,j} \le M_k + f^s_k,$$
$$\forall\ k,s,t \qquad : \phi^s \lambda^s_{t,k} \qquad [5.11]$$

Flow at channel by time

$$Q^{0,s}_{k,t} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^D_{i,j} + \sum_{i=1}^{n}\sum_{j}^{J} H^{t-u+1,s}_{i,j,k} g^S_{i,j} \le M_k + z^s_{k,t},$$
$$\forall\ k,s,t \qquad : \phi^s \varphi^s_{k,t} \qquad [5.12]$$

Inclination and declination threshold violations

$$z^s_{k,t+1} - z^s_{k,t} \le \Delta^{s+}_{k,t+1} + Vz^{s+}_{k,t+1}, \quad \forall\ k,s \text{ and } t' \le t \le t'' \qquad : \phi^s \chi^{s+}_{k,t+1} \qquad [5.13]$$

$$z^s_{k,t} - z^s_{k,t-1} \le -\Delta^{s-}_{k,t} + Vz^{s-}_{k,t}, \quad \forall\ k,s \text{ and } t^* \le t \le t^{**} \qquad : \phi^s \chi^{s-}_{k,t} \qquad [5.14]$$

$$f^s_k,\ z^s_{k,t},\ Vz^{s+}_{k,t}, \text{ and } Vz^{s-}_{k,t} \ge 0 \qquad : \theta^s_k,\ \varpi^s_{k,t},\ \vartheta^{s+}_{k,t},\ \vartheta^{s-}_{k,t} \qquad [5.15]$$

$g_{i,j}^{D}$ and $g_{i,j}^{S}$ (free) [5.16]

Explanation

[5.6] The objective function maximizes the expected total economic surplus for trading IC allowances, less the expected flood cost, due to flood damage under a finite set of rainfall events at different control points. Changes in the objective are the appropriate measure of changes in welfare (assuming the market is sufficiently competitive).

[5.7]–[5.11] Constraints have the same meaning as conditions [4.3]–[4.7] in Sto_MarketIC1.

[5.12] For each scenario *s*, the total flows at control point *k* in time *t* should be lower than the flow channel capacity M_k. However, flows may violate the capacity M_k and $z_{k,t}^{s}$ estimates these flows by time and scenarios across control points.

This arbitrary representation accounts for a reference baseline of flows for the initial IC allowances (existing land uses) from participants. This constraint also accounts for a flow gradient $\left(\Delta_k^{t,s}\right)$ for the changes in IC allowances. These changes in flows may violate threshold flows for hastening the peak time of flooding or lengthening flooding. Thus, participants willing to pay for these threshold violations could change IC allowances. Those who reduce these flows and reduce cost violations will receive a payment for their change in IC allowances.

[5.13] For each scenario *s*, increases in flows at control point *k* per unit time should be lower than a threshold. Violations for greater flows are estimated by $Vz_{k,t+1}^{s+}$, which is linked to flood costs. Thus, any change to reduce this cost would tend to be granted in the market in period $t' \leq t \leq t''$.

[5.14] For each scenario *s*, reductions in flow at control point *k* and time *t* should be greater than a threshold. Violations for reductions in flows are calculated by $Vz_{k,t}^{s-}$ which in turn is linked to the flood duration cost in period $t^* \leq t \leq t^{**}$.

[5.15] and [5.16] correspond to non-negativity conditions; and, IC allowance must be non-negative. This will naturally limit the final allowance allocations, $g_{i,j}^{D}$ and $g_{i,j}^{S}$.

Sto_MarketIC2 uses a canonical formulation to estimate dual prices, as in previous formulations. This formulation can be extended to allow participants to bid for a set of preferences for their IC allowances as in Sto_MarketIC1. In this case, as in Sto_MarketIC1, additional constraints should be included.

5.4 Price analysis

The clearing prices associated with constraint [5.11] have the same meaning as constraint [4.7] of the previous Sto_MarketIC1 model. That is, the dual variable $\phi^s \lambda^s_{t,k}$ represents the marginal flood damage.

The dual price $\phi^s \varphi^s_{k,t}$ in constraint [5.12] represents the marginal value for violating threshold capacities of hastening peak flood and lengthening flood duration at control point *k* in scenario *s*. This price will be non-zero for those flows at or above the thresholds.

The following sections will focus on flood cost implications when the solution violates different threshold capacities. Firstly, the section analyses the dual price related to maximum depth in the channel. Finally, the section will analyse the dual price when most boundaries for flood components are violated along control points, period and scenarios. The dual price is formulated in Appendix B.

5.4.1 Maximum depth in channels and floodplains

The first analysis of dual variables considers flood violations in floodplains and channels, and assumes no violation in threshold inclinations and declinations across scenarios. This would correspond to a catchment where the SO is concerned only about stage-flood levels. Therefore, $Vz^{s+}_{k,t+1}$ and $Vz^{s-}_{k,t}$ = 0, then $\phi^s \chi^{s+}_{k,t+1}$, and $\phi^s \chi^{s-}_{k,t}$ = 0; consequently, $\phi^s \varphi^s_{k,t}$ =0. Thus, the relevant dual equations are as follows:

$$\sum\nolimits_{t=1}^{T} \phi^s \lambda^s_{k,t} + \theta^s_k = \phi^s \frac{\partial C^f_k\left(f^s_k\right)}{\partial f^s_k}, \quad \forall\ k,s \qquad : \ f^s_k \qquad [5.17]$$

In Equation [5.17], the marginal value equals the marginal increment in flood damage for an extra unit of peak flow at control point *k* and scenario *s*.

Participants will face prices for their changes in IC allowances. These prices account for the expected changes in flood costs due to participants' contributions to maximum flood depth. Considering a flood distribution with a single maximum peak across scenarios at channel places, the dual variable $\lambda_{k,t}^{s*}$ will be non-zero in time *t**, so individual flow impacts $H_{i,j,k}^{t*,s}$ will be priced in the time *t**–*u*+1. Therefore, the SO will charge $\mu_{i,j}^{D}$ and will pay $\mu_{i,j}^{S}$ to participants for their changes IC allowances based on dual variables as follows:

$$\mu_{i,j}^{D} = \sum_{s=1}^{S} \sum_{k=1}^{K} H_{i,j,k}^{t*,s} \phi^{s} \frac{\partial C_k^f \left(f_k^s \right)}{\partial f_k^s}, \quad \forall\ i,j \qquad [5.18]$$

$$\mu_{i,j}^{S} = \sum_{s=1}^{S} \sum_{k=1}^{K} H_{i,j,k}^{t*,s} \phi^{s} \frac{\partial C_k^f \left(f_k^s \right)}{\partial f_k^s}, \quad \forall\ i,j \qquad [5.19]$$

5.4.2 Depth, rapid flooding and flood duration

The dual conditions account for all violations including: peak depth, lengthening duration and hastening peak time of flooding across scenarios and control points. By complementary slackness, $Vz_{k,t+1}^{s+}$ and $Vz_{k,t}^{s-}$ > 0; hence variables $\vartheta_{k,t}^{s+}$ and $\vartheta_{k,t}^{s-}$= 0 respectively. The last means constraints [5.13] and [5.14] are violated; therefore, the corresponding dual conditions will be as follows:

$$\phi^{s} \chi_{k,t}^{s+} = \phi^{s} \frac{\partial\ C_k^{v+} \left(Vz_{k,t}^{s+} \right)}{\partial Vz_{k,t}^{s+}}, \quad \forall\ k,s, \text{ and } t' \leq t \leq t'' \qquad : Vz_{k,t}^{s+} \qquad [5.20]$$

$$\phi^{s} \chi_{k,t}^{s-} = \phi^{s} \frac{\partial\ C_k^{v-} \left(Vz_{k,t}^{s-} \right)}{\partial Vz_{k,t}^{s-}}, \quad \forall\ k,s \text{ and } t^* \leq t \leq t^{**} \qquad : Vz_{k,t}^{s-} \qquad [5.21]$$

The dual condition [5.20] represents the marginal flood damage for an additional unit of flow, which in turn hastens floods at time *t*, control point *k* and scenario *s*, in the period *t'* to *t''*.

The dual condition [5.21] represents the cost for an extra unit of flow at time *t*, control point *k* and scenario *s*, in the period *t** to *t***. These flows will lengthen the flood duration, and so they will also increase flood damage. Thus, those participants whose changes in IC allowances lengthen flood duration face a higher price.

The duals for slope costs measure the effect of increasing damage between time *t*–1 and *t*, and for the reducing flow effect in the next time *t* and *t*+1, given the increments in flow (depth) at time t, $\left(\frac{\partial\ C_k^{v+}\left(Vz_{k,t}^{s+}\right)}{\partial Vz_{k,t}^{s+}}-\frac{\partial\ C_k^{v+}\left(Vz_{k,t+1}^{s+}\right)}{\partial Vz_{k,t+1}^{s+}}\right)$. Similarly, the threshold violation accounts for the increment in the flooding duration, given the increasing flow in time t+1 $\left(\frac{\partial\ C_k^{v-}\left(Vz_{k,t}^{s-}\right)}{\partial Vz_{k,t}^{s-}}-\frac{\partial\ C_k^{v-}\left(Vz_{k,t+1}^{s-}\right)}{\partial Vz_{k,t+1}^{s-}}\right)$. Note that period length Δt is constant. Thus, changes in slope are measured by the changes in flows or depth.

Figure 5-5 illustrates some hypothetical effects for enlarging flows and cost based on changes in slope, and on the reducing effect in the following period. The increment in flows over time t would reduce the slope to ${Vz_{k,t+1}^{s+}}^{**}$ between period t and t+1, which in turn would reduce the flood cost $\frac{\partial\ C_k^{v+}\left(Vz_{k,t+1}^{s+}\right)}{\partial Vz_{k,t+1}^{s+}}$. However, this change increases the slope to ${Vz_{k,t}^{s+}}^{**}$ and the flood cost to $\frac{\partial\ C_k^{v+}\left(Vz_{k,t}^{s+}\right)}{\partial Vz_{k,t}^{s+}}$, between period t–1 and t.

Consequently, the new dual condition that accounts for the above violations is Equation [5.22], which now includes the threshold violations. By arranging and substituting, the dual variable $\phi^s\varphi_{k,t}^s$ will become as follows:

$$\phi^s\varphi_{k,t}^s=\phi^s\left(\frac{\partial\ C_k^{v+}\left(Vz_{k,t'}^{s+}\right)}{\partial Vz_{k,t'}^{s+}}-\frac{\partial\ C_k^{v+}\left(Vz_{k,t'+1}^{s+}\right)}{\partial Vz_{k,t'+1}^{s+}}\right)+\phi^s\left(\frac{\partial\ C_k^{v-}\left(Vz_{k,t^*}^{s-}\right)}{\partial Vz_{k,t^*}^{s-}}-\frac{\partial\ C_k^{v-}\left(Vz_{k,t^*+1}^{s-}\right)}{\partial Vz_{k,t^*+1}^{s-}}\right)$$

$$\forall k,s,t \quad \text{for time } t' \text{ and } t^* \text{ in period } t' \text{ and } t^* \qquad [5.22]$$

The dual variable $\phi^s\varphi_{k,t}^s$ represents the marginal value for an additional unit of flow at time t, control point k, and scenario s. The dual accounts for the marginal damage due to hastening peak flood, and the damage for lengthening flood duration.

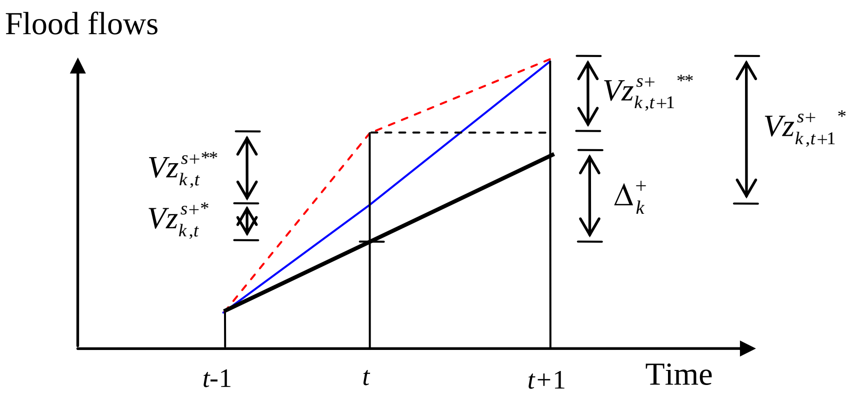


Figure 5-5 Effect of changes in flows which lengthen flooding between time t–1 and t+1.

The dual variables in Equations[5.17], [5.20] and [5.21] are a set of simultaneous equations. Any change in IC allowance, and so in flow would modify all the dual variables. Any change in flows would affect the main dual condition as well as other related dual conditions.

Those participants whose changes in IC allowances increase these violations will face a greater price. Those who reduce these violations will face equivalent reducing prices. For instance, participants could reduce the stage-flood level, but would hasten flooding. On the other hand, other participants could be incentivised to delay their runoffs and flows, because the cost of lengthening duration is less than the cost of hastening the peak time of flooding and increasing peak flow.

The dual variables $\mu^{D}_{i,j}$ and $\mu^{S}_{i,j}$, represented by the Equations [5.23] and [5.24], account for all previous threshold violations. Thus, the duals correspond to the expected total marginal flood costs for changes in flow patterns at channels. Any change in flood distribution will be penalised and priced.

$$\mu^{D}_{i,j} = \sum_{s=1}^{S} \sum_{k=1}^{K} \overbrace{H^{t^{*},s}_{i,j,k}\,\phi^{s}\,\frac{\partial C^{f}_{k}\left(f^{s}_{k}\right)}{\partial f^{s}_{k}}}^{Peak\ flow} +$$

$$\sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t}^{T} H^{t-u+1,s}_{i,j,k}\,\phi^{s} \left(\overbrace{\left(\frac{\partial C^{v+}_{k}\left(Vz^{s+}_{k,t'}\right)}{\partial Vz^{s+}_{k,t'}} - \frac{\partial C^{v+}_{k}\left(Vz^{s+}_{k,t'+1}\right)}{\partial Vz^{s+}_{k,t'+1}} \right)}^{Hastening\ flooding} \right) +$$

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t}^{T} H_{i,j,k}^{t-u+1,s}\,\phi^{s}\left(\overbrace{\left(\frac{\partial C_k^{v-}\left(Vz_{k,t^*}^{s-}\right)}{\partial Vz_{k,t^*}^{s-}}-\frac{\partial C_k^{v-}\left(Vz_{k,t^*+1}^{s-}\right)}{\partial Vz_{k,t^*+1}^{s-}}\right)}^{Lengthening\ flooding}\right)$$

$\forall$ *k,s,t* for time *t'* and *t** [5.23]

$$\mu_{i,j}^{S}=\sum_{s=1}^{S}\sum_{k=1}^{K}\overbrace{H_{i,j,k}^{t^*,s}\,\phi^{s}\,\frac{\partial C_k^{f}\left(f_k^{s}\right)}{\partial f_k^{s}}}^{Peak\ flow}+$$

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t}^{T} H_{i,j,k}^{t-u+1,s}\,\phi^{s}\left(\overbrace{\left(\frac{\partial C_k^{v+}\left(Vz_{k,t'}^{s+}\right)}{\partial Vz_{k,t'}^{s+}}-\frac{\partial C_k^{v+}\left(Vz_{k,t'+1}^{s+}\right)}{\partial Vz_{k,t'+1}^{s+}}\right)}^{Hastening\ flooding}\right)+$$

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t}^{T} H_{i,j,k}^{t-u+1,s}\,\phi^{s}\left(\overbrace{\left(\frac{\partial C_k^{v-}\left(Vz_{k,t^*}^{s-}\right)}{\partial Vz_{k,t^*}^{s-}}-\frac{\partial C_k^{v-}\left(Vz_{k,t^*+1}^{s-}\right)}{\partial Vz_{k,t^*+1}^{s-}}\right)}^{Lengthening\ flooding}\right)$$

$\forall$ *k,s,t* for time *t'* and *t** [5.24]

5.4.3 Theoretical illustration for participants' price allowances

This section illustrates flooding patterns that, in theory, could be observed across scenarios. The illustration shows how dual prices include the flooding components related to flood damage, and how participants who affect flooding have to face those prices. These prices depend on the probability of each storm scenario, participants' coefficients, and flood costs across control points (see Equations [5.23] and [5.24]). Assuming that flood cost functions represent real social costs, the prices that participants would face may be considered socially efficient.

Let us analyse a developer who desires to change the IC allowance on their property. Assume that their current flow impact is just one channel control point across storm scenarios, no one exercises any market power, and most storm scenarios present similar flooding patterns at the control point. Accordingly, with the new imperviousness, flows will increase the maximum flood depth and flows, and the price for changing the impervious level from *m* to *j* will be estimated as follows:

$$\mu_{i,j}^{D} = \sum_{s=1}^{S}\sum_{k=1}^{K} H_{i,j,k}^{t*,s}\,\phi^{s}\,\frac{\partial C_k^f\left(f_k^s\right)^{**}}{\partial f_k^s}\ ,\ \forall\ i,j \qquad [5.25]$$

Figure 5-6-*A* illustrates the change in flood pattern, given the increment in IC allowance in the property, and so in the clearing prices $\frac{\partial C_k^f\left(f_k^s\right)^{*}}{\partial f_k^s}$ to $\frac{\partial C_k^f\left(f_k^s\right)^{**}}{\partial f_k^s}$ in the storm scenario *s* and control point *k*. When participants reduce or change IC allowance, the price falls from $\frac{\partial C_k^f\left(f_k^s\right)^{**}}{\partial f_k^s}$ to $\frac{\partial C_k^f\left(f_k^s\right)^{*}}{\partial f_k^s}$ due to the changes on the flood impacts. Notice that flood damage has accounted for the stage-flood depth. Consequently, the price will be as follows:

$$\mu_{i,j}^{D} = \sum_{s=1}^{S}\sum_{k=1}^{K} H_{i,j,k}^{t*,s}\,\phi^{s}\,\frac{\partial C_k^f\left(f_k^s\right)^{*}}{\partial f_k^s}\ ,\ \forall\ i,j \qquad [5.26]$$

Patterns *B* and *C* illustrated in Figure 5-6 depict changes in flooding due to shifting the time of the stage-flood. In the case of *B*, the maximum flow depth remains constant, but flood costs related to the slope's violation increase. Consequently, allocation could change, which will be explained next. Participants who previously traded with similar prices would face a lower or greater price; the new prices will depend on their contribution to the new stage-flood. Furthermore, those prices could be lower or greater than before, as $H_{i,n,k}^{t**,s} > H_{i,n,k}^{t*,s}$ or $H_{i,n,k}^{t**,s} < H_{i,n,k}^{t*,s}$ at the new stage-flood time respectively. Thus, their bids (supply) cannot be accepted nor can they trade.

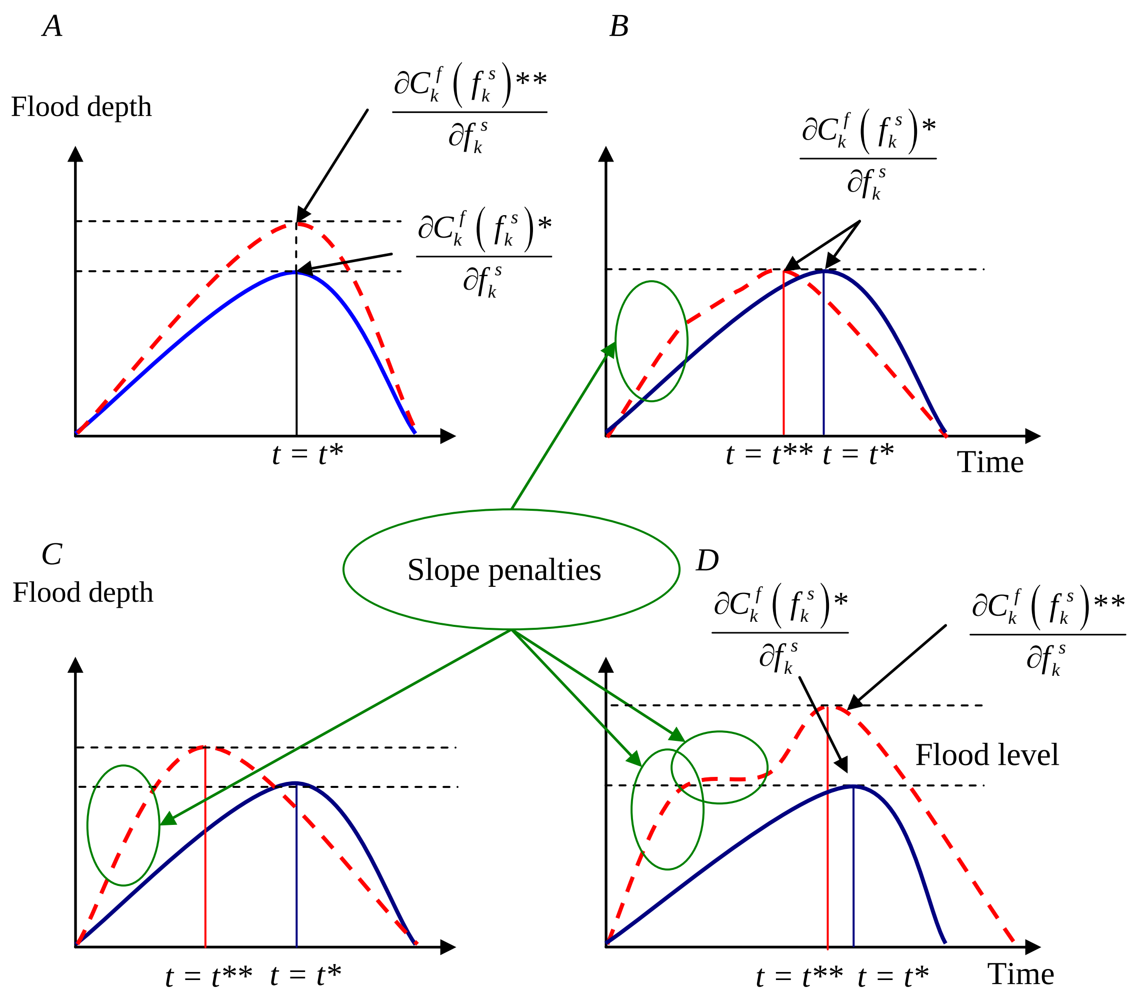


Figure 5-6 Hypothetical changes in flood patterns. Solid lines represent initial flood and dashed lines are resulting floods. Circles represent the area where slopes are violating the thresholds for lengthening flood peaks and duration. *A* corresponds to changes in peak, *B* changes the peak time, *C* changes in maximum peak and peak time, and *D* changes in peak time, maximum peak and flood duration.

A similar situation can be observed when IC allowances change, causing a slope violation for hastening peak flooding. A participant who desires to change IC allowances and violate thresholds could finish paying the following price $\mu_{i,j}^{D}$.

$$\mu_{i,j}^{D} = \sum_{s=1}^{S} \sum_{k=1}^{K} H_{i,j,k}^{t^*,s} \phi^s \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s} +$$

$$\sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t}^{T} H_{i,j,k}^{t-u+1,s} \phi^s \left(\frac{\partial C_k^{v+}\left(Vz_{k,t'}^{s+}\right)}{\partial Vz_{k,t'}^{s+}} - \frac{\partial C_k^{v+}\left(Vz_{k,t'+1}^{s+}\right)}{\partial Vz_{k,t'+1}^{s+}} \right) \quad [5.27]$$

This dual variable $\mu_{i,j}^{D}$ for IC allowance accounts for the changes in flow at the new peak time, from t^* to t^{**}, and for the marginal changes in flows which will shift slopes $Vz_{k,t}^{s+}$ and related flood cost $\frac{\partial\ C_k^{v+}\left(Vz_{k,t'}^{s+}\right)}{\partial Vz_{k,t'}^{s+}} - \frac{\partial\ C_k^{v+}\left(Vz_{k,t'+1}^{s+}\right)}{\partial Vz_{k,t'+1}^{s+}}$ over times, and storm scenarios. In this case, a participant may have paid previously for the previous flood conditions. With the new flood patterns, their changes in allowances are not increasing the peak, so he/she could increase imperviousness and pay less for these changes. Actually, the participant could be paying only for hastening the peak time of flooding (flash flood).

With pattern *C* (Figure 5-6), the floodplain has similar effects with shifting peak and raised flood cost. In this flood pattern, participants face a greater flood cost for increasing the maximum peak damage from $\frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s}$ to $\frac{\partial C_k^f\left(f_k^s *\right)}{\partial f_k^s *}$, where $\left(\frac{\partial C_k^f\left(f_k^s *\right)}{\partial f_k^s *} > \frac{\partial C_k^f\left(f_k^s\right)}{\partial f_k^s}\right)$. Participants also face higher-slope costs for hastening peak flood time, t to t^{**}. Furthermore, allocations would also change under the new flood patterns, similar to the previous *B* illustration.

Finally, the final IC allowances with pattern *D* would hasten the stage-flood time, increase the maximum depth, and lengthen the flood duration. Thus, if a developer desires to change IC allowances to reach this flood pattern, he/she should pay for these changes as well as for the expected flood damage. If most storm scenarios remain unchanged, and the flooding does not change significantly, the dual variable used to charge those changes in IC allowances will correspond to those calculated by Equation [5.23].

Two numerical examples are used to show the penalties for hastening peak flow and flood duration. Both examples account also for peak flows. A participant wants to change imperviousness in their property, which theoretically keeps the stage flood in the area across storm scenarios, but flows could hasten peak time or could lengthen duration. To simplify the penalty analysis, we use only one scenario. Figure 5-7 illustrates the hydrograph curves at the control point. The initial flow condition has a peak flow of 36 m^3/sec at time 5 with incremental flows of 4 m^3/sec before reaching the peak, and decreasing flows of 4 m^3/sec after reaching the peak. Channel capacity is 20 m^3/sec. The SO defines the penalising period as those before and after time 5, and penalises by

\$10/(m^3/sec) any flows above channel capacity (based on changes in the initial hydrograph). The SO additionally penalises peak flow by \$5/(m^3/sec). The illustration considers a market model that accounts for Equations [5.4] and [5.5].

In the first example, a participant wants to increase IC allowances in 10 ha and bids \$15/ha for 5 ha, \$10/ha for 3 ha, and \$5/ha for 2 ha, which hastens peak from time 5 to time 4 if whole area increases imperviousness. If the SO does not include penalty for hastening the participant could change the 10 ha and pays nothing. However, if the market accounts for a hastening peak, the participant could increase imperviousness only in 8 ha, paying \$56 in total. The peak flow is also reduced by 0.8 m^3/sec at the control point.

In the second example, the participant wants to change the hydrograph curve, to lengthen duration from time 9 up to time 13. The participant keeps the same bid preferences. In this case, if the SO does not include the penalty for duration, the participant could change imperviousness, but pays nothing. However, if the penalty for lengthening is included, the participant then faces this cost. In this case, the participant changes 5 ha and pays \$62.5 in total. Thus, even though both examples include penalties, the participant could change imperviousness, but he/she faces the cost for hastening or lengthening. In both examples with penalties, the participant could not change imperviousness in their whole property.

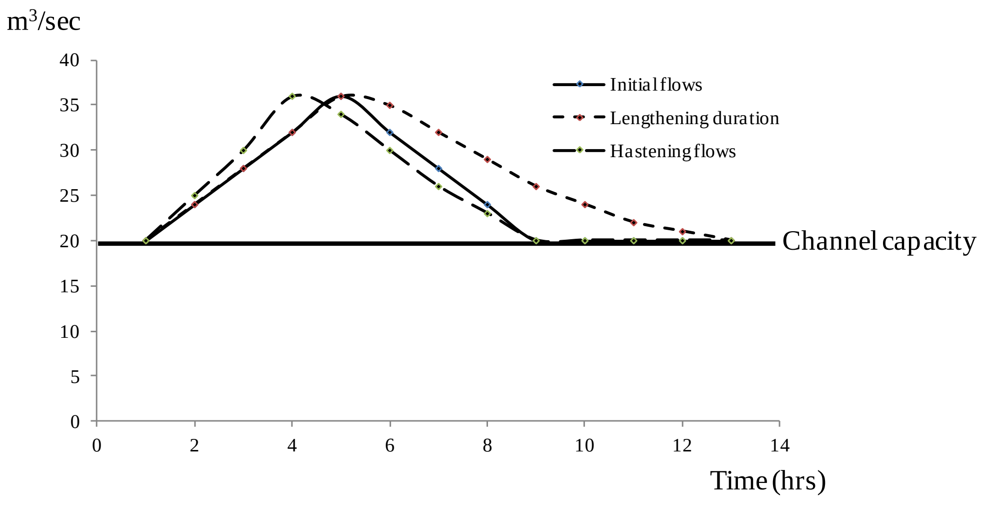


Figure 5-7 Initial and wanted flows at a hypothetical control point. Channel capacity is 20 m^3/sec.

5.4.4 Initial IC allowances and the SO's revenue

This section extends the discussion regarding initial IC allowances from Sto_marketIC1 described in Chapter 4. With model Sto_MarketIC2, the SO reaches similar revenue positions related to initial allowances and preferences from participants in the market.

The Sto_MarketIC2 formulation establishes that allowances are a set of rights to impact on flows. The initial IC allowances, in physical terms of flooding, correspond to the set of total flows $\sum_{i=1}^{N}\sum_{j=1}^{J}\bar{H}_{i,j,k}^{t-u+1,s} A_{i,j} = \hat{f}_{k,t}^{s}$, $\forall s,k,t$. These impacting rights can be decomposed into a set of valued capacity rights, in terms of maximum peak flows and flood depth, and marginal increments of flows and depths by time such as $\Delta_{k,t}^{s+}$ and $\Delta_{k,t}^{s-}$ at control points for hastening the peak time of flooding and lengthening flooding in each storm scenario (channels and floodplain areas or receptor constraints). Thus, each landholder has some initial allowance corresponding to a set of individual peak flow impacts on receptor constraints $\sum_{j=1}^{J}\bar{H}_{i,j,k}^{t-u+1,s} A_{i,j} = \tilde{f}_{i,k,t}^{s}$, $\forall$ s,k,t. Each landholder also has a set of initial rights for changing flow levels, such as $\sum_{j=1}^{J}\left(\bar{H}_{i,j,k}^{t-u+2,s} - \bar{H}_{i,j,k}^{t-u+1,s}\right)A_{i,j}$ for time t=1,…,T, scenario s=1,...,S, and control point k=1,…,K. These rights link to flood components which account for the flood distribution.

When participants shift the flood distribution, the SO may be a net receiver or payer; for instance, the SO may be a net receiver if she/he sells peak flow capacity $M_k - \sum_{i=1}^{N}\sum_{j=1}^{J}\bar{H}_{i,j,k}^{t-u+1,s} A_{i,j}$ at the peak flow time t*. The SO may also sell hastened flood time capacity $\Delta_{k,t+1}^{s+} - \left(z_{k,t+1}^{s} - z_{k,t}^{s}\right)$ and flood duration capacity $\left(z_{k,t-1}^{s} - z_{k,t}^{s}\right) - \Delta_{k,t}^{s-}$ at some scenarios. So, the SO may be a net receiver if participants shift their flows and increase peak flows across storm scenarios. In this case, participants are prepared to pay high prices for the violated flow thresholds that account for the expected flood damage from increasing peaks at control points. The SO receives the increase in the expected flood damage and revenue for selling loose capacities.

The SO may be a net payer, when participants offer to change IC allowances which reduce stage-floods at prices lower than the flood cost for the expected flood damage, and when other participants demand with sufficiently low prices. For instance, a participant may offer a price $P_{i,j,b}^{S}$ lower than the flood cost for the changes in the expected flood damage.

Additionally, the SO may face revenue neutrality, in which he/she does not sell or buy any net change in violating capacities at control points. In this case, the opportunity cost for changing participants' IC allowance matches the expected marginal flood damage for the current flood distribution. As a result, the flood distribution is retained at the end. The SO can also reach revenue neutrality when he/she is a net payer at some control points, and net receiver in others, while matching the condition $RE \approx 0$ at the end.

5.5 Final remarks and conclusion

Chapter 5 described model Sto_MarketIC2, and how its equilibrium prices follow changes in flood distribution. The flood distribution cost is approximated in terms of the maximum peak flow that could be reached in the flood area, for hastening peak flooding, and flood duration. Thus, Sto_MarketIC2 accounts for spatial and temporal flood damage in the catchment. Participants who change IC allowances, and hence the flood distribution, will internalise the changes in the expected flood damage.

The market establishes a set of rights for boundaries which account for the final flood distribution in the catchment. These rights represent flows that could affect the catchment when extreme storms are faced. These rights also represent physical routing of flows that link flood components such as velocity and duration with damage.

The SO could use the proposed constraints and thresholds to manage desired flood conditions. The cost could account for hastening flooding in a specific period, based on a desired depth increase, as discussed in this chapter. However, the SO could also establish a flow threshold, grant IC allowances for flows below a specified level, and penalise any change above this threshold. The final IC allowances in the catchment will take into account the desired condition.

Participants will observe that increasing IC allowances could hasten flooding, so they would receive a payment to reduce imperviousness of their properties. However, other participants could receive a payment or pay less for increasing imperviousness if flows are moved to loose threshold conditions. The catchment could reach impervious levels which reduce flash flood conditions.

In the same way, a flood cost function could account mainly for flood duration. So the catchment could reach IC allowances which could reduce the flood duration. The final condition depends on the desired condition that the SO wants to reach or keep in the catchment. The SO would not consider the cost for lengthening flow duration if the concerns were peak depth and hastening flooding. Any IC allowance that delay flows would be granted under this condition.

With participants located upstream and downstream of a control point, participants' impacting flows contribute to flood damage based on the decomposed flood damage components. Upstream participants could notice that reducing imperviousness may reduce peak and delayed flows, but those flows lengthen flood duration. Downstream participants, located close to the flooding areas, could notice that increasing imperviousness could raise flows, increasing flow velocity and contributing to hastening peak flow. These flows would be penalised. Hastening peak flows contribute to reduce the warning time and flash flood could be noticed often in the area.

Thus, participants trade under the proposed thresholds and costs which account for the desired flood in the catchment. Participants prepared to pay for violating thresholds and so raise flood costs could change IC allowances.

If a flood disaster affects the catchment, the cost would probably be greater than the expected damage that participants were able to internalise. Even under this market formulation, the risk for the new flood physical distribution is not fully internalised. The market design may include a hedging mechanism to keep society safe from frequent flooding. The next chapter proposes a hedge based on the conditional value at risk.

Chapter 6

6 HEDGING VIA CONDITIONAL VALUE AT RISK (CVaR)

6.1 Introduction

The market models, Sto_MarketIC, proposed in Chapters 4 and 5 considered the stochasticity of rainfall events. However, they do not consider extreme flood risk explicitly, which the SO may face when participants' trade changes the flood distribution. Furthermore, Sto_MarketIC was modelled as a risk neutral design, which implies that people in the flood area are risk neutral. The final flood level was defined by clearing the market. Changes in flood damage were penalised and estimated by the TSSP recourse, which determines expected flood costs based on damage within the catchment. The damage can be viewed as a socioeconomic cost, such as for flood warning, mitigation and damage to infrastructure and contents, and for the purpose of initiating evacuation procedures.

The Sto_MarketIC clearing model was able to estimate the expected damage for the changes in the flood distribution across a range of storm events. However, if an extreme event occurs, the disaster damage could be far higher than the expected damage cost. For example, the expected cost of flooding could be $1 million; however, it could exceed $10 million in a single disaster. In this chapter, this effect is accounted for, with a view to include the community risk of extreme flood events within the catchment.

The IC allowances implicitly correspond to flood rights for a specific flood distribution. Thus, in terms of flooding, people could desire a specific risk position with respect to flooding. On the other hand, participants in the upper catchment area could own flood rights in the catchment. Accordingly, it is proposed that a risk measure represent the flood risk that the community desires within a given catchment. This risk should account for the probable maximum floods that could occur in the area, acknowledging that there is always a nonzero probability of a higher flood (Ely and Peters 1984; Stedinger and Griffis 2008). Those participants who may attempt to change this profile would internalise the risk. Accordingly, the SO would be concerned with flooding in excess of the maximum flood levels set by the community.

Figure 6-1 illustrates the link between the rainfall intensity distribution (A), flood damage (B) and flood distribution (C) with different final IC within the catchment, which represents the flood disaster related to the final IC, and assuming normal AMC type II. The filled area in B represents a disaster relating to a storm event, and the filled area in C represents the SO’s risk assessment for possible flood disasters.

So far, flood rights in terms of risk positions in the market have not been discussed. People, particularly those residing on flood plains may desire to have rights *per se* as a means of being safe from any flood event. This situation has motivated authorities to focus on providing flood protection via structural policies rather than managing impervious cover. A new status quo of IC allowances will be reached in the catchment, which may allow achievement of a risk profile based on what the community may desire to ensure flooding is avoided within a given catchment.

This chapter presents how the SO could manage risk in the market clearing formulation, to hedge against changes in flood damage associated with extreme storm events. It is unlikely that severe flood damage associated with extreme storm events can be totally eliminated, but the market could be expected to reduce such losses. Such a limit represents the risk position as downside risk. This chapter proposes to use Conditional Value at Risk "CVaR" as the risk profile. Section 6.2.4 will analyse CVaR further. Thus, the SO can hedge against a range of extreme storms and anticipated damage. The risk position for extreme storm events could be used for the design of suitable infrastructure in the long term such as roads and bridges, as stressed by Yeou-Koung (2005).

The implication could be viewed as equivalent to an electricity market which includes generators that increase the risk of failure for frequency, voltage and blackouts. Thus, the grid may have a higher probability of failure, and consequently increase the cost of system collapse. Transpower (2008) noticed such an effect with wind generators as part of the grid. Indeed, this risk of failure may be more than the desired risk in the system.

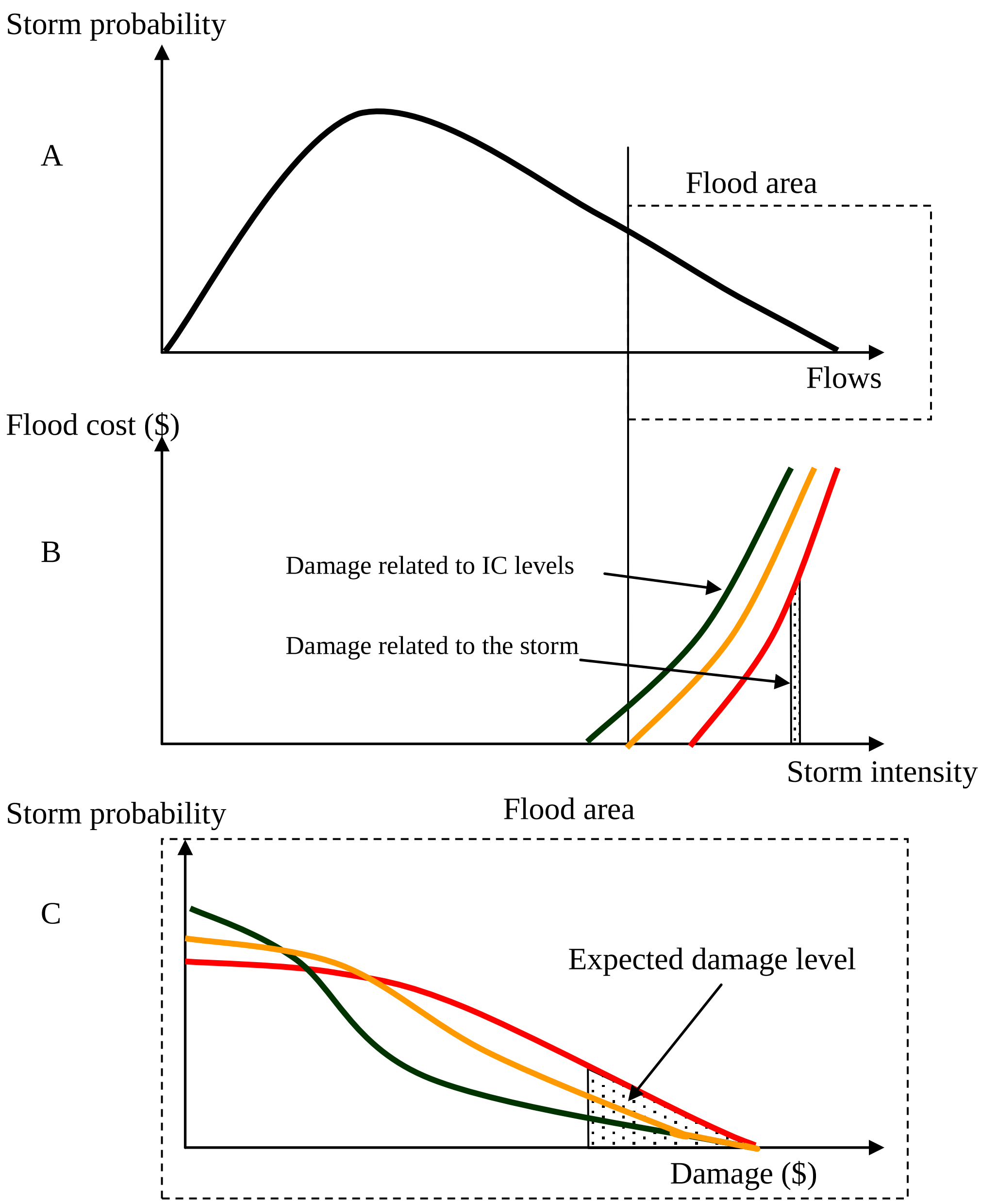


Figure 6-1 Rainfall and flood distribution under different final IC allowances. A is the storm probability distribution, B is the final damage related to IC allowances and storm intensity, and C is the storm probability and resulting flood damage.

Uncertainty related to flooding determines the risk that is perceived by the different agents in the market (participants, SO and people in the flood area). In fact, any decision that modifies flood distribution in the catchment implies changes in risk positions which in turn are transferred to landholders. However, in this research it is assumed that the SO is capable of representing the risk of catchment participants who wish to avoid flood damage.

Possible hedged positions (and actions) that people in the flood area could have with insurance are not considered. In addition, it is assumed the risk of flood damage depend on rainfall uncertainty and the IC levels within the catchment. The SO would determine which flood distributions would be preferred over others, and whether these preferences would be included in the market clearing model.

Because not all storms cause flood damage, the market design deals a situation with non-zero probability of no flood and no damage cost. This may affect the implementation and application of the risk concept, which the SO should carefully analyse. The SO may wish to maintain a flood distribution in the catchment, and because he/she mis-estimated the risk position, the flood damage could be more than the desired condition. On the other hand, the model could be over-constrained, and make the solution infeasible. These points will be further discussed in this chapter.

Section 6.2 resumes the decision-making under risk, and introduces concepts of risk aversion, stochastic dominance, risk measures and CVaR. Section 6.3 presents a form to include CVaR in the market formulation. Section 6.4 describes the market model. Section 6.5 discuses dual prices with CVaR. Section 6.6 presents examples. Section 6.7 discuses misestimating CVaR values. Section 6.8 discuses possible issues with hedging via CVaR. Section 6.9 is final remarks and conclusions.

6.2 Decision-making under risk

The variability of random variables generates risk (Fishburn 1980; Kall and Wallace 1994; Birge and Louveaux 1997). Many approaches have been proposed to manage and measure risk, and the most suitable method depends on the particular situation.

Extensive research on risk has been based on expected utility theory, stochastic dominance, downside risk, and other investment decision approaches, such as the mean-variance (Von Neumann and Morgenstern 1947; Markowitz 1952; Pratt 1964; Arrow 1971; Fishburn 1977; 1980; Rockafellar and Uryasev 2000; Levy 2006). Von Newman and Morgenstern (1947) developed the theory of expected utility and risk preferences, where the decision maker is characterized according to their utility function *U*, which is based on a distribution of outcomes with preferences over lotteries. Arrow (1971) and Pratt (1964) showed that the risk premium changes when the decision maker is risk averse, and this premium can be decreasing, increasing, or constant with wealth. Markowitz (1952) proposed a mean-variance measure to trade off risk and return. Fishburn (1977; 1980) defined a risk measure based on the downside variance, which is the lower partial moment of the distribution from a boundary position *t*, or a weighted probability function of polynomial deviations below a specified target. Thus, if the polynomial's order is greater than 1, the decision maker will be risk-sensitive. Additionally, Fishburn introduced the concept of stochastic dominance as a way to analyse risky decisions with partial knowledge of *U* and with the moments of the probability distribution. Eppen et al. (1989) noted that the measure of downside risk or "tail risk" is a particular case of the mean-risk dominance proposed by Fishburn (1977), which measures the risk faced due to losses. In a catchment based market, the risk due to losses is where the SO faces extreme flood related or disasters. Rockafellar et al. (2000; 2001) and Pflug (2000) introduced the concept of CVaR as a measure of downside risk.

6.2.1 Risk aversion

The utility function and preferences of a risk averse decision-maker will be as follows:

- The utility function *U* represents risk preferences with a non-negative first derivative condition $U' \geq 0$ and a non-positive second derivative $U'' \leq 0$. However, the utility function with regards to losses has a non-positive first derivative condition $U' \leq 0$.
- The utility function is continuous over a convex set of preferences.

  Jensen's inequality holds: the expected utility is equal to or smaller than the utility of the expected return. $E\left[U\left(w_0+x\right)\right]= pU\left(w_0+x_1\right)+ \quad (1-p)U\left(w_0+x_2\right)$ and $E\left[U\left(w_0+x\right)\right] \leq U\left(w_0+px_1+(1-p)x_2\right)= U\left(w_0+E\left[x\right]\right)$

  A risk avoider will not be a *fair payer*, and will not play a *fair game*. That means avoiders do not pay the expected prize of a game. However, they have the willingness to pay for a positive risk premium π to insure their wealth level. By Jensen's inequality $E\left[U\left(w_0+x\right)\right] \leq U\left(w_0+E\left[x\right]\right)$, there would be a premium π where $E\left[U\left(w_0+x\right)\right]=U\left(w_0+E\left[x\right]-\pi\right)$. Thus, risk avoiders may be willing to pay an expected value with a view to reducing risk to obtain a similar utility position as close as possible to their expected utility with the game.

6.2.2 Stochastic dominance

Stochastic dominance is used in decision making theory to order revealed preferences. This term refers to gambling where there is a probability distribution over possible outcomes such as IC allowances; thus, floods can be treated as though they were games. However, stochastic dominance does not give a complete ordering for higher moments. Let us see the way that stochastic dominance conditions and preferences may affect the IC market.

U_1 (utility) dominates U_2 if preferences for one flood distribution dominate another flood distribution in U_2, for all flood scenarios x. Thus, the expected value of U_1, i.e., $E[U_1(x)]$ is greater than the expected value of U_2, $E[U_2(x)]$, for all x (flood distribution) with at least one strict inequality.

There are several types of stochastic dominance (Levy 2006). These include the following:

First degree stochastic dominance (*FSD*) rules rely on probability distribution functions. For instance, let us assume that G and F are two cumulative distributions of two impervious decision levels, which relate to flood distribution, then F dominates G if $F(x) \leq G(x)$ for all flood levels x, and consequently $\mathrm{E}_F[U(x)] \geq \mathrm{E}_G[U(x)]$ for any monotonic $U(x)$.

Measures associated with *FSD* include measures such as means, geometric means and the left tail condition, which is closely related to CVaR (Levy 2006). The necessary condition of left tail (losses) of the distribution is $\mathrm{Min}F(x) \leq \mathrm{Min}G(x)$ where the cumulative distribution area G is above the distribution F for each flood level x in the tail. That means, for each final flood event, the decision maker would prefer distribution F rather than G. However, according to Levy (2006), sometimes the left tail condition does not guarantee first stochastic dominance.

Second degree stochastic dominance (*SSD*) is closely related with risk preferences. If F and G are two flood conditions with density function $f(x)$ and $g(x)$ respectively, then F dominates G in the second degree if and only if $SSD \equiv \int_a^x \left[G(t) - F(t)\right] dt \geq 0$.

Alternatively the condition can be formulated as $\int_a^x \left[G(t) - F(t)\right] dt \geq 0$, and so

$E_F\left[U(x)\right]-E_G\left[U(x)\right]\geq 0$, for all x with at least one strict inequality for some x_0. Therefore given a decreasing risk-averse utility function ($U'(x)\leq 0$), a risk avoider prefers $F(x)$ to $G(x)$ in the range (a,b) if $\int_a^b\left[G(t)-F(t)\right]U'(x)dx\geq 0$. The risk premium corresponds to the amount of income that the SO is willing to pay for accepting a risky flood situation.

The final ICs (a portfolio) whose flood damage distributions dominate would be those with positive changes in utility, especially in extreme flood damage situations. Figure 6-2 shows distribution probabilities *F* and *G* with a *SSD* of *F* upon *G*. Since $U'(x)\geq 0$ and $U''(x)\leq 0$, extreme losses in distribution *G* will generate marginal disutility values which will be higher than those produced in *F*. Thus, the dominance of *F* would be expected for particular impervious levels in the catchment.

Third degree stochastic dominance (*TSD*) is related to preferences for positive skew and $U'''(x)$. However, *TSD* degree stochastic dominance are outside the scope of this thesis. Hence, this chapter will focus on risk measurement methods which are in accordance with *FSD* and *SSD*, which assumes risk averse The next section presents how risk measures could be modelled in the market formulation.

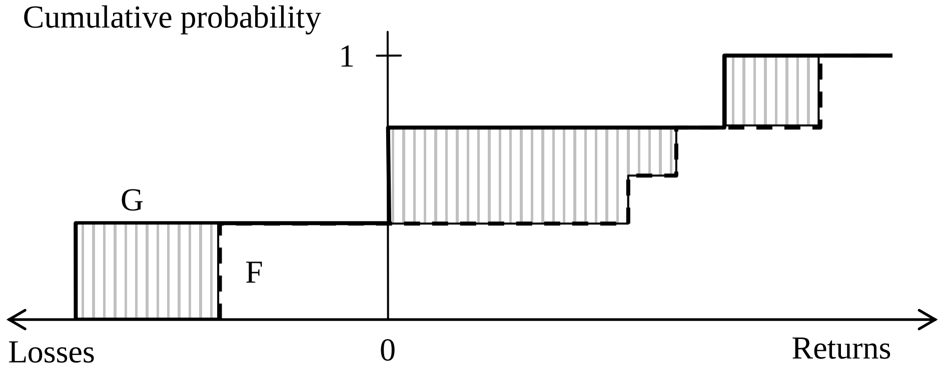


Figure 6-2 Illustration of two cumulative probabilities *G* and *F* where *F* is *SSD* to *G*. The grey areas correspond to differences in cumulative probabilities.

The stochastic dominance was introduced to support the idea that CVaR is a risk measure, which allows making decision in accordance with the first and second degree. Therefore, CVaR could be used in a market model as a flood risk profile that the community desires within a given catchment. However, the concept of stochastic dominance is not explicitly used in the market formulation.

6.2.3 Risk measure

Researchers and practitioners use various approaches to include risk in decisions problems. Such approaches include Mean Value-Variance (Markowitz 1952), Sharpe´s single factor (Sharpe 1964), Value at risk "VaR" (see, for instance, Duffie and Pan 1997; Linsmeier and Pearson 2000), Mean Absolute-Deviation (Konno and Yamazaki 1991), and CVaR (Pflug 2000; Rockafellar and Uryasev 2000; 2001). Equation [6.1] represents a general maximization (minimization) including risk measures.

$$\max_{x} \left\{ E\left[f\left(x,w\right)\right] - \rho D\left[\left[f\left(x,w\right)\right]\right] : x \in X \right\} \text{ or}$$

$$\min_{x} \left\{ E\left[f\left(x,w\right)\right] + \rho D\left[\left[f\left(x,w\right)\right]\right] : x \in X \right\} \qquad [6.1]$$

Where $f\left(x,w\right)$ is the function that represents the decision problem with uncertainty, D is a measure of risk, perhaps dispersion or variability, and ρ is a weighted trade-off between expected flood damage and risk measure. A second-order Taylor's approximation allows obtaining an expected value-variance (E-V) from a utility function where, the variance measure, both upwards and downwards deviations, are treated the same. However, in the market the focus will be on the extreme downwards side of the distribution, because most decision makers would be concerned about the losses. Ahmed (2006) proved that mean-variance criteria lead to computational intractability with stochastic programming such as two-stage stochastic linear programming with non-convex formulations.

On the other hand, non convexity could be noticed if a non linear utility function in damage is used. This could underestimate risk for the SO, so prices would not reflect changes in damage, and non-supporting prices could reached in the market. (This was discussed in Chapter 4). More imperviousness could be allowed in the catchment, increasing flooding problems.

A similar decision problem would occur when the measure of risk corresponds to the value at risk (VaR). VaR measures the maximum loss with a specified confidence level α (Pflug 2000). VaR fails to be consistent with subadditivity, because in a portfolio with two instruments the VaR may be greater than the sum of individual VaRs (Rockafellar and

Uryasev 2002). When studying alternative risk measures, Uryasev (2000b) noted that VaR is difficult to optimise due to non-convexity with multiple local extrema. Rockafellar and Uryasev (2002) pointed out that VaR does not handle losses beyond the threshold amount indicated as VaR and cannot discriminate worse situations.

Ogryczak and Ruszczynski (1999) pointed out that the mean-variance approach presents difficulties with stochastic dominance rules and with the task of modelling all risk preferences and consistencies which "may lead to inferior decisions." On the other hand, Ogryczak and Ruszczynski (2002) noted that under the worst conditional expectation, a mean-risk measure may be consistent with stochastic dominance. The authors also pointed out that asymmetric risk approaches such as downside risk measures would be in "harmony with the stochastic dominance order". In particular, the Conditional Value at Risk (CVaR) meets this condition, as well as other important properties related with *coherence* (Rockafellar and Uryasev 2000; Uryasev 2000b; Rockafellar and Uryasev 2001; Acerbi and Tasche 2002). *Coherence* of risk measure satisfies four axioms or properties of translation invariance, positive homogeneity, subadditivity, and monotonicity (for further details, see e.g., Artzner et al. 1999).

Comparing CVaR with a non-linear transformation of damage, CVaR preserves convexity (Rockafellar and Uryasev 2002). As stated in previous paragraphs, changes in the expected flood damage related to imperviousness level is convex, therefore a convex condition could also be reached with CVaR. CVaR has better performance and is considered a better measure of risk than Mean Value-Variance, VaR and Mean Absolute Deviation (Uryasev 2000a; Rockafellar and Uryasev 2002).

Comparing CVaR with the nonlinear transformation of damage, CVaR might control extreme changes at the tail of the flood damage distribution, but a non-linear transformation of flood damage could not avoid this change in flood damage. However, both costs will be faced for each participant in the Sto_MarketIC_Risk. This price implication will be further discussed in Section 6.5.

6.2.4 CVaR

CVaR measures the conditional expectation of losses that exceed the value at risk (VaR) at a confidence level α (Pflug 2000; Rockafellar and Uryasev 2000; Uryasev 2000b; Rockafellar and Uryasev 2001). Thus, a confidence level α of 0.95 indicates a 0.05 chance that the expected losses exceed a level $X. Recalling the flooding issue, the flood damage would be greater than a given boundary of cost. Thus, larger α values would lead to conservative impervious levels in the catchment due to the higher weight on the worse scenarios. Figure 6-3 shows storm probabilities and flood losses where the $CVaR_\alpha$ measures the expected flood losses above the confidence level α in the storm distribution. Note that the sum of probabilities related to flooding is not 1, since not all storm scenarios produce flood damage, only extreme storms do so.

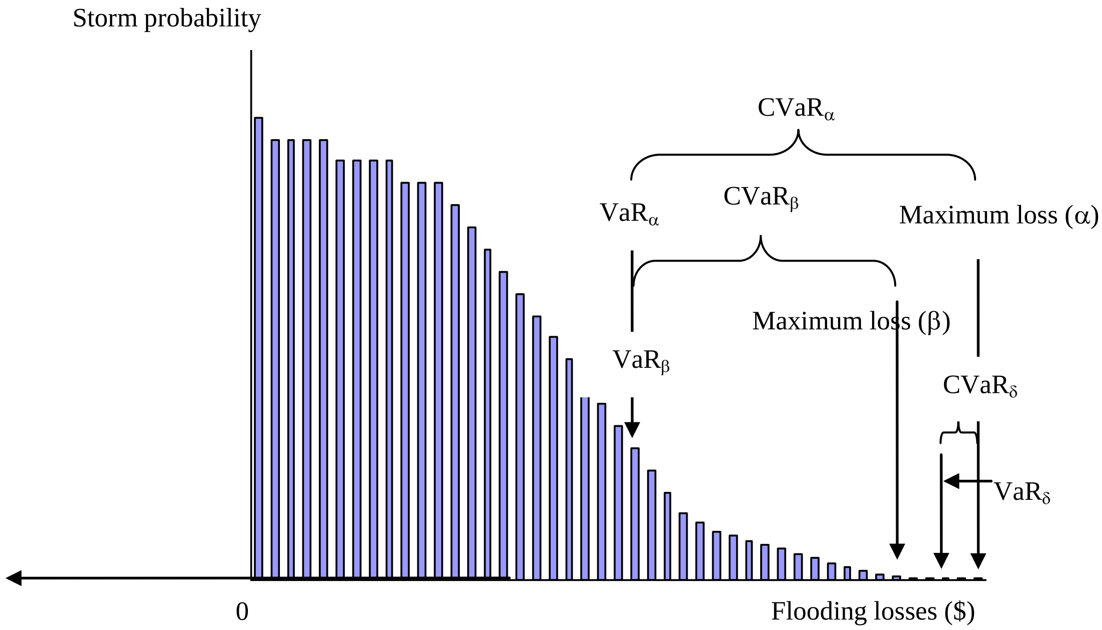


Figure 6-3 Hypothetical storm probabilities and flood losses under a final IC allowance in the catchment. α is the confidence level, $CVaR_\alpha$ is the conditional value at risk, and VaR_α is value at risk for confidence level α.

CVaR has been used in many areas to hedge against extreme losses, especially in portfolio management for trading operation in stock markets (see, e.g., Krokhmal et al. 2005; Mansini et al. 2007; Trindade et al. 2007; Sarykalin et al. 2008; Zheng 2009; Uryasev et al. 2010) and for making insurance decisions (Mulvey and Erkan 2006; Liu et al. 2008). Carrion et al. (2007) proposed CVaR for a large consumer participant in the electricity market. Here, the participant makes decisions about bilateral contracts, production and participation in the spot market via minimising the expected value of the procurement cost while limiting CVaR. Noyan (2010) proposed CVaR for selecting facility locations and inventory with uncertain demand, and included CVaR for disaster under emergency supplies. Piantadosi et al. (2008) proposed to use CVaR to manage storm water, to find optimal pumping policies, avoiding losses and reducing the risk of environmental damage.

The CVaR measure has been widely used for its consistency as a risk measure and it is in harmony with respect to stochastic dominance of order 1 and order 2. CVaR meets properties of *coherence* of functional preferences such as translation invariance, positive homogeneity, superadditivity, monotonicity, and consistency (for further details, see e.g., Artzner et al. 1999).

Street (2010) demonstrated consistency and convexity in an equivalent utility-probability function in CVaR, and the author pointed out that a utility function, measured as "probability-dependent utility", would bring an equivalence in terms of risk averse preferences between CVaR and the "certainty equivalent". Zheng (2009) pointed out that CVaR is a measure of skewed and tailed distributions (stochastic dominance of order 3).

A measure of CVaR at level α can be represented as follows:

$$CVaR_{\alpha} = (1-\alpha)^{-1} \int_{P(x,\eta)}^{\infty} Q(x,w)\, p(w)\, dw \qquad [6.3]$$

Where $Q(x,w)$ is a random variable that represents losses, $p(w)$ is the probability distribution, η is the value at risk η_{α} = min { $\eta_{\alpha} \in \Re$: $P(x,\eta) \geq \alpha$ } and $P(x,\eta)$ is the cumulative distribution function. Thus, CVaR can be represented as follows:

$$CVaR_{\alpha} = \inf_{\eta \in \Re} \left\{ \eta + \frac{1}{1-\alpha} E\left(\left[Q(x,w)-\eta\right]\right)_{+} \right\} \qquad [6.4]$$

Uryasev (2000b) presented an analytical representation for discretised density function $p(w)$ which is only available for *s* scenarios. This function is convex and solvable by LP:

$$\hat{F}_{\alpha}(x,\eta) = \eta + \nu \sum_{s}^{S} \left(Q(x,w)-\eta\right)_{+} \qquad [6.5]$$

The function $\hat{F}_{\alpha}(x,\eta)$ is linear with respect to the vector $Q(x,w)$ whereas ν is a constant equal to $\left((1-\alpha)S\right)^{-1}$, and storm scenario $s \in w$. Additionally, Uryasev (2000b) used auxiliary variables υ_s to substitute the terms $\left(Q(x,w)-\eta\right)_{+}$ into the expression $\hat{F}_{\alpha}(x,\eta)$, and imposed auxiliary constraints represented by $\upsilon_s \geq Q(x,w_s)-\eta$ with $\upsilon_s \geq 0$ The author showed an approach to approximate CVaR$_\alpha$ by a set of linear constraints C_α and confidence levels α (scenarios) by using the following constraint:

$$\eta + \nu \sum_{s=1}^{S} \upsilon_s \leq C_\alpha \qquad [6.6]$$

CVaR could be used as the flood risk position in the market clearing model. This point will be proposed and discussed in the next sections.

6.3 Including risk in the market clearing model

A stochastic representation of Equation [6.1] that incorporates risk profile preferences would quantify the trade-off between the mean cost of risk and the economic surplus from trading ICs. The market model includes only CVaR as profile risk in the market formulation. This model would be consistent with the SO risk preferences and the potential changes in the flood distribution in terms of CVaR.

Piantadosi et al. (2008) analysed policies to manage urban storm water using CVaR. The authors minimised CVaR to find an optimum pumping policy for a connected storage dams in Australia, while penalties accounted for environmental damage.

Ahmed (2006) and Noyan (2010) minimise cost, explicitly including CVaR in the objective function. Similarly, Street (2010) proposed an equivalent formulation where the risk measure is a convex combination with a weight of $\rho^* = \dfrac{\rho}{1-\rho}$ from ρ in equation [6.1], and so the objective becomes max: $c^T x - \left(1-\rho^*\right) E\left[Q\left(x,w\right)\right] - \rho^* CVaR_\alpha\left(Q\left(x,w\right)\right)$.

Following the approaches of Ahmed (2006), Noyan (2010) and Street (2010), the market clearing formulation could include risk by the following options: (i) maximising the economic trading value with flood costs as recourse, while constraining the risk position (Uryasev 2000b); and (ii) maximising the value of trading ICs while penalising for changes in flood distribution and the risk position for extreme events. This risk position is defined as the expected maximum flood damage level above a confidence level that the SO would expect to face. (For more detail describing CVaR in the objective and constraints, see Krokhmal et al. (2005) and Fábián (2008)). The CVaR position is equivalent to having a risk position in terms of storms resulting in different flood severities within the catchment.

Mulvey et al. (2006), Fábián (2008) and Uryasev et al. (2010) used comparable ideas of maximum expected losses in a constraint.

According to this framework, the two proposals above could be applied to the IC market; however, this research will focus on constraints with CVaR, i.e., in option (i).

A simplified risk profile condition could be established by constraining the market model rather than including a risk profile in the objective. In this case, the SO knows the risk as a CVaR profile at each control point and at the catchment level. This constrained option will be proposed and extended later in this chapter.

However, constraining the risk positions may lead to an obvious difficulty that may make the market solution infeasible. At this stage the market accepts this issue. Thus, if the market solution remains infeasible, the SO could relax the constraints.

The constraints used to establish and limit risk profile positions for control points and catchments are as follows:

- At control point

$$C_k^s\left(f_k^s\right)-\eta_\alpha^k \le \upsilon_k^s, \ \forall\ k,s$$

$$\eta_\alpha^k+\frac{\sum_{s=1}^{S}\phi^s\,\upsilon_k^s}{1-\alpha}\le W_\alpha^k, \ \forall\ k$$

- At catchment level

$$\sum_{k=1}^{K}C_k^s\left(f_k^s\right)-\bar{\eta}_\alpha \le \bar{\upsilon}^s, \ \forall\ k$$

$$\bar{\eta}_\alpha+\frac{\sum_{s=1}^{S}\phi^s\bar{\upsilon}^s}{1-\alpha}\le \bar{W}_\alpha \quad ,$$

The auxiliary variables υ_k^s and $\bar{\upsilon}^s$ relate losses in the TSSP recourse for violating thresholds at different control points and catchments, across time and storm scenarios. If the damage cost for maximum depth is $Q\left(x,w_s\right)_k$, the variable υ_k^s will account for the damage for changes in depth at control point *k* and scenario *s*. This variable accounts for the changes in damage above the confidence level α. The variable $\bar{\upsilon}^s$ will account for the damage for changes in depth of the catchment in scenario *s*.

Equation [6.6] can be expressed as $\upsilon_k^s \geq C_k^s\left(f_k^s\right) - \eta_\alpha^k$, where $C_k^s\left(f_k^s\right)$ is the damage for the maximum depth in scenario *s* and control point *k*, and υ_k^s is the damage beyond η_α^k. If the damage function included another flood component, such as velocity (hastening peak time of flooding), the damage would be $C_k^s\left(f_k^s, Vh_{k,t}^{s+}\right)$ and so the constraint would be $\upsilon_k^s \geq C_k^s\left(f_k^s, Vh_{k,t}^{s+}\right) - \eta_\alpha^k$ Thus, if the SO is concerned regarding management of flash floods, CVaR could model the requirements incorporating the related damage costs. The auxiliary variable υ_k^s could account for flood components and associated damage, allowing for the required trade-offs. The final point and its implications will be discussed in the section on duality and flood cost.

Using CVaR requires an accurate measure of the probability distributions of storms for the close relationship between expectation and risk position. CVaR's focus is on the extreme damage tail which requires analysing the extreme storm events. Additionally, the market design assumes a storm distribution which implies that the catchment is affected by storms with similar intensities, and durations over a specified time period. The model ignores possible climate changes that could modify the parameters and probabilities are not accounted for in the market.

If the market is cleared using option (ii), the objective function would be as follows:

$$\max \quad E\left[f\left(x,w\right)\right] - \rho CVaR_\alpha\left(f\left(x,w\right)\right) \qquad [6.7]$$

Then, rearranging and using the property of translation invariance of CVaR, the objective becomes:

$$\text{Max } \left(1-\rho\right)c^T x - E\left[Q\left(x,w\right)\right] - \rho CVaR_\alpha\left(Q\left(x,w\right)\right) \qquad [6.8]$$

Additional auxiliary constraints as stated in the previous paragraph are needed:

$$Q\left(x,w_s\right) - \eta_\alpha \leq \upsilon^s,\ \forall s \qquad [6.9]$$

$$\eta_\alpha + \frac{1}{1-\alpha}\sum_{s=1}^{S}\phi^s \upsilon^s = CVaR_\alpha\left(Q\left(x,w\right)\right) \qquad [6.10]$$

Equation [6.10] corresponds to the conditions of Equation [6.5] and Equation [6.6]. These conditions could represent the risk at the catchment level. Equation [6.9] and [6.10] can be generalised for each control point. The contribution to the risk position from control points becomes:

$$\eta_\alpha + \frac{1}{1-\alpha}\sum_{k=1}^{K}\sum_{s=1}^{S}\phi^s \upsilon_k^s = CVaR_\alpha\left(Q\left(x,w\right)\right) \qquad [6.11]$$

Equation [6.11] meets the superadditivity property of CVaR and accounts for the risk position at the catchment. The variable υ_k^s is estimated at each control point *k* and scenario *s*.

6.4 The proposed market clearing model

This section proposes a model which will clear trading of IC allowances, while accounting for the violated boundaries along control points and storm scenarios, considering the SO's risk profile position expressed as CVaR. The market model will catch any violation of thresholds by linking flood components with the corresponding damage. Any change in flood distribution over a desired profile is considered in the estimation of the SO's downside risk (CVaR).

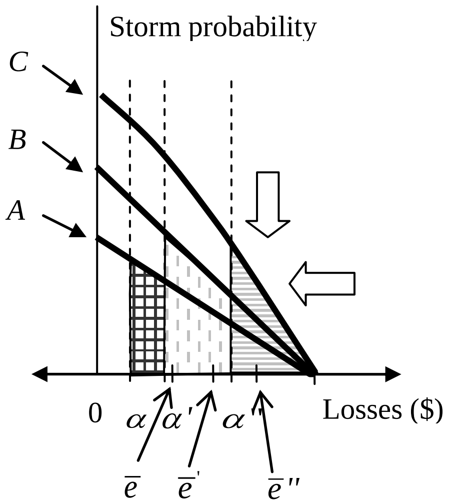


Figure 6-4 Hypothetical flood probability density functions with IC allowances *A*, *B* and *C*; α'', α' and α correspond to the confidence levels related to VaR; $\bar{e}''$, $\bar{e}'$ and $\bar{e}$ are the conditional expected values below the confidence level α .

Figure 6-4 illustrates trade of ICs and the corresponding final flood distribution. It assumes that the initial condition of the catchment IC is *C* or *B* with equivalent CVaR-*C* ($\bar{e}''$) and

CVaR-*B* ($\bar{e}\,'$) which are greater than the preferred CVaR condition ($\bar{e}$). In this scenario, the trade combination would be moved towards IC allowances which may reach level *A*. However, if the last level was the policy of the SO, the trades may not be revenue positive and the SO may be a net payer to reach the risk position. Participants who increasing the risk would face an increased price for their IC allowances. This issue will be discussed in the next sections and in Chapter 7.

Parameters

α = Confidence level coefficient (0,1) which corresponds to the probability level of the threshold of losses.

ρ = Risk coefficient in the range $(0,\infty)$ which corresponds to a non-negative trade-off between risk preferences and the expected damage. If ρ = 0, the SO is risk neutral, and ρ > 0 means the SO is risk averse.

ϕ^s = Probability of a storm in scenario *s*. This parameter satisfies the following properties: $0 \leq \phi^s \leq 1$, and $\sum_{s=1}^{S} \phi^s = 1$.

W_α^k = Maximum expected damage that the SO accepts at control point *k* for extreme storm events. This value corresponds to the α–CVaR at control point *k*.

$\bar{W}_\alpha$ = Maximum expected damage in the catchment that the SO accepts for extreme storm events. This value corresponds to the α–CVaR at the catchment level.

Decision variables

$\mu_{i,j}^D$ = Buying price to firm *i* for changing IC allowance to type *j* ($/ha).

$\mu_{i,j}^S$ = Selling price to firm *i* for changing to IC allowance type *j* ($/ha).

f_k^s = Flow above threshold capacity which takes into account the maximum peak flow (depth) in scenario *s* at control point *k* (volume/time).

η_α^k = The value at risk at confidence level α at control point *k*.

$\bar{\eta}_\alpha$ = The value at risk at confidence level α in the catchment.

υ_k^s = Losses above η_α^k in scenario *s*. The losses correspond to the flood damage above a confidence level α in the storm distribution or the equivalent flood damage distribution. The losses are in each scenario *s* which belongs to the range $1-\alpha$ of the storm distribution.

$\bar{\upsilon}^s$ = Losses above $\bar{\eta}_\alpha$ in scenario *s* at the catchment. The losses correspond to the flood damage above a confidence level α in the storm distribution or the equivalent flood damage distribution. The losses in scenario *s* belong to the range $1-\alpha$ of the storm distribution.

Dual variables

$\phi^s\lambda_{t,k}^s$ = The price to impact (receive) at control point *k*, time *t* and scenario *s* (\$/volume/time).

$\phi^s\Phi_k^s$ = The marginal cost for increasing the risk of flooding at control point *k* and scenario *s*. This is the marginal risk-damage for an additional unit of flooding at scenario *s*, or for a marginal change in the flood distribution at the loss tail.

$\phi^s\bar{\Phi}^s$ = The marginal cost for increasing the risk of flooding in the catchment and scenario *s*. This is the marginal risk of damage for an additional unit of flooding at scenario *s* across control points, or for a marginal change in the flood distribution at the loss tail, at the catchment level.

σ_k^α = Marginal opportunity cost (in terms of economic surplus) for trading and changing the flood distribution, when the SO allows an extra unit of loss in the α–CVaR at control point *k*. This value corresponds to the marginal cost of allowing changes in the SO risk position related to the flood at control point *k*.

$\bar{\sigma}^\alpha$ = Marginal opportunity cost if the SO allows an additional unit of loss in the loss tail (CVaR). This value implicitly corresponds to the adjusted risk for allowing higher marginal cost of some flood points within the catchment.

Market clearing model: Sto_MarketIC_Risk1. This model implicitly accounts for CVaR in the constraints.

Maximize

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}P_{i,j,b}^{D}\ \ qbuy_{i,j,b}-\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}P_{i,j,b}^{S}\ \ qsell_{i,j,b}$$
$$-\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}C_{k}^{s}\left(f_{k}^{s}\right) \qquad [6.12]$$

Subject to:

$$0\le qbuy_{i,j,b}\le D_{i,j,b}^{\max},\ \forall\ i,j,b \qquad : \beta_{i,j,b}^{-},\beta_{i,j,b}^{+} \qquad [6.13]$$

$$0\le qsell_{i,j,b}\le S_{i,j,b}^{\max},\ \forall\ i,j,b \qquad : \gamma_{i,j,b}^{-},\gamma_{i,j,b}^{+} \qquad [6.14]$$

$$g_{i,j}^{D}=\sum_{b=1}^{B}qbuy_{i,j,b},\ \forall\ i,j \qquad : \mu_{i,j}^{D}\ \text{(free)} \qquad [6.15]$$

$$g_{i,j}^{S}=\sum_{b=1}^{B}qsell_{i,j,b},\ \forall\ i,j \qquad : \mu_{i,j}^{S}\ \text{(free)} \qquad [6.16]$$

$$Q_{k,t}^{0,s}+\sum_{i=1}^{N}\sum_{j=1}^{J}H_{i,j,k}^{t-u+1,s}\ g_{i,j}^{D}+\sum_{i=1}^{N}\sum_{j=1}^{J}H_{i,j,k}^{t-u+1,s}\ g_{i,j}^{S}\le M_{k}+\ f_{k}^{s},$$
$$\forall\ t,s,k \qquad : \phi^{s}\lambda_{t,k}^{s} \qquad [6.17]$$

$$C_{k}^{s}\left(f_{k}^{s}\right)-\eta_{\alpha}^{k}\le\upsilon_{k}^{s},\ \forall\ k,s \qquad : \phi^{s}\Phi_{k}^{s} \qquad [6.18]$$

$$\eta_{\alpha}^{k}+\frac{\sum_{s=1}^{S}\phi^{s}\,\upsilon_{k}^{s}}{1-\alpha}\le W_{\alpha}^{k},\ \forall\ k \qquad : \sigma_{k}^{\alpha} \qquad [6.19]$$

$$\sum_{k=1}^{K}C_{k}^{s}\left(f_{k}^{s}\right)-\bar{\eta}_{\alpha}\le\bar{\upsilon}^{s},\ \forall\ s \qquad : \phi^{s}\bar{\Phi}^{s} \qquad [6.20]$$

$$\bar{\eta}_{\alpha}+\frac{\sum_{s=1}^{S}\phi^{s}\bar{\upsilon}^{s}}{1-\alpha}\le\bar{W}_{\alpha}, \qquad : \bar{\sigma}^{\alpha} \qquad [6.21]$$

$$f_{k}^{s},\ \upsilon_{k}^{s}\ \text{and}\ \bar{\upsilon}^{s}\ge 0 \qquad : \theta_{k}^{s},\ \Theta_{k}^{s},\ \bar{\Theta}^{s} \qquad [6.22]$$

$$x_{i,j}^{D},\ x_{i,j}^{S},\ g_{i,j}^{D},\ g_{i,j}^{S},\ \bar{\eta}_{\alpha}\ \text{and}\ \eta_{\alpha}^{k}\ \text{free} \qquad [6.23]$$

Explanation

[6.12] The objective function of Sto_MarketIC_Risk1 maximises the total economic surplus for trading IC allowances, less the expected flood damage across rainfall events. The objective function measures changes in welfare by trading rather than the absolute measure of welfare (assuming the market is sufficiently competitive).

[6.13]-[6.17] Constraints have the same meaning as [4.3]–[4.7] in Sto_MarketIC1. By definition, the lower bounds in [6.13], [6.14] and [6.22] have negative dual variables, so they are represented canonically forcing duals to be positive, 0 $\geq -qbuy_{i,j,b}$ and 0 $\geq$ $-qsell_{i,j,b}$, so $-\beta_{i,j,b}{}^{(-)} \geq 0$ and $-\gamma_{i,j,b}{}^{(-)} \geq 0$ respectively; thus, positive $\beta^{-}_{i,j,b}$, $\gamma^{-}_{i,j,b}$, θ^{s}_{k} and Θ^{s}_{k} will be conveyed. The model should include an extra condition to specify the total area and its impervious level, $\sum_{z=1}^{Z}\sum_{j=1}^{J} A^{0,z}_{i,j} = A^{Tot}_{i}$, where z is an index to indicate the location of a particular block within the property as well as its total area (A_i^{Tot}).

[6.18] For each scenario s, this constraint calculates the loss beyond the value at risk. The dual $\phi^{s}\Phi^{s}_{k}$ is the opportunity cost for allowing an extra unit of CVaR at the confidence level 1−α at control point k.

[6.19] For each control point k, the total losses beyond the value at risk across scenarios should be lower than a maximum expected loss. The dual variable σ^{α}_{k} is the opportunity cost of allowing another unit of CVaR.

[6.20] For each scenario s, this constraint calculates the loss beyond the value at risk at the catchment level. The dual $\phi^{s}\bar{\Phi}^{s}$ is the opportunity cost for allowing an extra unit of CVaR across storm scenarios s, which belongs to confidence level 1−α of the storm distribution at the catchment level.

[6.21] This constraint defines the total maximum expected damage that the SO is willing to face at a confidence level α in case of extreme events across all control points. The dual variable $\bar{\sigma}^{\alpha}$ is the opportunity cost for having an extra unit of boundary, in terms of CVaR.

[6.22] Non-negative conditions of IC allowances, flows above threshold capacities, and losses above η^{k}_{α} and $\bar{\eta}_{\alpha}$.

[6.23] Buying and selling quantities of IC allowances and the initial IC allowance are non-negative. This condition limits the final allocation of $g^{D}_{i,j}$ and $g^{S}_{i,j}$. A similar condition occurs with η^{k}_{α}.

The next section will present a discussion of dual variables and the related flood costs for violating any flood boundary within the catchment. A detailed dual formulation of Sto_MarketIC_Risk1 is presented Appendix A.

6.5 Price analysis

This section presents an analysis of duals, flood costs and the SO's risk position in the market clearing models, with a view to show the implications of including risk in the market as well as its effects on prices faced by participants. Risk should be transferred to those who increase flood damage and the SO's risk. Contrarily, participants should be rewarded for reducing risk and flood damage in the catchment.

As previously stated, the right to IC allowance corresponds to a set of impacting flows received at control points across scenarios, given an initial imperviousness in the catchment. With a market formulated by Sto_MarketIC_Risk1, the rights also account for the SO's risk positions. This implies that landholders participating with established IC allowances have rights to the SO's risk positions. These rights are represented by $\sum_{s=1}^{S}\phi^{s}\sum_{i=1}^{N}\sum_{j=1}^{J}H_{i,j,k}^{t-u+1,s}A_{i,j}^{I}$, which can be decomposed in terms of the peak flows, $\sum_{s=1}^{S}\phi^{s}\sum_{i=1}^{N}\sum_{j=1}^{J}\bar{H}_{i,j,k}^{t-u+1,s}A_{i,j}^{I}$, the flows at the risk position $\sum_{s\in(1-\alpha)}^{S}\phi^{s}\sum_{i=1}^{N}\sum_{j=1}^{J}\tilde{H}_{i,j,k}^{t-u+1,s}A_{i,j}^{I}$ at control point *k*, and the total risk position at the catchment levels, $\sum_{s\in(1-\alpha)}^{S}\phi^{s}\sum_{i=1}^{N}\sum_{j=1}^{J}\hat{H}_{i,j,k}^{t-u+1,s}A_{i,j}^{I}$. $\tilde{H}$ and $\hat{H}$ represent the flows above the confidence level α in the storm scenarios (related to flood scenarios). Additionally, the IC allowance also can be seen as a set of valued impacts.

The next sub-section presents duals for the model Sto_MarketIC_Risk1 and for risk implications of peak flows. The discussion is then expanded to the price analysis for the SO with different risk positions.

6.5.1 Price analysis and CVaR

The dual prices $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ represent the reduction in economic surplus when participant *i* is permitted an extra area with IC allowance type *j*. The marginal reduction in surplus is also linked to the SO's risk position. The dual analyses reveals that these prices are generated by the flow impacts that violate thresholds, such as maximum peak flow (maximum depth), and raising the risk positions at control point *k* and at the catchment level. The next dual equation allows decomposing the clearing price into damage and SO risk.

$$\sum_{t=1}^{T} \phi^s \lambda_{t,k}^s - \phi^s \Phi_k^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} - \phi^s \bar{\Phi}^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} + \theta_k^s = \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}, \forall k,s: \; f_k^s \qquad [6.24]$$

When flooding occurs at control point *k*, the result is $f_k^s > 0$, and a maximum peak is reached, the shadow price will be active at that time *t*; hence, $\phi^s \lambda_{t,k}^s > 0$. Then $\theta_k^s = 0$ by complementary slackness, and the clearing price $\phi^s \lambda_{t,k}^s$ matches the marginal flood damage as well as the marginal risk positions in terms of CVaRs ($\phi^s \Phi_k^s$ and $\phi^s \Phi^s$). To simplify this analysis, it is assumed that flooding has just one peak across storm scenarios; thus, shadow prices would be binding just at the stage-flood time for each scenario. In addition, the SO's binding risk positions generate a price which is represented with the following conditions:

$$-\sum_{s=1}^{S} \phi^s \Phi_k^s + \sigma_k^\alpha = 0, \forall k \qquad : \eta_\alpha^k \qquad [6.25]$$

$$-\phi^s \Phi_k^s + \frac{\phi^s \sigma_k^\alpha}{1-\alpha} - \Theta_k^s = 0, \forall k,s \qquad : \upsilon_k^s \qquad [6.26]$$

$$-\sum_{s=1}^{S} \phi^s \Phi^s + \bar{\sigma}^\alpha = 0 \qquad : \bar{\eta}_\alpha \qquad [6.27]$$

$$-\phi^s \bar{\Phi}^s + \frac{\phi^s \bar{\sigma}^\alpha}{1-\alpha} - \bar{\Theta}^s = 0, \forall s \qquad : \bar{\upsilon}^s \qquad [6.28]$$

Equation [6.25] and [6.27] represents the balance between the incremental flood damage above a confidence level α, and the risk at control point and catchment respectively. Thus, marginal damage across scenarios matches the marginal cost for an additional unit of damage in the loss tail for extreme disasters. If widespread flooding was present within the catchment, the SO could impose a maximum damage for W_α, which would force a single binding condition for the entire catchment, and a single price would be faced across all control points in response to the SO's risk position.

Equations [6.26] and [6.28] show the relationships between the shadow price for marginally increasing the damage in the tail loss distribution (above the confidence level α), and the marginal cost in terms of the risk positions at control points and at the catchment level. This loss will raise the SO's bounded risk position, signalling prices to participants within the system which are adjusted by $(1-\alpha)$. The clearing price at control point *k* and scenario *s* matches the marginal flood damage as well as the corresponding incremental adjusted risk position at the control point and at the catchment level when an additional flow unit of peak flow is permitted at the control point *k*, time *t* and scenario *s*.

Equations [6.25] and [6.26] account for the changes in the final portfolio of IC allowances and the corresponding expected changes in the flood distribution, particularly in the extreme tail of flood damage (see Figure 6-4). Arranging and substituting Equation [6.25] and [6.26] generates a dual relationship for Φ_k^s where there are two significant points. First, the shadow prices $\phi^s \Phi_k^s$ would be binding in the $(1-\alpha)$ right side of the losses distribution. (See Figure 6-4, but note the losses are shown on the left side, and the analysis and model places the losses on the right side.) Secondly, the dual value Φ_k^s takes the same value along binding scenarios, being adjusted by the probability of occurrence ϕ^s for those scenarios above the confidence level α. The next equation represents this condition.

$$\sum_{s=1}^{S} \phi^s \Phi_k^s = \left(1-\alpha\right) \Phi_k^s, \ \forall\ k \qquad [6.29]$$

Thus, any flooding changes in binding scenarios would have the same implication in terms of the risk. The SO's position would be affected in those scenarios that were allowed to have an additional unit of flood damage which is weighted according to the probability.

Similar implications can be observed and analysed from Equations [6.27] and [6.28] at the catchment level, particularly in the extreme tail of flood damage. Similarly for Equation [6.29] the dual value $\bar{\Phi}^s$ takes the same value along binding scenarios at the catchment level, which are adjusted for the probability of scenarios above the confidence level α. Thus, any change in flooding may affect the risk position at the catchment level and this price considers possible relationships between flooding at several control points. In this case, the condition is as follows:

$$\sum_{s=1}^{S} \phi^s \bar{\Phi}^s = (1-\alpha)\bar{\Phi}^s \quad [6.30]$$

Clearing prices depend on the risk positions when CVaRs' limits are binding at control points $\phi^s \sigma_k^\alpha \geq 0$, and/or catchment level $\phi^s \bar{\sigma}^\alpha \geq 0$. Thus, if $\upsilon_k^s > 0$ and $\upsilon^s > 0$, by complementary slackness $\Theta_k^s = 0$ and $\bar{\Theta}^s = 0$. Substituting [6.26] and [6.28] into [6.24] the clearing price will be as follows:

$$\phi^s \lambda_{t,k}^s = \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} + \frac{\phi^s \sigma_k^\alpha}{1-\alpha} \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} + \frac{\phi^s \bar{\sigma}^\alpha}{1-\alpha} \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}, \ \forall\ k,s,t \quad [6.31]$$

These clearing prices will account for the marginal flood increments, even at non-connected control points, due to those participants who raise their risk positions at the catchment level. The next section presents the shadow prices, in the equilibrium, that users would face for changing impervious levels when risk positions are bounded.

6.5.2 Participants' shadow prices

Within the market, participants face the price σ_k^α for the binding constraint associated with the risk position at control point *k*. At the catchment level this price is $\bar{\sigma}^\alpha$. Now let us see how the shadow prices $\mu_{i,j}^D$ and $\mu_{i,j}^S$ can be used to pay and to charge participants.

$$\mu_{i,j}^D = \sum_{s=1}^{S} \sum_{k=1}^{K} H_{i,j,k}^{t*,s} \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} \left[1 + \frac{\sigma_k^\alpha + \bar{\sigma}^\alpha}{1-\alpha}\right], \ \forall\ i,j \quad [6.32]$$

$$\mu_{i,j}^S = \sum_{s=1}^{S} \sum_{k=1}^{K} H_{i,j,k}^{t*,s} \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s} \left[1 + \frac{\sigma_k^\alpha + \bar{\sigma}^\alpha}{1-\alpha}\right], \ \forall\ i,j \quad [6.33]$$

The prices in [6.32] and [6.33] are the allowance prices, and they correspond to a set of valued impacts at control points, which generate marginal changes of flood damage by scenario, and affect the SO's risk position. Any change in IC allowance at any location upstream may change flows across scenarios, and consequently to affect the flood distribution. In addition, different risk positions can be generated and priced, which are analysed as follows.

If the catchment level risk constraint is non-binding, i.e., $\phi^s \bar{\sigma}^\alpha = 0$, but the CVaR positions are binding at several control points k=1,..,K-c (with c non-binding CVaR control points, so $\phi^s \sigma_k^\alpha \geq 0$); then participants face the following prices for the allowances.

$$\mu_{i,j}^D = \sum_{s=1}^S \sum_{k=1}^K H_{i,j,k}^{t*,s}\ \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}\left[1+\frac{\sigma_k^\alpha}{1-\alpha}\right], \forall\ i,j \qquad [6.34]$$

$$\mu_{i,j}^S = \sum_{s=1}^S \sum_{k=1}^K H_{i,j,k}^{t*,s} \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}\left[1+\frac{\sigma_k^\alpha}{1-\alpha}\right],\ \forall\ i,j \qquad [6.35]$$

Participants face prices based on their changes in the impacting flow patterns $H_{i,j,k}^{t-u+1,s}$ along control points and scenarios. Notice that $H_{i,j,k}^{t-u+1,s}$ can be positive or negative depending on the initial IC allowance. The prices represent marginal changes of the maximum peak flow damage, and the marginal cost of damage at the risk position and at binding control points.

The SO can include CVaR at all control points, or just those of concern. Therefore, the participants whose flows impact at binding control points and raise the CVaR positions would be charged for those flows.

If the risk position is binding at the catchment level, so $\phi^s \bar{\sigma}^\alpha \geq 0$, but control points do not match the CVaR positions, so $\phi^s \sigma_k^\alpha = 0$, then participants will observe the following prices for their allowances.

$$\mu_{i,j}^D = \sum_{s=1}^S \sum_{k=1}^K H_{i,j,k}^{t*,s} \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}\left[1+\frac{\bar{\sigma}^\alpha}{1-\alpha}\right], \forall\ i,j \qquad [6.36]$$

$$\mu_{i,j}^S = \sum_{s=1}^S \sum_{k=1}^K H_{i,j,k}^{t*,s} \phi^s \frac{\partial C_k^s\left(f_k^s\right)}{\partial f_k^s}\left[1+\frac{\bar{\sigma}^\alpha}{1-\alpha}\right], \forall\ i,j \qquad [6.37]$$

In this case, participants should realise that their changes in IC would shift impacting flow $H_{i,j,k}^{t-u+1,s}$, thus affecting the flood distribution and the resulting flood damage in extreme storm events. Since the SO has a limit for CVaR at the catchment level, participants should face the marginal cost for the SO's risk position.

If both the catchment and control point risk positions are binding, $\phi^s \bar{\sigma}^\alpha \geq 0$ and $\phi^s \sigma_k^\alpha \geq 0$, then participant allowances would be priced for [6.32] and [6.33]. Thus, they will pay/receive allowances based on their marginal changes of flows for varying flood distributions as well as for the expected damage above the confidence level α in the flood damage distribution.

Participants who change IC allowances may increase peak flows and flood damage, and so influence the risk position of the community (see equations [6.32]–[6.37]). Additionally, some participants may observe that increasing imperviousness shifts the peak time, impacting flows at control points. Thus, participants pay a small price or are rewarded for reducing a possible risk position by the SO. However, if the market model considers the flood cost for hastening the peak time of flooding (fast flood), a possible trade-off between the flood cost and the constrained risk may be observed. Thus, even though the new IC allowances may avoid peak flow times, such IC allowances could hasten inundation and hence participants cannot change IC allowances.

Participants face the price for influencing flood damage and binding risk positions of the SO. The signalled price may incentivise participants to manage IC allowances and avoid risk.

6.5.3 Hedging with a set of CVaR positions at control points

The previous analysis accounts for a CVaR value at each control point as a way to hedge against different flood scenarios for extreme events and to establish the corresponding risk positions. Variables should include a set of $\alpha=1\ldots\alpha'$ for $\upsilon_{k,\alpha}^{s}$, which estimates damage above VaR$_\alpha$ $\left(\eta_\alpha^k\right)$ at different confidence levels. Thus, the constraints [6.18], [6.19], [6.20] and [6.21] could include a set of CVaR positions at control points and catchments. For instance at control point the conditions are as follows:

$$C_k^s\left(f_k^s\right)-\eta_\alpha^k\leq\upsilon_{k,\alpha}^s,\ \forall\ k,s,\alpha \qquad :\ \phi^s\Phi_{k,\alpha}^s \qquad [6.38]$$

$$\eta_\alpha^k+\frac{\sum_{s=1}^{S}\phi^s\,\upsilon_{k,\alpha}^s}{1-\alpha}\leq W_\alpha^k,\ \forall\ k,\alpha \qquad :\ \sigma_k^\alpha \qquad [6.39]$$

The new dual conditions represent; the clearing price, the opportunity cost to the system and the risk positions, as follows.

$$\sum_{t=1}^{T}\phi^{s}\lambda_{t,k}^{s}-\sum_{\alpha=1}^{\alpha'}\phi^{s}\Phi_{k,\alpha}^{s}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}-\sum_{\alpha=1}^{\alpha'}\phi^{s}\bar{\Phi}_{\alpha}^{s}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}-\theta_{k}^{s}$$

$$=\phi^{s}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}},\ \forall\ k,s \qquad : \ f_{k}^{s} \qquad [6.40]$$

Equation [6.40] differs from [6.24] in that the new condition accounts, explicitly, for a set of risk positions at each control point. A position in the set is priced when CVaR_{α} levels are bounded. The price represents the cost to the system for changing risk and the flood damage distribution; accordingly, the dual price $\lambda_{t,k}^{s}$ accounts for all such risk positions. Thus, when assuming bounded conditions at one control point and catchment at one peak time per scenario, the new clearing condition is:

$$\phi^{s}\lambda_{t,k}^{s}=\phi^{s}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}+\sum_{\alpha=1}^{\alpha'}\frac{\phi^{s}\sigma_{k}^{\alpha}}{1-\alpha}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}+\sum_{\alpha=1}^{\alpha'}\frac{\phi^{s}\bar{\sigma}^{\alpha}}{1-\alpha}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}},\ \forall\ k,s,t \quad [6.41]$$

The clearing price accounts for the incremental flood damage, and for the trade-off between the cost of the risk positions at each control point, as well as for its contribution at the catchment level, adjusted by the incremental flood damage, when an extra unit of peak flow at peak time t^* is allowed to impact on control point k. Consequently, this would signal a price for participants to shift the flood distribution particularly at the tail distribution.

Equation [6.42] defines the prices to charge those participants.

$$\mu_{i,j}^{D}=\sum_{s=1}^{S}\sum_{k=1}^{K}H_{i,j,k}^{t*,s}\ \phi^{s}\frac{\partial C_{k}^{s}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}}\left[1+\sum_{\alpha=1}^{\alpha'}\frac{\sigma_{k}^{\alpha}+\bar{\sigma}^{\alpha}}{1-\alpha}\right],\ \forall\ i,j \qquad [6.42]$$

It is expected that at least one binding CVaR limit will price the system for each control point. This limit accounts for the tail flood distribution that the SO wants to keep in the catchment. Thus, the new impervious level conditions and BMPs in the catchment will be distributed such to reach the desired tail flood condition.

As similar to [6.31], the CVaR's set could over-constrain the market and make it infeasible. This could occur because the SO may desire to adjust an established status quo of flood damage via CVaR in the catchment. This will signal the market with high prices, and so participants will be limited in changing IC allowances. Consequently, the SO will be a net payer. This will be discussed further in the chapter.

Next, an example is presented showing how CVaR values are included in the formulation for a small hypothetical catchment. The CVaR positions will change final prices from participants.

6.6 Example applications

This section gives three theoretical examples to show implications of including CVaR as risk measure in the market model. The examples assume 10 participants, who are located at different places in the catchment (see Table 6-1 and Table 6-6) and a stream network carries flows from the participants, producing a spatial and temporal flows condition at different control points (see Figure 4-9). Different stage flood damage functions are used in the examples, which account for peak flows. Example 1 considers a linearised convex stage flood damage function in the recourse. Examples 2 and 3 account for a linearised, but non-convex stage flood damage, considering the SOS2 method. Finally, examples 1 and 2 compare trading (allocations and prices) and initial and final flooding under different storm scenarios and CVaR position. Example 3 considers a set of CVaR positions at each control point to evaluate the effect of CVaRs in allocations and prices of participants.

Example 1

This hypothetical example includes the SO's risk position in terms of CVaRs. The market timeframe is one year, so the recourse cost accounts for the expected damage during the period. It is assumed that; 10 participants each have 10 hectares; 5 participants desire to increase ICs (1, 2, 3, 4, and 5) and the other 5 participants desire to reduce ICs (6, 7, 8, 9, and 10). The catchment has three control points CP1, CP2 and CP3 and participants are impacting at different locations (see Figure 4-9). Control points CP1, CP2 and CP3 have threshold limits M_k of 41, 57 and 70 volume/time. Table 6-1 summarises participants' preferences as well as the information on ICs and impacting areas. For instance, participant 1 has 10 hectares and desires to change 9 hectares from forest to meadow; participant 9 has 10 hectares of concrete and desires to reduce 9 hectares to meadow. It is assumed the flow impact at control points will rise associated with the increasing impervious levels.

Table 6-1 Initial ICs and preferences of participants (Example 1).

Particip.	Initial IC	Initial area (ha)	Impact at control point	Option 1			Option 2			Option 3		
				ha	IC	$/ha	ha	IC	$/ha	ha	IC	$/ha
1	Forest (F)	10	1,2,3	4	M	$22	3	M	$6	2	M	$3
2	Meadow (M)	10	1,2,3	4	Cr	$12	3	Cr	$10	2	Cr	$8
3	Meadow (M)	10	2,3	4	Cn	$15	3	Cn	$13	2	Cn	$11
4	Meadow (M)	10	2,3	4	Cn	$14	3	Cn	$11	2	Cn	$10
5	Crop (Cr)	10	2,3	4	Cn	$13	3	Cn	$11	2	Cn	$8
6	Crop (Cr)	10	2,3	5	F	$2	3	F	$8	1	F	$9
7	Concrete (Cn)	10	2,3	5	M	$4	3	M	$7	1	M	$10
8	Concrete (Cn)	10	3	5	Cr	$5	3	Cr	$8	1	Cr	$9
9	Concrete (Cn)	10	3	5	M	$7	3	M	$10	1	M	$12
10	Crop (Cr)	10	3	5	F	$5	3	F	$10	1	F	$15

The market is established with 9 storms with probabilities of 0.8, 0.08, 0.05, 0.03, 0.017, 0.0125, 0.0075, 0.0025, and 0.0005, which generate different flood scenarios. The SO is concerned about extreme flood damage, and includes CVaR constraints for three control points (CP) and at the catchment level. The SO is considering CVaRs at confidence level $(1-\alpha)$=0.95 of $600, $1,480, and $2,710 for CP1, CP2 and CP3 respectively. Additionally, he/she is considering a CVaR, at the catchment level, of $10,000.

The damage function related to total flows at CP1 is quadratic and at CP2 and CP3 are cubic, and the damage functions are $\$CP1 = 1.28 f^2$, $\$CP2 = 0.0008 f^3$ and $\$CP3 = 0.0004 f^3$, where f is the peak flow at the control point. The damage functions are convexified.

The resulting trades for this example were 26.65 hectares to buyers and 15.95 hectares to sellers. The clearing prices in the market varied according to; control point, scenario and time peak. Table 6-2 summarises the clearing prices at control points. The manager would receive net revenue of $12.68, because their risk position allows increasing impervious level in the zone close to CP1; however, the SO receives less revenue if this is risk neutral ($103.44). For instance, participant 3 bought 4 hectares (concrete) and paid $58; participant 7 sold 9 hectares and received $132 (for the change to meadow). Table 6-3 summarises these transactions. Positive and negative payments are charged and received for participants respectively.

Table 6-2 Clearing prices at control points (CP)

CP	Scenario	Time	Clearing price: With Risk position (CVaR) ($/m^3/time)	Clearing price: Risk neutral ($/m3/time)	CP	Scenario	Time	Clearing price: With Risk position (CVaR) ($/m^3/time)	Clearing price: Risk neutral ($/m^3/time)
1	5	7	$0.131	$0.089	2	8	9	$0.714	$0.299
1	6	7	$0.284	$0.131	2	9	8	$0.131	$0.089
1	7	7	$0.158	$0.284	3	2	9	$0.035	$0.200
1	8	7	$0.096	$0.158	3	3	10	$0.163	$0.035
1	9	6	$0.030	$0.096	3	4	10	$0.640	$0.205
2	3	9	$0.598	$0.116	3	5	10	$1.675	$0.423
2	4	9	$2.027	$0.332	3	6	10	$1.490	$0.567
2	5	9	$1.982	$0.479	3	7	10	$1.450	$0.505
2	6	9	$1.920	$0.470	3	8	10	$0.938	$0.530
2	7	9	$1.113	$0.490	3	9	9	$0.659	$0.289

Alternatively, if the market design is risk neutral, i.e., equivalent to a risk neutral SO (not accounting CVaR risk positions); then, participants' prices would be reduced, which may generate increasing IC. Thus, the SO sells more ICs for 45 hectares and buys 10 hectares and the final total revenue is $103.44. This revenue could be used to mitigate damage in the flood area. For instance, participant 3 bought 9 hectares (concrete) and paid $41.3; participant 7 sold 5 hectares and received $22.93 (for changing to meadow) (see Table 6-3).

The final flood distribution across control points changes depending on the SO's risk position (neutral or risky). In the example, the risk positions are bounded at CP2 and CP3 and they generate shadow prices of $0.2154 and $0.1353 respectively, but the constraints for CP1 and catchment risk levels are not binding. The flood distribution changed at CP1 and CP2 with both risk positions, given the slacks of the CVaRs at those points at the beginning of the auction. Flows at CP2 increased, changing the flood distribution and binding the CVaR. At CP3 the risk position is binding in the final flood distribution. The catchment and control points are hedged against changes in flood at the extreme tail of the flood distribution, and the clearing prices for the allowances account for these risk positions (see Table 6-3).

Table 6-3 Transactions for sellers and buyers

Particip.	Initial trading area (ha)	SO with risk position as CVaR			SO risk neutral		
		Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)
1	10	7	0	$26.99	9	0	$12.17
2	10	9	0	$34.70	9	0	$12.17
3	10	4	0	$58.67	9	0	$41.27
4	10	0	0	$0	9	0	$41.27
5	10	6.25	0	$68.69	9	0	$30.95
6	10	0	5	-$36.67	0	5	-$11.46
7	10	0	9	-$132.00	0	5	-$22.93
8	10	0	1.54	-$7.70	0	0	$0
9	10	0	0	$0	0	0	$0
10	10	0	0	$0	0	0	$0
Total	100.00	26.25	15.54	$ 12.68	45.00	10.00	$ 103.44

Table 6-4 summarises the initial and final peak flows at control points with different risk positions. When CVaR positions are binding, the final flood distribution does not change significantly (CP2 and CP3); however, flood distribution was shifted with assumed risk neutrality. For instance, at CP3 in scenario 9, flooding was anticipated to increase from 530 to 573.5 m^3/time, while in reality it had no significant shift, being from 530.0 to 530.1 m^3/time with CVaR.

Table 6-4 Initial and final stage-flooding in m^3 with a SO risk and neutral risk

Control Point	Scenario	Initial flooding (m^3)	Final flooding with CVaR (m^3)	Final flooding with risk neutral (m^3)	Control Point	Scenario	Initial flooding (m^3)	Final flooding with CVaR (m^3)	Final flooding with risk neutral (m^3)
1	5	0	0	0.04	2	8	189.0	190.4	215.1
1	6	1.0	4.2	4.6	2	9	353.0	355.4	396.5
1	7	9.4	13.2	13.72	3	2	2.0	2.0	7.22
1	8	22.0	26.8	27.4	3	3	50.0	50.0	58.7
1	9	64.0	72.0	73	3	4	98.0	98.0	110.18
2	3	25.0	25.5	33.7	3	5	146.0	146.0	161.66
2	4	57.8	58.5	69.98	3	6	170.0	170.0	187.4
2	5	90.6	91.5	106.26	3	7	218.0	218.0	238.88
2	6	107.0	107.9	124.4	3	8	290.0	290.0	316.1
2	7	139.8	140.9	160.68	3	9	530.0	530.1	573.5

Example 2

In this example, 10 participants are located at different locations in a small catchment as shown in Figure 4-9. Participants have 20 ha each with different IC allowances. Table 6-5 shows the initial allowances for the non-trading and trading areas from each participant. Participants desire to change IC allowances amounting to 10 ha each; however, they each have different demands and bid prices, see Table 6-6.

Table 6-5 Initial IC allowance from participants

Particip.	Non-trading IC	Non-trading area (ha)	Trading Area, Initial IC	Changing area (ha)
1	Forest (F)	10	Meadow (M)	10
2	Forest (F)	10	Crop (Cr)	10
3	Crop (Cr)	10	Crop (Cr)	10
4	Concrete (Cn)	10	Meadow (M)	10
5	Concrete (Cn)	10	Crop (Cr)	10
6	Crop (Cr)	10	Meadow (M)	10
7	Concrete (Cn)	10	Concrete (Cn)	10
8	Concrete (Cn)	10	Concrete (Cn)	10
9	Crop (Cr)	10	Crop (Cr)	10
10	Meadow (M)	10	Crop (Cr)	10

Table 6-6 Participants' preferences for changing IC allowances

Particip.	Initial IC	Changing area (ha)	Impact at control point	Option 1			Option 2			Option 3		
				ha	IC	$/ha	ha	IC	$/ha	ha	IC	$/ha
1	Meadow (M)	10	1,2,3	5	Cr	$15	3	Cr	$12	2	Cr	$10
2	Crop (Cr)	10	1,2,3	5	Cn	$15	3	Cn	$12	2	Cn	$10
3	Crop (Cr)	10	2,3	5	Cn	$9	3	Cn	$7	2	Cn	$5
4	Meadow (M)	10	2,3	5	Cn	$20	3	Cn	$18	2	Cn	$15
5	Crop (Cr)	10	2,3	5	Cn	$15	3	Cn	$12	2	Cn	$10
6	Meadow (M)	10	2,3	6	F	$2	3	F	$8	1	F	$9
7	Concrete (Cn)	10	2,3	6	M	$4	3	M	$7	1	M	$10
8	Concrete (Cn)	10	3	6	Cr	$5	3	Cr	$8	1	Cr	$9
9	Crop (Cr)	10	3	6	F	$7	3	F	$10	1	F	$12
10	Crop (Cr)	10	3	6	F	$5	3	F	$10	1	F	$15

A channel and stream network carries flows from the participants, producing a spatial and temporal condition of the flows. The flows impact at specific areas and (potentially) cause flooding problems. Similar impacting flows are assumed at control points which vary according to the impervious levels and scenarios. Control points CP1, CP2 and CP3 have threshold limits M_k of 70, 230 and 250 volume/time. Fourteen scenarios with different intensities are used in the market, which are perfectly correlated over control points. The probabilities are 0.399, 0.2, 0.15, 0.1, 0.07, 0.04, 0.02, 0.01, 0.005, 0.003, 0.0012, 0.0008, 0.0005, and 0.0005, for storms which have similar intensities across the small catchment. Those impact coefficients and probabilities are used in the market clearing formulation.

Each flood area has a different topography which causes non-uniform incremental flood depth damage. Figure 6-5 illustrates the initial flood distribution, and the flood damage cost for peak flows. These conditions at control points will be used in the market model, which has linearised and non-convex damage functions. SOS2 conditions will force a convex combination between adjacent points (see section 4.10 Chapter 4). Cplex 9.1 reached the solution in 14 seconds.

Before clearing, the expected damage at control points 1, 2, and 3 are \$441.30, \$1,093.30, and \$1,905.20, respectively. The SO needs to hedge against extreme events which is achieved by evaluating various CVaR options. The SO does not desire to face losses greater than \$11,056, \$21,040 and \$31,805 at the confidence level α=0.97 CVaR for control points 1, 2 and 3; and neither at the catchment level above \$110,000. This will bind the risk positions that the SO desires while being a net payer.

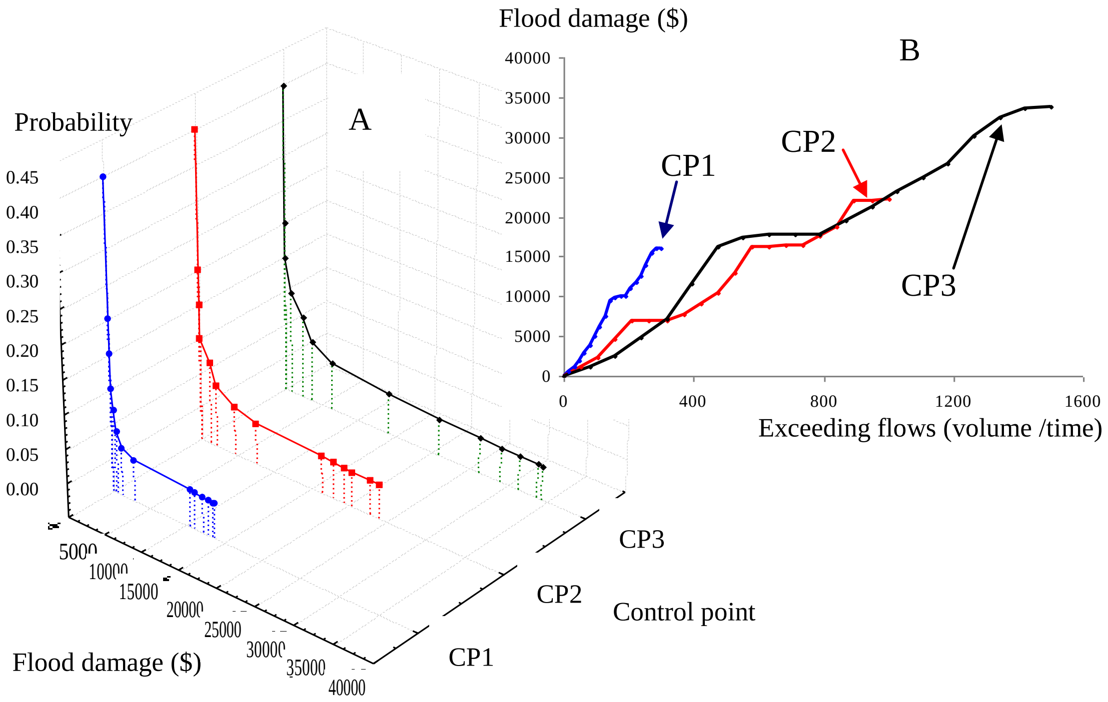


Figure 6-5 Initial flood distribution (A) and flood damage cost (B). CP1, CP2 and CP3 are control points 1, 2 and 3 respectively.

If the market is cleared using the risk positions, the final expected damage will change at each control point, and catchment. The risk positions tighten the control point constraints, reducing the total expected flood damage. The SO may desire this condition; however, the SO could be in the final position as a net payer. The expected damage was reduced from \$1,093.30 to \$1,079.90 at CP2, from \$1,905.20 to \$1,864.96 at CP3, and from \$3,439.90 to \$3,386.26 at the catchment level. The flood distribution at CP1 was maintained and the expected damage did not change (\$441.36). However, if the SO cleared the market as risk neutral, the final expected damage at the catchment level is increased to \$3,504.12 and at CP1, CP2 and CP3 to \$456.49, \$1,114.96, and \$1,932.68 respectively. Consequently, the SO is a net revenue receiver amounting to \$64.93.

The risk positions are bounded at CP1, CP2, and at the catchment level with dual prices of \$0.087, \$0.00018 and \$0.00027 respectively. It is interesting to analyse the situation of the participants, given they could trade with such binding constraints. Table 6-7 summarises the final trade when the SO includes CVaR constraints. In this case, participants 4 and 5 increase IC allowances of 6.05 and 5 ha, and pay \$108.90 (\$18/ha) and \$67.5 (\$13.5/ha), and participants 6 and 7 reduce IC allowances of 6 and 10 ha, and receive \$27 (\$4.50/ha) and \$180 (\$18/ha) (located in zone 2). Participants 1 and 10 can change IC allowances in 0.022 and 4.71 ha, and other participants cannot change IC in the catchment. Participant 1 could not increase IC, because the risk position is binding at CP1, and no participants are offering to reduce an equivalent change in flood damage and risk. However, under a risk neutral condition, participant 1 could change 5 ha from meadow to crop.

The catchment is almost hedged in terms of extreme events and the SO pays \$53.88.

When a linearised and non-convex flood damage cost was used in Chapter 4, the SO's revenue depended on the participants' preferences and the damage function. In this example 2, the SO would be a net payer of \$53.88, because the catchment became pervious at CP2 and CP3, and also because control point risk positions were tightened at CP1, CP2, and at the catchment level.

Table 6-7 Transactions for sellers and buyers

Particip.	Initial trading area (ha)	SO risk controlling			Risk neutral		
		Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)
1	10	0.022	0	$0.33	5	0	$60.45
2	10	0	0	$0	0	0	$0
3	10	0	0	$0	0	0	$0
4	10	6.05	0	$108.89	8	0	$143.23
5	10	5	0	$67.50	5	0	$67.14
6	10	0	6	-$27.00	0	6	-$26.86
7	10	0	10	-$180.00	0	10	-$179.03
8	10	0	0	$0	0	0	$0
9	10	0	0	$0	0	0	$0
10	10	0	4.71	-$23.60	0	0	$0
	100.00	11.07	20.71	- 53.88	18.00	16.00	$ 64.93

To reach the desired position at CP2 and at the catchment level the SO is a net payer. Consequently, the flood distribution was shifted. However, at CP1 the flood distribution was almost maintained at the same level. If the SO relaxes CP2 and the catchment risk position levels, the SO becomes almost revenue neutral with $2.54. This is the result of the SO selling expected flood damage at CP3.

The expected damage changes from the initial $3,439.90 to the final $3,386.26. The reduction is due to the expected damage ($3,439.9 – $3,386.26 = $53.64), and the payment of $53.83 corresponds to the surplus for the SO and the supra rent for the linearisation; however, this difference is almost zero ($53.64 – $53.83= -$0.19).

Example 3

The market clearing is now illustrated with a set of CVaR positions at each control point (see Section 6.5.3). The flood damage cost functions are linearised and non-convex, hence the SOS2 method is used. The same assumptions from example 2 are used, but the clearing model accounts for 3 CVaR positions for each control point and catchment level. These risk positions are established at the confidence levels α=0.95, α=0.97 and α=0.99. Table 6-8 summarises the hypothetical risk positions in the market clearing formulation.

Table 6-8 CVaR positions at different confidence levels

	$CVaR_{\alpha=0.95}$ ($)	$CVaR_{\alpha=0.97}$ ($)	A-$CVaR_{\alpha=0.99}$ ($)	B-$CVaR_{\alpha=0.99}$ ($)
Control point 1	$8,000	$11,056	$26,300	$23,400
Control point 2	$19,000	$21,205	$48,000	$36,000
Control point 3	$30,000	$31,805	$67,224	$50,000
Catchment	$90,000	$110,000	$145,000	$145,000

When clearing the market with the previous assumptions ($CVaR_{\alpha=0.95}$, $CVaR_{\alpha=0.97}$, and A-$CVaR_{\alpha=0.99}$) and participants' preferences, the final area traded should be the same as the previous example, given that the risk positions are bounded only at $CVaR_{0.97}$. (The A-$CVaR_{0.99}$ is not bounded). Prices and allocations for the other participants would be the same. However, small differences can be observed if unscaled infeasibilities were observed with Cplex due to ill-conditioned constraints. CVaR constraints could be tightened so that a solution can be reached, but there would be a violation of reduced cost (indicating non-optimality) or of a bound (indicating infeasibility) (Fourer et al. 2003; ILOG 2008). This point was presented in previous paragraphs.

If the model was additionally cleared using B–$CVaR_{0.99}$ rather than with A–$CVaR_{0.99}$ (see Table 6-8), it would have more restrictive conditions for the extreme events above the 99% confidence level, which may be interpreted as a more risk-averse market. In this condition, the total trade would change as well as the prices at control points. Participants 1 and 4 would increase IC allowance for 0.02 and 4.19 ha, and they would pay $0.33 ($16.50/ha) and $83.83 ($20/ha); participants 6, 7 and 10 would reduce 6, 10 and 6 ha, and they would receive $30 ($5/ha), $200 ($20/ha) and $36.19 ($6/ha) respectively. Therefore, for instance, clearing price would increase for participants 6 from $4/ha to $5/ha and participant 7 from $18/ha to $20/ha for the binding risk position respectively.

Risk constraints bind at CP1 and $CVaR_{0.97}$ with 0.0591 ($/$ at $CVaR_{\alpha}$=0.97), and at CP3 and $CVaR_{0.99}$ with 0.015 ($/$ at B-$CVaR_{\alpha}$=0.99). Clearing prices at control points changed due to the changes in the marginal flood damage as well as for the effects of the extra cost to the system, when allowing an extra unit of risk position at the confidence level α. Thus, the final flood distribution is shifted toward less damage. However, this has a cost to the system which corresponds to the reduction in damage, and for this reason the SO is exposed as a net payer of $182.02. The final expected damage is $3,279.31 from an initial expected damage of $3,439.90, i.e., the difference is $160.59. However, the total payment from the SO of $182.02 is more than the reduction in the expected flood damage $160.59). This difference is due to the catchment risk position and flood damage linearisation.

6.7 Misestimating confidence level (α) and CVaR value

Prices can be affected when the confidence level (α) and CVaR values are mis-estimated. The mis-estimation was discussed indirectly in previous examples when the market had slack and tightened CVaR conditions. Misestimating CVaR can allow more IC levels or in an alternative situation can result in the SO becoming a net payer or a net receiver. For instance, IC allowances can be changed which would shift the flood distribution, but the SO desired to maintain the current flood distribution. Such a case was presented in example 1 (Section 6.6), when at CP1 the CVaR constraint was slack, which allowed incrementing the IC allowances in this area. Otherwise, the SO could be a net payer, as shown in example 3 where the binding CVaR condition increased prices and probably reduced IC allowances in the zone. In both situations, the goals may be to maintain the current flood damage distribution.

Similar effects could be observed when misestimating the confidence level which may result in CVaR constraints being slack or binding.

6.8 Possible issues in hedging against changes in damage via CVaR

This section discusses possible issues that could be noticed when a CVaR position is included in the market formulation. The section addresses misestimating and over-tightening CVaR positions, revenue condition for the SO, market infeasibility, and raising prices for participants. Numerical examples are used to explain these problems with CVaR.

Extreme events, such as 200 mm of rainfall in 24 hours would require extra analysis for the probable effects on prices with CVaR. For instance, suppose the disaster cost is unchanged despite lower IC allowances. In this case, if CVaR were used, the model may be infeasible, or clearing prices would increase and consequently the SO could be a net payer if participants previously had the IC allowances. If the extreme disaster does not change, even though IC allowances were reduced and the model is accounting for CVaR, then desired CVaR is not reachable and the model is infeasible. An equivalent issue was discussed previously in this chapter. Even if all participants could implement practices to reduce runoff from their properties and consequently the impacting flows, the cost of the disaster does not change. This could be the case of hedging against flows above 1,450 volume/time at CP3 shown in Figure 6-5.

Additionally, the SO may be over-allocated according to a new risk position, but previously CVaRs were loose at some control points. In this case, the SO would be a net payer to reduce impacting flows. Another comparable situation could be faced with binding CVaR conditions, if the SO desires to reduce CVaRs even more. Because participants have rights for the previous risk conditions, the SO would be a net payer. But the condition may not be reachable if any offer combination for reducing ICs (and its equivalent in reducing damage) were above the desired CVaR condition and the solution would be infeasible.

Figure 6-3 illustrated a $CVaR_{\delta}$ accounting for the flood damage at the extreme events. The catchment is a small area to simplify the analysis, so the storm is affecting the catchment with similar intensity and hence probability. Examples 3 to 16 storm scenarios are extended with the following probabilities: 0.3987, 0.2, 0.15, 0.1, 0.07, 0.04, 0.02, 0.01, 0.005, 0.003, 0.0012, 0.0008, 0.0005, 0.0005, 0.0002 and 0.0001. The last two scenarios reach the asymptotic flood damage levels (see Figure 6-5). Above 290 volume/time and 1450 volume/time at CP1 and CP3, the flood damage would reach the maximum of \$16,000 and \$33,750 at these control points respectively. Thus, any storm event and IC allowance combination (even with the maximum permeability) will trigger damage at these limits. $CVaR_{\delta=0.9998}$ of \$35,201 and \$74,251 at CP1 and CP3 correspond to lower bounds which limit trade, see Table 6-9. Any lower value would generate an infeasible solution, because no IC allowance combination can be lower than the bound. (This could also correspond to a fundamental capacity, even if all properties were pervious, disaster would reach this level.)

If the catchment was badly constrained with CVaR, the market could be infeasible and the SO would be a net payer until new IC allowances allowed the desired flood distribution. Meanwhile, this over-constrained condition produces an extra cost on the system which would require careful evaluation. Prices are raised and participants are limited for most activities within the catchment.

Suppose different CVaR values are estimated for the tail distribution as shown in Table 6-9. Under the previous catchment condition and participants' preferences (example 3), participants 1, 4, 5, 6, 7 and 10 change IC allowances in 1.11, 5.98, 5, 6, 10, and 6 ha. Payments from the SO are -$16.70 ($15.05/ha), -$107.7 ($18.01/ha), -$67.50 ($13.50/ha), $27 ($4.50/ha), $180 ($18/ha) and $30.02 ($5/ha) (negative value means the SO receives from participants). The flood tail distribution is kept at control points, and the SO is a net payer with $45.12. The expected flood damage is $3,439.12.

Table 6-9 CVaR at different confidence levels

Control Point	$CVaR_{0.95}$	$CVaR_{0.97}$	$CVaR_{0.99}$	$CVaR_{0.999}$	$CVaR_{0.9995}$	$CVaR_{0.9998}$
CP1	$8,405	$11,389	$23,265	$29,800	$32,165	$35,201
CP2	$17,020	$21,599	$37,115	$47,819	$48,525	$48,889
CP3	$23,614	$32,417	$52,325	$70,220	$72,110	$74,251
Catchment	$95,700	$110,000	$140,000	$159,000	$165,000	$170,000

However, a completely different condition is observed if the SO over-tightens the CVaR constraint. For instance, $CVaR_{0.99}$ of $49,325 at CP3 would raise clearing prices due to the risk position components, 0.146 ($/$ at CVaR) at $CVaR_{0.99}$ boundary (reduced by $3,000). Most offers from participants are accepted, and the surrounding areas become permeable. Participants 6 to 10 reduce IC allowances in 9.75, 10, 6, 10 and 9 ha and receive $87.75 ($9/ha), $360 ($36/ha), $35.50 ($5.93/ha), $141.30 ($14.10/ha) and $127.20 ($14.13/ha) at CP2 and CP3 respectively. Prices rise for all participants, in particular participant 6 from $4.50 to $9, participant 7 from $18 to $36, and participant 10 from $5 to $14.13. Consequently, the flood tail distribution is shifted at CP3 and CP2. At CP2 and CP3, the expected damage decreases from $1,108 to $1,009.21, and from $1,927.60 to $1,700.80 respectively; at CP1 the expected damage does not change. The SO is a net payer of $746.20 and the final expected damage is $3,165.11 from $3,487.41. The expected reduction in damage is only $322.30, and the difference corresponds to bounded risk positions at CP1 and CP3 with $CVaR_{0.99}$.

Previous cases accounted for risk positions that tighten the catchment and control points, raising prices. This is because the current flood condition needs to be reduced and so the SO is exposed to this condition. Notice that if the market was cleared risk neutral, the flood distribution would be shifted, and the expected flood damage may increase.

Another way to manage extreme flooding could be to impose penalties on participants as in CO_2 market (Holt et al. 2007). This could be an effective policy while the SO could be revenue adequate. However, this additional constraint includes another price at each participant. Now, each participant faces this individual price, which may limit their decision about imperviousness and BMPs, and their trade. This penalty produces an additional cost to the system faced for each participant. The SO should evaluate this option to manage flooding; however this idea is not further discussed and could be part of future research.

6.9 Final remarks and conclusion

This chapter presented a method to hedge against changes in floods via CVaR. Various measures of risk were examined resulting in the CVaR method being favoured due to its coherence properties. Additionally, the inclusion of the CVaR risk profile into a market clearing model was demonstrated. The risk position depends on the community's desired catchment conditions as represented by the SO. The desired catchment conditions have price and allocation effects for the IC allowances and final trading outcomes.

The chapter showed how prices are affected by SO's risk constraints. The SO may choose a CVaR position at each control point and for the entire catchment. The CVaRs' constraints were shown to work as caps for the market design. Prices were shown to be higher as the risk CVaRs were tightened at control points and at the catchment level.

This chapter showed that participants face risk, producing a cost to the system that is transferred to those participants who impact on binding risk conditions. Participants who do not change their impacts at the tail flood distribution incur cost risk, but these participants are not charged.

Incorporating a risk position could produce an extra cost to the system if the risk is over-constrained. Participants may face a high price for most IC allowances, but they could be incentivised to adopt BMPs in their properties. A market with this condition may work as one-sided with the SO paying for any peak flow reductions to reach the risk position.

The SO could be a net receiver if loose CVaR risk positions become bounded.

This chapter focused discussion on stage-flood, but the CVaR positions could also account for different aspects of floods that contribute to damage, such as duration (see Chapter 5).

Until now, this thesis has used only simple and theoretical numerical examples. In the next chapter, the proposed market models will be illustrated using more comprehensive cases.

Chapter 7

7 CASE STUDY

7.1 Introduction

This chapter presents applications of the proposed market models as described in Chapters 3, 4, 5 and 6. The case studies are based on the L2 river catchment located in the Ellesmere catchment, Canterbury, New Zealand.

Data from the study area are used to simulate the impact of storm events on IC allowances [land use] and flow consequences. Hydrological and hydraulic parameters from the area are used to simulate impacts resulting from changes in IC allowances. Storms are designed using HIRDS (NIWA 2002) and are the main source of uncertainty in this illustration. However, it is recognised that other sources of uncertainty such as flow and flood modelling parameters and possible climate change could produce more intensive storms and resulting flood damage (Yang and Read 1999). Hydrological and hydraulic simulators HEC-HMS (HEC 2008a) and HEC-RAS (HEC 2008b) are used to simulate runoffs from hypothetical properties and hydraulic flows in the L2 river. To simplify the analysis and reporting, the properties were agglomerated into 24 areas.

In the L2 catchment, the SO could use the Det_MarketIC, Sto_MarketIC and Sto_MarketIC_Risk formulations, but each has a different price, allocation and possible revenue implications that will each be illustrated. The formulations include different flood components (depth, hastening inundation and duration), flood cost types (maximum stage-flood, changes in flows and violating flow thresholds) and flood damage approximations (convexified and linearised non-convex damage functions). Prices and allocations are analysed under different market formulations and flood damage approximations. The first illustration considers a hypothetically established market under an extreme storm for the Det_MarketIC. Secondly, rainfall uncertainty is included in Sto_MarketIC, with discussion on the effects of prices, allocation, flood distribution, and hedging of floods in the catchment. The Sto_MarketIC includes the proposed flood components and related flood costs. Finally, the Sto_MarketIC_Risk is evaluated, where the SO desires to reach a risk position against extreme storm disasters, which could correspond to an acceptable risk that the community desires for the flood areas.

In Section 7.2 is introduced the catchment location, the properties, and the hydrological and hydraulic assumptions. Subsection 7.2.1 presents participants' properties and their preferences for land uses. Subsection 7.2.2 presents the storm design. Subsection 7.2.3 describes the catchment hydrology. Subsection 7.2.4 presents the hydraulic flows in the network. Section 7.3 describes the initial flood conditions at the flooding places. In Section 7.4 there are examples (cases) using the proposed market formulations. Section 7.5 is final remarks and conclusions.

7.2 Property, hydrological and hydraulic assumptions

The catchment area is divided into 4 sub-catchments of 1,004.31, 500.00, 3,089.94, and 3,677.00 ha respectively (see Figure 7-1). Six properties per subcatchment are assumed each with similar areas, but with different IC allowances and impervious surfaces (see Appendix C). HIRDS provides rainfall depth for different average recurrence intervals (ARI) for a specific location, and HIRDS was also used to design realistic storm events for modelling. Runoff from properties and routing flows in channels were estimated with HEC-HMS and HEC-RAS. HEC-RAS was also used to model flows at different channel sections as well as possible exceeding flows. ‘Exceeding flows’ as previously defined are flows above channel capacity or flows which will overflow the channel banks.

7.2.1 Participant properties and preferences

Figure 7-1 shows the initial IC allowance areas, the related to curve number *CN*, areas, and concentration time (T_c) for each of the six properties. T_c (hr) is estimated based on the SCS lag method (NRCS 1975; SCS 1985; NRCS 2010). The SCS lag method developed by Mockus (NRCS 2010) considers the hydraulic length of the watershed in meters *I*, catchment lag in hours *L*, a parameter *S* based on the curve number (*CN*), and average watershed slope in percentage (*Y*). T_c is as follows:

$$T_c = \frac{L}{0.6},\; L = \frac{I^{0.8}\left(S+1\right)^{0.7}}{734.4556\; Y^{0.5}} \text{ and } S = \left(\frac{1,000}{CN}\right) - 10$$

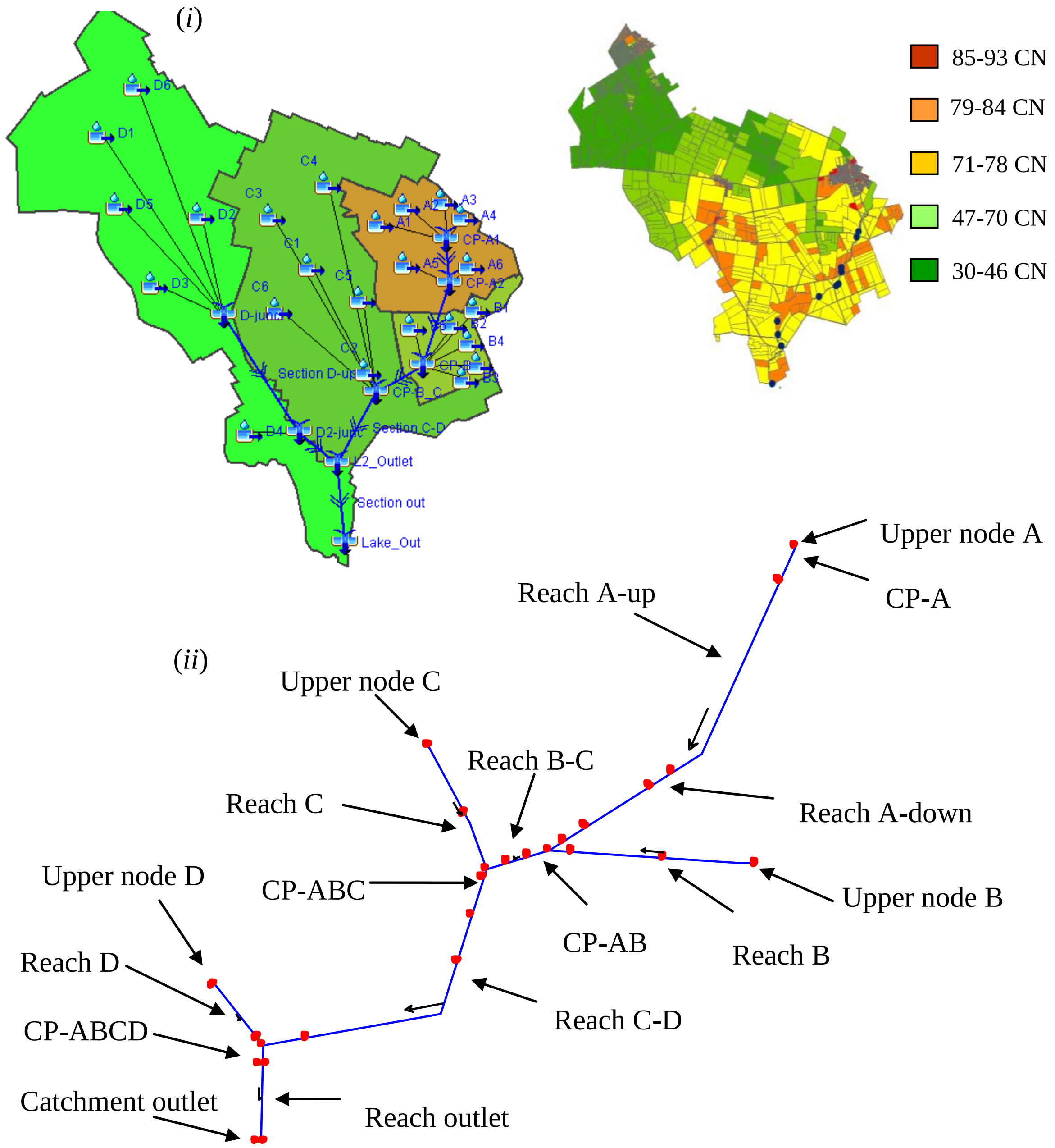


Figure 7-1 Reach sections. (*i*) corresponds to the L2 catchment with the individual injecting upper places and (*ii*) represents the channel network. Red points correspond to cross-sections within each reach.

Table 7-1 also shows the participant's preferences. For instance:

Table 7-1 Initial and desired conditions of participants at different subcatchments.

Property	Area (ha)	Initial Condition			Desired Condition			Base flows (1) (m^3/sec)
		CN	Impervious area (%)	*Tc* (min)	*CN*	Impervious area (%)	*Tc* (min)	
A1	167.385	85	65	856	80	60	1,010	0.253
A2	167.385	85	65	856	80	60	1,010	0.253
A3	167.385	85	65	856	80	60	1,010	0.253
A4	167.385	80	65	1,010	74	60	1,207	0.253
A5	167.385	74	9	1,207	80	9	1,010	0.253
A6	167.385	74	9	1,207	80	9	1,010	0.253
B1	83.333	78	10.5	813	74	10.5	913	0.075
B2	83.333	82	10.5	717	80	10.5	764	0.075
B3	83.333	75	10.5	887	74	10.5	913	0.075
B4	83.333	78	10.5	813	70	10.5	1,020	0.075
B5	83.333	80	10.5	764	85	10.5	648	0.075
B6	83.333	80	10.5	764	85	10.5	648	0.075
C1	514.99	53	5	3,273	60	5	2,743	0.088
C2	514.99	78	9	1,684	60	9	2,743	0.088
C3	514.99	53	5	3,273	60	5	2,743	0.088
C4	514.99	53	5	3,273	60	5	2,743	0.088
C5	514.99	74	9	1,893	60	9	2,743	0.088
C6	514.99	74	9	1,893	50	9	3,531	0.088
D1	612.83	78	65	1,806	80	65	1,698	0.235
D2	612.83	53	5	3,509	53	5	2,940	0.235
D3	612.83	53	5	3,509	70	5	2,940	0.235
D4	612.83	47	5	4,088	53	5	3,509	0.235
D5	612.83	47	5	4,088	60	5	3,509	0.235
D6	612.83	78	65	1,806	60	65	2,266	0.235

(1) A proportional base flow was calculated based only on properties' size and the flows measure in the river and catchment.

- Participant A1 has 167.3 ha (urban area) with a hypothetical hydraulic length of 1,293m, *CN* 85 and slope 0.01%, and their T_c is 856 approx min. With the purpose to shift IC in the areas, this participant desires to reduce IC allowances to *CN* 80 by improving grass cover in the area, and so T_c is approx 1,010 min.

- Participant B4 (farm area) has a property with bare soil, and desires to crop, and hence to reduce IC allowances to *CN* 70. Consequently, the T_c would increase to approximately 1,020 min.

- Participant C5 (farm area), with row crops, wishes to change IC allowances to meadows, and hence to reduce *CN* to 60. In this case the T_c would increase to approximately 2,743 min.

- Participant D3 (farm area) with pasture (*CN* 53) desires to crop using a BMP (contoured), and hence would change IC to *CN* 70. The T_c would be reduced to approximately 2,940 min.

7.2.2 Storms

Twenty-four hour type I, II and III storms were designed with ARIs of 2, 5, 20, 30, 40, 50, 70, 80, 100, 125, and 150. The Sto_MarketIC uses 28 scenarios to estimate impacting flows from properties and flood components. These scenarios represent the storm distribution and the chosen scenarios accounts for the frequency curve as accurately as possible.

The storm design accounts for intensity-duration-frequency (IDF) curves (Durrans 2010) and the peak time. Thus, storms account for the maximum intensity and the peak time. Table 7-2 shows the duration-intensity and estimate for the Ellesmere area from HIRDS. The storm intensity at the first 0.5 hour corresponds to 43.567 mm/hr ($e^{3.382942-0.560905\times\ln(0.5)-0.001858\times\ln(0.5)^2}$ =43.467, see B in Table 7-3). Column E in Table 7-3 corresponds to the 24-hours ARI 100 type I design storm used to simulate flows from properties and flows at control points in the L2 catchment.

Table 7-2 Estimates for the duration-intensity for an ARI 100 years.

Duration (min)	Duration (hrs)	Accumulation Depth (mm)	Incremental Depth (mm)	Intensity (mm/hr)	Ln(Intensity) (mm/hr)	Ln(Duration) (hrs)
0	0	0	0	0	0	0
10	0.17	6.6	6.6	39.60	3.68	-1.79
20	0.33	9.2	2.6	27.60	3.32	-1.10
30	0.50	11.1	1.9	22.20	3.10	-0.69
60	1.00	15.5	4.4	15.50	2.74	0.00
120	2.00	21.3	5.8	10.65	2.37	0.69
360	6.00	35.5	14.2	5.92	1.78	1.79
720	12.00	49	13.5	4.08	1.41	2.48
1440	24.00	67.6	18.6	2.82	1.04	3.18
2880	48.00	84.8	17.2	1.77	0.57	3.87
4320	72.00	96.9	12.1	1.35	0.30	4.28

Annual exceedance probabilities were estimated from HIRDS, which reports frequencies for different duration and storm depths across New Zealand. Storm probabilities can be estimated by adjusting historical storms to a distribution such as EV1 and EV2 which was described in section 2.3, Chapter 2, or with a first-order two-state Markov chain for a daily time-series (Brown 2007). For this illustration, exceedance probabilities are calculated using frequencies from HIRDS, followed by specific storm distribution types, i.e., I, II and III are assumed to have probabilities of 0.2, 0.7 and 0.1 respectively. For example a 24-hour ARI 50 type II storm, may have an exceedance probability of 1.4%. Low intensity storms such as ARI 2 would not produce significant flood problems, and the discretised storm probabilities will sum to 1.

Table 7-3 Twenty four hour type I (peak at the 1/3 storm time) ARI 100 storm for the L2 catchment.

Duration (hrs) A	Intensity (mm/hr) B	Cumulate depth (mm) C= A x B	Incremental depth (mm) D= Ct-Ct-1	Redistributed rain (mm) E	Duration (hrs) A	Intensity (mm/hr) B	Cumulate depth (mm) C= A x B	Incremental depth (mm) D= Ct-Ct-1	Redistributed rain (mm) E
0.5	43.567	21.784	21.784	1.635	12.5	7.077	88.464	1.558	1.558
1	29.457	29.457	7.674	1.723	13	6.922	89.987	1.524	1.524
1.5	23.430	35.145	5.687	1.826	13.5	6.776	91.478	1.491	1.491
2	19.917	39.834	4.689	1.948	14	6.638	92.938	1.460	1.460
2.5	17.559	43.898	4.064	2.096	14.5	6.508	94.369	1.431	1.431
3	15.842	47.525	3.627	2.279	15	6.385	95.772	1.403	1.403
3.5	14.521	50.824	3.299	2.515	15.5	6.268	97.149	1.377	1.377
4	13.467	53.866	3.042	2.834	16	6.156	98.501	1.352	1.352
4.5	12.600	56.701	2.834	3.299	16.5	6.050	99.830	1.329	1.329
5	11.872	59.362	2.662	4.064	17	5.949	101.136	1.306	1.306
5.5	11.250	61.877	2.515	5.687	17.5	5.853	102.420	1.284	1.284
6	10.711	64.266	2.389	21.784	18	5.760	103.684	1.264	1.264
6.5	10.238	66.545	2.279	7.674	18.5	5.672	104.928	1.244	1.244
7	9.818	68.728	2.182	4.689	19	5.587	106.154	1.225	1.225
7.5	9.443	70.823	2.096	3.627	19.5	5.506	107.361	1.207	1.207
8	9.105	72.842	2.018	3.042	20	5.428	108.551	1.190	1.190
8.5	8.799	74.790	1.948	2.662	20.5	5.352	109.724	1.173	1.173
9	8.519	76.674	1.885	2.389	21	5.280	110.882	1.157	1.157
9.5	8.263	78.501	1.826	2.182	21.5	5.210	112.023	1.142	1.142
10	8.027	80.273	1.773	2.018	22	5.143	113.150	1.127	1.127
10.5	7.809	81.997	1.723	1.885	22.5	5.078	114.263	1.113	1.113
11	7.607	83.675	1.678	1.773	23	5.016	115.361	1.099	1.099
11.5	7.418	85.310	1.635	1.678	23.5	4.955	116.447	1.085	1.085
12	7.242	86.905	1.595	1.595	24	4.897	117.519	1.072	1.072

7.2.3 Hydrology of properties

Once storms are defined, they are used to simulate flows from properties under different IC allowances. Then the SCS method for losses and unit hydrographs (SCS 1985) is used to estimate flows from each property. The flows are routed through the channel network, which has uniform base flows (see last column in Table 7-1). Routing flows were simulated using the Kinematic wave channel routing equation with HEC-HMS, assuming trapezoidal sectional shapes. Table 7-4 presents channel reaches and shapes used in the simulations.

Posterior impacting flows were simulated from each property at injecting locations, which were also simulated into the nodal channel system, and at junctions. It was assumed that once flows arrived at the junctions (which can also be control points), the flows would be proportional to the injected flows. This would correspond to impacts at short distances such as in the L2. Thus, a set of discretised individual flows at different places in the network can be calculated by time and scenario, under different IC allowances from each participant. The channel network and the injection points from each participant are illustrated in Figure 7-1.

Table 7-4 Channel reaches, cross-sections and properties.

Channel reaches	Length (m)	Slope (m/m)	Manning coefficient	Sectional shape	Base width (m)	Side slope (xH:1V)
Channel reach A-up	1,005	0.00024	0.02	Trapezoid	3	1
Channel reach A-down	2,313	0.00024	0.03	Trapezoid	4	1
Channel reach B-C	2,647	0.00024	0.03	Trapezoid	5	1
Channel reach C-D	3,337	0.00024	0.03	Trapezoid	5	1
Channel reach D-up	3,155	0.00024	0.02	Trapezoid	5	1
Channel reach D-down	900	0.00024	0.02	Trapezoid	10	1
Channel reach outlet	700	0.003	0.03	Trapezoid	10	1

7.2.4 Hydraulic flows in the network

Hydraulic simulations were carried out with HEC-RAS to evaluate flood areas, and to estimate flood components in the channel network under different flows resulting from the simulated storms. The injecting flows correspond to runoff flows, at each upper node, from each section (see injecting node from section A in Figure 7-1 *ii*). The water level at the L2 catchment outlet was defined by the total channel flows, the outlet cross-section and a friction factor of 0.01 (Manning's *n* factor). Red dots represent cross-sections along each reach (Figure 7-1 *ii*). Each reach has at least 2 cross-sections.

HEC-RAS simulates both super-critical and sub-critical flows for each cross-section. Given the condition in the L2 river, with sudden change in channel dimensions and slopes, all flows will be sub-critical in nature (see e.g., Haan et al. 1994; Mascarenhas et al. 2005; Birbil et al. 2009). With critical flows, similar flows may produce different depths and velocities, or different flows f_1 and f_2 produce similar depth *h*, and looping conditions can be observed in the rating curve (see Figure 7-2). However, this evaluation is considered difficult (Birbil et al. 2009). Sub-critical flows were used to estimate channel depth. The minimum flow that reached the first channel bank (levee) was used to determine channel capacity and also used to identify flows that resulted in particular flood levels.

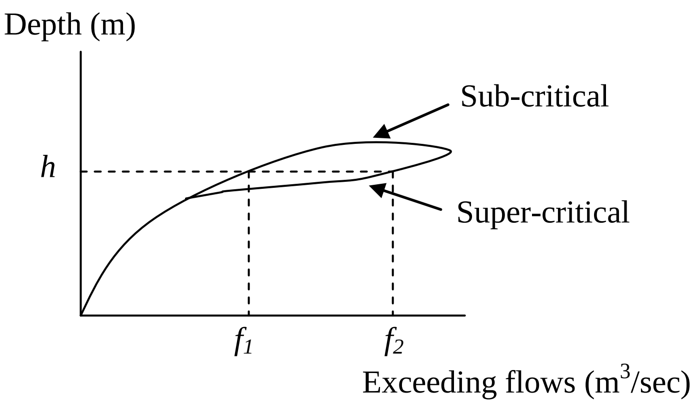


Figure 7-2 Rating curve (depth vs. flows).

7.3 Initial flood condition

7.3.1 Flows and flow capacities

Channels are defined by their cross-sectional shape as shown in Figure 7-3. When the simulated flow levels exceed either the left or right bank, a flood condition occurs (Figure 7-3 B). In the Figure 7-3 B, the bank station on the right is 1.11 m from the channel bottom, but given the non-uniform sectional shape, this point would be equivalent to a 1 m hydraulic depth, and flows above 4.2 m^3/sec will generate flooding in the area. When one of the sides is exceeded, HEC-RAS keeps the flow balance, and thus assumes an artificial vertical boundary at the bank station (see Figure 7-3 B).

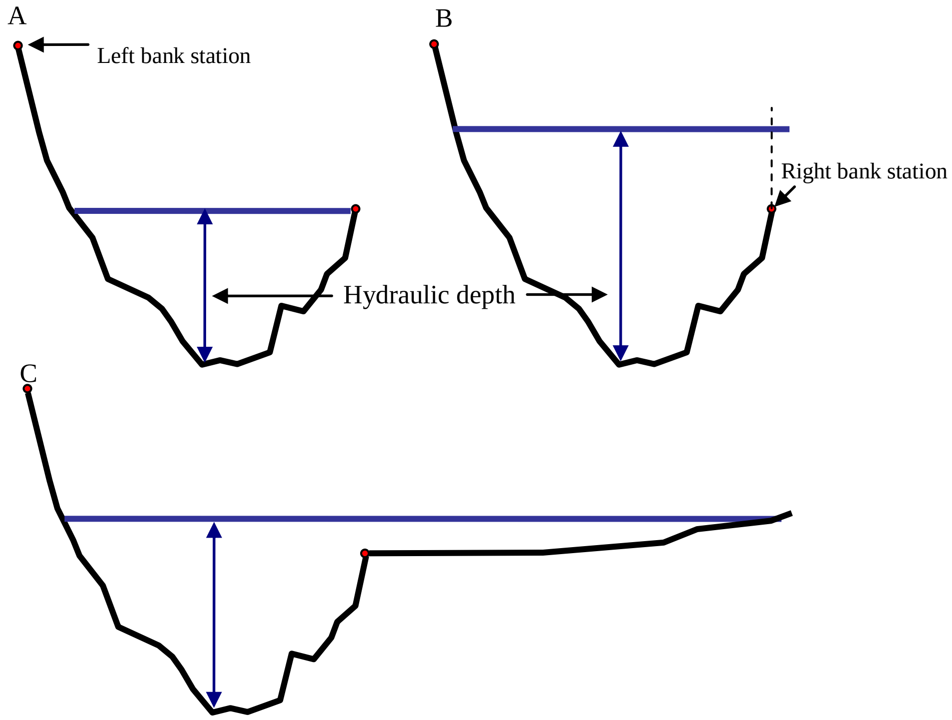


Figure 7-3 Cross section of control point CP-A with two hydraulic depth levels from HEC-RAS. Blue line in A, B and C represents hydraulic depths.

The case study relates channel flows to flooding at surrounding and sectional places. A new channel depth was calculated using Manning's equation and an extended floodplain cross-section as Figure 7-3 C was used to estimate the new flood depth at the cross-section. A similar approximation was illustrated in Chapter 2.

Figure 7-1 illustrates five assumed flood areas, identified as CP-A, CP-AB, CP-ABC, CP-D and CP-ABCD. These control points are close to the junction areas. Although flows may exceed the banks of other points along channels, only points at these areas were studied. These points are assumed to be more likely selected by the SO for managing and monitoring floods.

Table 7-5 presents the flows that reach the bank station before overtopping the control points. For instance, flooding becomes an issue at CP-A with minimum flows M_k of 4.2 m^3/sec.

Table 7-5 Control points

Control Points	Maximum flow before overtopping bank, M_k (m^3/sec)	Maximum hydraulic depth (m)
CP-A	4.2	1.03
CP-AB	4.36	1.24
CP-ABC	5.7	1.19
CP-D	3.3	1.46
CP-ABCD	8.1	1.25

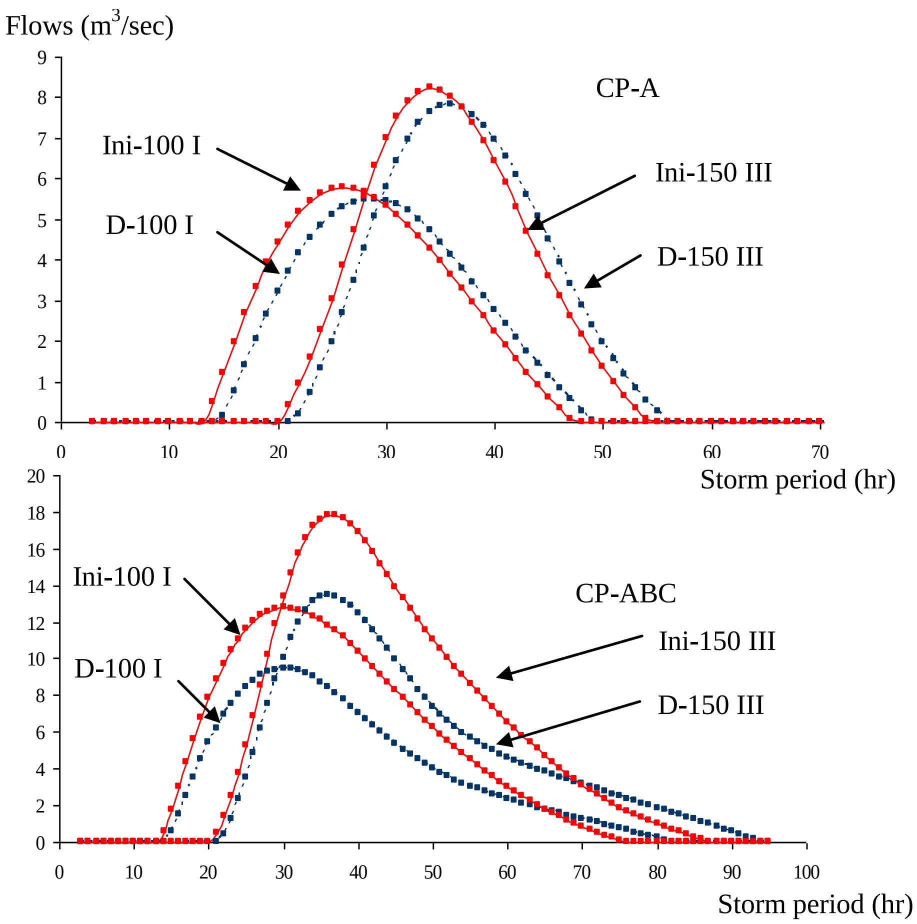


Figure 7-4 Initial (Ini) and desired (D) conditions of exceeding flows above banks at control points A and ABC under two scenarios (100 and 150 year) and storm distribution types I and III.

Table 7-6 presents the initial flood distribution in terms of exceeding flows at control points. Figure 7-4 illustrates these flows at two control points and storm scenarios. For instance, according to the current IC allowances in the catchment, the initial exceeding flows reach peaks of 5.773 m^3/sec at CP-A for 100 type I, with a flood duration of 34 hrs. The desired conditions would shift exceeding peaks from 5.773 to 5.499 m^3/sec at CP-A.

Table 7-6 Initial (Ini) and desired (D) exceeding flows above capacity at control points and storm scenarios (scen)

Scen	(Ini) Exceeding peak flows (m^3/sec)					(Ini) Duration (hrs)					(D) Exceeding peak flows (m^3/sec)					(D) Duration (hrs)				
	CP-A	CP-AB	CP-ABC	CP-D	CP-	CP-A	CP-AB	CP-ABC	CP-D	CP-	CP-A	CP-AB	CP-ABC	CP-D	CP-	CP-A	CP-AB	CP-ABC	CP-D	CP-
2-II	0.343	1.574	1.520	0.962	2.759	9	19	20	30	31	0.068	1.202	0.579	0.770	1.626	4	18	13	30	25
10-II	1.932	3.958	4.597	2.336	6.881	20	27	35	43	45	1.623	3.434	2.982	2.168	4.986	20	27	27	50	46
20-I	2.390	4.606	5.662	3.044	8.743	26	32	44	51	53	2.144	4.181	3.860	2.952	6.592	26	32	35	61	58
20-II	2.898	5.433	6.541	3.184	9.470	24	30	42	50	51	2.576	4.822	4.505	3.068	7.114	24	30	34	60	56
20-III	3.022	5.708	6.823	3.170	9.735	23	29	42	50	50	2.680	5.060	4.754	3.067	7.278	23	30	33	60	55
30-I	2.924	5.408	6.756	3.548	10.248	27	34	47	55	57	2.675	4.942	4.721	3.501	7.838	27	34	41	66	63
30-II	3.483	6.325	7.732	3.705	11.050	25	32	45	55	55	3.156	5.675	5.447	3.630	8.426	25	32	39	65	62
30-III	3.619	6.632	8.046	3.688	11.344	25	31	44	55	55	3.268	5.938	5.717	3.627	8.605	24	31	38	65	61
40-I	3.569	6.384	8.095	4.161	12.092	29	35	51	61	61	3.313	5.873	5.775	4.184	9.379	29	36	47	71	69
40-II	4.185	7.409	9.190	4.340	12.982	28	33	49	60	60	3.852	6.702	6.591	4.326	10.032	28	35	47	71	67
40-III	4.338	7.748	9.535	4.322	13.311	27	33	48	61	60	3.977	6.997	6.893	4.321	10.225	27	34	46	69	68
50-I	4.012	7.058	9.032	4.588	13.364	30	37	53	64	65	3.753	6.515	6.505	4.659	10.442	30	37	53	75	73
50-II	4.671	8.158	10.203	4.778	14.320	28	35	51	64	63	4.332	7.414	7.389	4.809	11.146	28	35	51	72	71
50-III	4.832	8.518	10.566	4.760	14.668	28	34	50	64	63	4.467	7.730	7.712	4.806	11.345	28	34	50	69	71
70-I	4.803	8.263	10.704	5.354	15.647	32	38	58	71	70	4.539	7.665	7.822	5.524	12.363	32	38	61	77	78
70-II	5.531	9.487	12.011	5.570	16.718	31	36	55	70	68	5.185	8.681	8.812	5.690	13.142	31	36	58	73	76
70-III	5.710	9.884	12.408	5.546	17.096	30	36	55	70	69	5.333	9.033	9.173	5.687	13.356	30	37	59	70	74
80-I	5.140	8.778	11.417	5.684	16.631	33	38	59	74	72	4.874	8.158	8.389	5.898	13.186	33	40	63	77	80
80-II	5.901	10.061	12.786	5.912	17.742	31	36	58	73	70	5.550	9.224	9.428	6.073	14.004	31	38	62	73	77
80-III	6.083	10.472	13.202	5.884	18.143	30	36	57	71	71	5.700	9.588	9.804	6.068	14.222	31	37	61	70	74
100-I	5.773	9.742	12.774	6.308	18.486	34	39	62	78	77	5.499	9.085	9.460	6.609	14.757	35	41	68	77	82
100-II	6.590	11.129	14.248	6.552	19.681	32	37	61	75	75	6.232	10.243	10.588	6.798	15.627	33	39	66	74	78
100-III	6.785	11.567	14.694	6.520	20.099	32	37	60	72	74	6.394	10.634	10.988	6.789	15.854	32	38	66	71	74
125-I	6.466	10.810	14.268	6.996	20.528	36	41	65	79	81	6.186	10.097	10.641	7.403	16.479	35	41	72	78	82
125-II	7.342	12.299	15.860	7.258	21.810	34	39	64	75	79	6.981	11.359	11.858	7.601	17.420	34	40	70	75	78
125-III	7.550	12.774	16.335	7.223	22.264	33	39	64	72	76	7.150	11.779	12.285	7.594	17.650	33	40	70	71	75
150-I	7.083	11.756	15.601	7.614	22.352	36	42	69	79	83	6.798	11.003	11.701	8.117	18.032	36	43	75	78	83
150-II	8.011	13.341	17.295	7.890	23.715	35	40	68	75	79	7.642	12.355	13.000	8.327	19.020	34	40	73	75	78
150-III	8.230	13.838	17.795	7.852	24.188	34	40	67	72	76	7.823	12.794	13.451	8.317	19.256	34	40	73	72	75

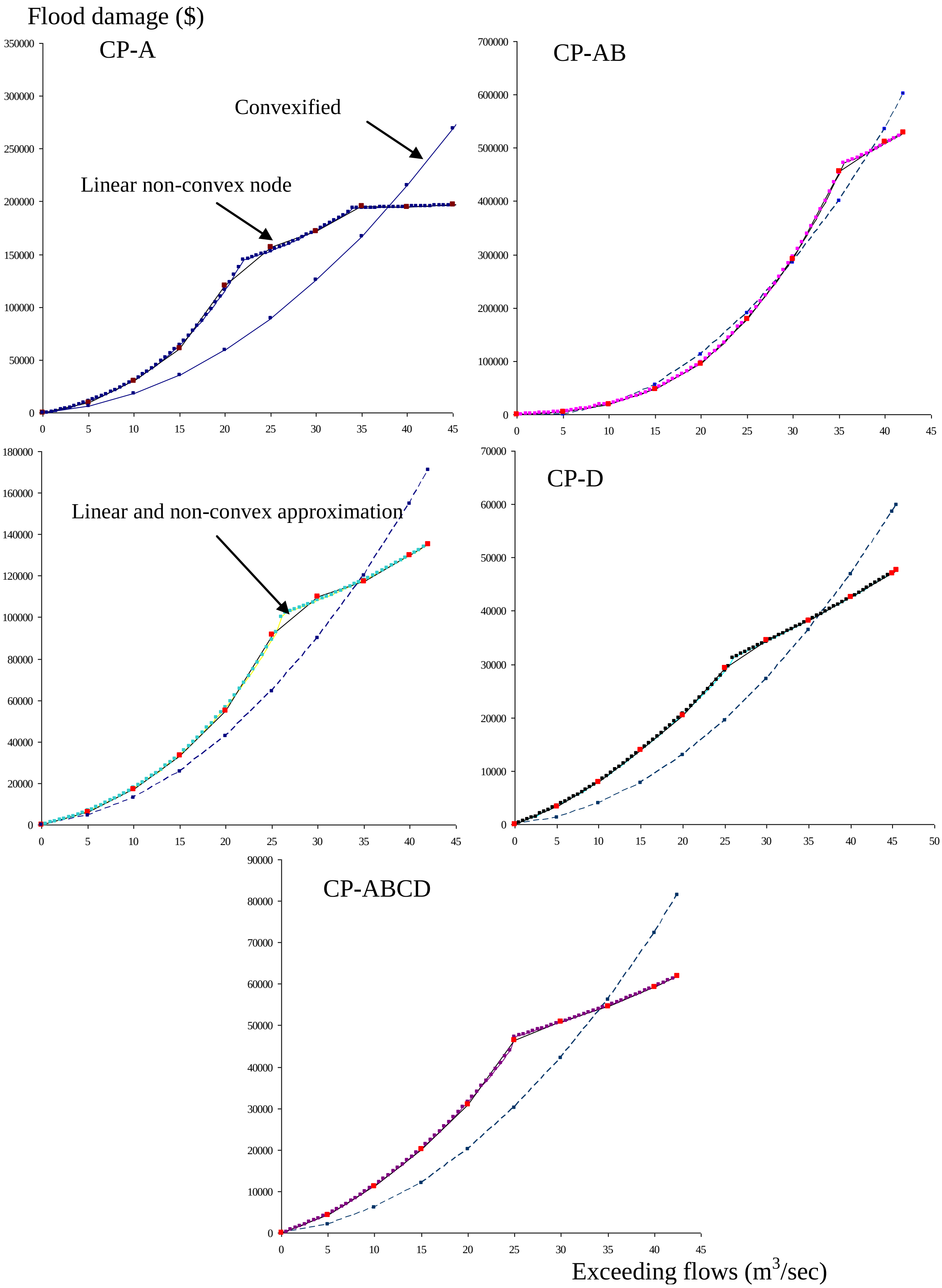


Figure 7-5 Stage-flood damage at control points with two flood damage function approximations: convexification, and linear and non-convex. CP-A, CP-AB, CP-ABC, CP-D and CP_ABCD are control points

Flood duration would be changed from 34 to 35 hrs at CP-A for 100 type I storm respectively. The new desired conditions would reduce peaks and would lengthen flood conditions.

7.3.2 Flood cost

Peak flows cause flood damage at control points. The relationships between damage and flows are non-linear. Note that flood damage was estimated for hypothetical flood conditions, assuming exponential flood damage curves at each flood section and at each control point (see Appendix C for the approximation values). The market timeframe is a year, so the recourse flood cost accounts for the expected damage for the storm throughout the year.

To use the flood damage cost function, the following two approaches are applied: i) piecewise linear convexified, and ii) linearised and non-convex. For the first approach, the functions are convexified as shown in Figure 7-5. The dashed blue lines correspond to convex approximations. For the second approach, the SOS2 method is applied to the flood damage cost function at each control point. The red points are estimated nodes for the linear and non-convex approximations as stated in Chapter 4.

7.4 Outcomes for proposed market models

The proposed market models assume competitive conditions associated with the modelling of 28 storm scenarios (Table 7-6). It is assumed that all participants comply with the outcomes from the market. The SO monitors flood places, channels and river sections in the catchment, which may occur particularly if a market was not established.

Participants bid in 3 tranches (see Table 7-7), and the bid decisions are private. Bids correspond to the opportunity cost for changing IC allowances, which are related to the capped points in an extreme scenario (Det_MarketIC) and the opportunity cost of changing the flood distribution (Sto_MarketIC). The same bids are used in all cases.

Table 7-7 Participant bids for tranche 1, 2 and 3.

Particip.	Area (ha)	*CN*	Impervious area (%)	Tranche 1		Tranche 2		Tranche 3	
				ha	$/ha	ha	$/ha	ha	$/ha
A1	167.385	80	60	100	$2	50	$3	17.385	$5
A2	167.385	80	60	100	$2	50	$3	17.385	$5
A3	167.385	80	60	100	$2	50	$3	17.385	$5
A4	167.385	74	60	100	$2	50	$3	17.385	$5
A5	167.385	80	9	100	$10	50	$6	17.385	$3
A6	167.385	80	9	100	$11	50	$6	17.385	$3
B1	83.333	74	10.5	50	$2	20	$4	13.333	$6
B2	83.333	80	10.5	50	$2	20	$4	13.333	$6
B3	83.333	74	10.5	50	$2	20	$4	13.333	$6
B4	83.333	70	10.5	50	$2	20	$4	13.333	$6
B5	83.333	85	10.5	50	$10	20	$7	13.333	$3
B6	83.333	85	10.5	50	$12	20	$7	13.333	$3
C1	514.99	60	5	250	$11	200	$5	64.99	$2
C2	514.99	60	9	250	$2	200	$5	64.99	$6
C3	514.99	60	5	250	$11	200	$7	64.99	$2
C4	514.99	60	5	250	$10	200	$7	64.99	$2
C5	514.99	60	9	250	$1	200	$3	64.99	$5
C6	514.99	50	9	250	$1	200	$3	64.99	$5
D1	612.83	80	65	250	$7	200	$6	162.83	$2
D2	612.83	53	5	250	$6	200	$4	162.83	$1
D3	612.83	70	5	250	$8	200	$5	162.83	$1
D4	612.83	53	5	250	$9	200	$5	162.83	$1
D5	612.83	60	5	250	$7	200	$4	162.83	$2
D6	612.83	60	65	250	$1	200	$3	162.83	$5

7.4.1 Det_MarketIC

The market formulation is cleared under an established storm scenario, similar to using credits for reducing peak flows in the USA relating to extreme storm events such as those that occur on a 1:50 or 1:100 year basis. This market model will account for a 100 ARI type I storm in 24 hours, which is chosen randomly. This storm design is normally used for designing policies (Doll et al. 1998). The storm corresponds to 122.5 mm depth based on HIRDS estimation.

The market will account for capped control points. Although capped points are established, floods may still occur in the area. The initial peak flows at control points based on the status quo of IC allowances (land uses) are 9.97, 14.10, 18.47, 9.61, and 26.57 m^3/sec, with respective flood durations of 34, 39, 62, 78, and 77 hrs at CP-A, CP-AB, CP-ABC, CP-D, and CP-ABCD. The following three trading cases are analysed.

Case 1: Initially, the catchment has peak flow conditions for a 100 year type I storm with control point caps, L_k, of 9.97, 14.10, 18.47, 9.61 and 26.57m^3/sec for CP-A, CP-AB, CP-ABC, CP-D and CP-ABCD respectively. That means the market is fully allocated and the capped control points reach the same level than the stage flows from the status quo of initial IC allowances.

In this case, most participants trade and change IC allowances. Table 7-8 summarises the trade outcomes and final payments. For instance, participants A1, B4 and C5 sell 100, 15.7 and 14.26 ha to reduce IC allowances, and they receive \$200, \$31.54 and \$14.26 respectively. Participant D3 buys 450 ha to increase IC allowance to *CN* 70 and pays \$2,169.46. The SO receives a net \$655.71 due to shifted peak flow times, and so the SO sells flows from these new peak times. At CP-A and CP-AB, the capacities are binding at time 27 from 26, at CP-ABC at time 31 from 30, at CP-D at time 47 from 43. At ABCD, flows are not binding. The clearing prices at CP-A, CP-AB, CP-ABC, and CP-D are \$720.35, \$521.40, \$533.12, and \$4,300.50/m^3/sec respectively.

Case 2: the market is capped at channel capacities, M_k, in an attempt to prevent flooding from any extreme storm. Channel capacities at control points CP-A, CP-AB, CP-ABC, CP-D, and CP-ABCD are 4.2, 4.36, 5.7, 3.3, and 8.1 m^3/sec respectively. That means, stage flows at control points should be reduced via imperviousness by 5.77, 9.74, 12.77, 6.31, and 18.47 m^3/sec at CP-A, CP-AB, CP-ABC, CP-D, and CP-ABCD respectively. This situation, the solution with Det_MarketIC is not feasible. Even though most participants would reduce IC allowances, and the SO would try to reduce flows to reach channel capacity, flooding would still occur in the catchment. Thus, if the SO tries to manage the flooding and to hedge against a range of storms via over-constraining the market, the solution may be not feasible. A strategy is required to hedge against a range of storms such as investment in levees or banks. Even with scaling initial IC allowances, the solution may not be feasible.

Case 3: the market is capped for the exceeding peak flows, L_k, at CP-A 5.2, CP-AB 9.1, CP-ABC 11.5, CP-D 5.5 and CP-ABCD 17 m^3/sec for a 100 year type I storm. That means, the stage flow should be reduced by 0.87 m^3/sec at CP-A, 0.6 m^3/sec at CP-AB, 1.27 m^3/sec at CP-ABC, 0.81 m^3/sec at CP-D, and 1.57 m^3/sec at CP-ABCD.

In this case, the solution is feasible. The SO desires to reduce flood levels at control points. Most reducing IC bids are accepted. The SO will pay a net \$18,882.80. This could be expensive (maybe far over the SO's budget). Participants A1 and C5 reduce total allowances, selling 150 and 250 ha and receive approximately \$600 and \$319 respectively.

Participants B4 and D3 do not change IC allowances (see Table 7-8), but they were trading in case 1. Other participants such as C4 and C6 trade similar IC allowances in similar terms (buying 514 and selling 250 ha respectively). Compared to case 1, C4 pays an extra \$25, and C6 receives an extra \$86. The clearing prices at CP-A, CP-AB, CP-ABC, and CP-D are \$4,938.70, \$0, \$647.79, and \$20,564.80/m^3/sec respectively, much higher than in case 1. CP-AB and CP_ABCD were unbounded in this case.

Table 7-8 Trading result from application of Det_MarketIC for trading cases 1 and 3

Particip.	Trading case 1				Trading case 3			
	Area bought (ha)	Area sold (ha)	Paid (\$)	Received (\$)	Area bought (ha)	Area sold (ha)	Paid (\$)	Received (\$)
A1	0	100	\$0	\$200.00	0	150	\$0	\$600.21
A2	0	25.034	\$0	\$50.07	0	150	\$0	\$600.21
A3	0	100	\$0	\$200.00	0	150	\$0	\$600.21
A4	0	150	\$0	\$545.66	0	167.385	\$0	\$1,633.67
A5	150	0	\$544.81	\$0	0	0	\$0	\$0
A6	150	0	\$544.81	\$0	18.552	0	\$204.07	\$0
B1	0	0	\$0	\$0	0	0	\$0	\$0
B2	0	0	\$0	\$0	0	0	\$0	\$0
B3	0	0	\$0	\$0	0	0	\$0	\$0
B4	0	15.774	\$0	\$31.55	0	0	\$0	\$0
B5	83.333	0	\$37.72	\$0	83.333	0	\$1.94	\$0
B6	83.333	0	\$37.72	\$0	83.333	0	\$1.94	\$0
C1	514.99	0	\$75.17	\$0	514.99	0	\$100.41	\$0
C2	0	0	\$0	\$0	0	99.431	\$0	\$198.86
C3	514.99	0	\$75.17	\$0	514.99	0	\$100.41	\$0
C4	514.99	0	\$75.17	\$0	514.99	0	\$100.41	\$0
C5	0	14.266	\$0	\$14.27	0	250	\$0	\$319.81
C6	0	250	\$0	\$299.69	0	250	\$0	\$385.53
D1	612.83	0	\$193.52	\$0	450	0	\$1,177.87	\$0
D2	450	0	\$454.73	\$0	250	0	\$1,182.88	\$0
D3	450	0	\$2,169.46	\$0	0	0	\$0	\$0
D4	450	0	\$492.63	\$0	432.697	0	\$2,163.48	\$0
D5	529.498	0	\$1059.00	\$0	0	0	\$0	\$0
D6	0	612.83	\$0	\$3,762.95	0	612.83	\$0	\$19,577.70
Total	4,503.96	1,267.90	\$5,760.91	\$5,105.19	2,862.89	1,829.65	\$5,036.41	\$23,919.20

The three cases illustrate market effects when the SO uses different caps for the catchment. These caps affect prices, allocations, the final IC allowances in the catchment, and the SO faces different revenue conditions.

7.4.2 Sto_MarketIC

This section illustrates the Sto_MarketIC formulation. Twenty nine storm scenarios are defined using HIRDS, which uses a generalised extreme value distribution, to estimate rainfall frequency and rainfall depth. In case 4, the SO considers only the maximum stage-flood. Two types of functions are used for modelling this peak damage function: convexified flood damage cost (approximation type A) and a linearised non-convex damage function (approximation type B). Secondly, the SO extends the penalties due to the hastened peak flood time (factor *i*), and flood duration (factor *ii*). Case 5 uses flood damage approximations type A (x.A) and B (x.B) with flood constraints and thresholds type I (see section 5.2.2.1 Chapter 5). Case 6 uses flood damage approximations type A (x.A) and B (x.B) with flood constraints and thresholds type II (see section 5.2.2.2 Chapter 5). Finally, in case 7, the SO is concerned only for hastening peak time of flooding, and uses flood damage approximation type B to analyse prices and allocations.

7.4.2.1 Sto_MarketIC for maximum stage-flood cost

Case 4: This case penalises only the maximum stage-flood, which is assumed to be the main flood component. The initial estimated expected damage in the catchment corresponds to $10,839.20 and $18,711.80 for trading cases 4.A and 4.B respectively. After clearing the market, the final expected damages are $9,284.41 and $15,471.20 for trading cases 4.A and 4.B respectively. The catchment is now hedged against a wider range of storms and consequently the expected damage is reduced in both cases by $1,554.79 and $3,240.60. However, the total net payment from the SO is only $1,425 and $2,933 in trading cases 4.A and 4.B respectively. The SO is a net payer, and due to linearisation effects, the SO is paying less than the real estimated damage with an extra rent effect for linearisation. Probably a more accurate approximation would be necessary.

Participants A1, B4 and C5 increase IC allowances with both types of flood damage approximations. However, they receive different payments (see Table 7-9). For instance, participant D3 increases IC allowances in 250 ha with both damage type approximations, but final payments vary by $124.20.

In cases 4.A and 4.B, the flood distribution was shifted and reduced, and the system cost was between $1,425 and $2,933 respectively, compared to $18,882.80 for Det_MarketIC. With Det_MarketIC, the flood damage is not clear, nor internalised by participants. If the equivalent final exceeding flows are used from the Sto_MarketIC outcomes in the "100 ARI type I storm" scenario, to cap the control points in the Det_MarketIC, and bid participant's preferences are kept, the system cost is $1,688.70. However, the Det_MarketIC final allocation is different from cases 4.A and 5.B (as will be seen in the next section). (For further details regarding possible differences in the solution between a deterministic and stochastic formulation, see Birge and Louveaux (1997), and Kall and Wallace (1994)). On the other hand, if participants reduce bid/offer prices by 90%, similar allocations could be reached with Det_MarketIC, but the system cost would be only $168.87. Therefore, participants may not fully internalise flood damage in the Det_MarketIC if prices are set too low. This point was discussed in Chapter 3.

Table 7-9 Trading results from the application of Sto_MarketIC1 for trading case 4

Particip.	Trading case 4.A (convexified)				Trading case 4.B (non-convex)			
	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)
A1	0	114.51	$0	$343.53	0	150	$0	$599.60
A2	0	150	$0	$450.00	0	150	$0	$599.60
A3	0	100	$0	$300.00	0	150	$0	$599.60
A4	0	150	$0	$709.08	0	167.385	$0	$1,106.18
A5	150	0	$652.54	$0	100	0	$600.00	$0
A6	150	0	$652.54	$0	120.377	0	$722.26	$0
B1	0	0	$0	$0	0	0	$0	$0
B2	0	0	$0	$0	0	0	$0	$0
B3	0	0	$0	$0	0	0	$0	$0
B4	0	50	$0	$161.73	0	50	$0	$198.55
B5	83.333	0	$61.98	$0	83.333	0	$78.85	$0
B6	83.333	0	$61.98	$0	83.333	0	$78.85	$0
C1	514.99	0	$80.52	$0	514.99	0	$121.29	$0
C2	0	250	$0	$542.79	0	250	$0	$817.44
C3	514.99	0	$80.52	$0	514.99	0	$121.29	$0
C4	514.99	0	$80.52	$0	514.99	0	$121.29	$0
C5	0	250	$0	$326.17	0	250	$0	$493.20
C6	0	250	$0	$383.78	0	250	$0	$580.57
D1	612.83	0	$145.69	$0	612.83	0	$263.75	$0
D2	612.83	0	$26.65	$0	612.83	0	$49.97	$0
D3	612.83	0	$134.70	$0	612.83	0	$258.91	$0
D4	612.83	0	$23.17	$0	612.83	0	$42.71	$0
D5	612.83	0	$41.87	$0	612.83	0	$77.55	$0
D6	0	250	$0	$251.38	0	250	$0	$474.78
Total	5,075.79	1,564.51	$2,046.68	$3,472.46	4,996.16	1,667.39	$2,540.72	$5,473.52

7.4.2.2 Sto_MarketIC with additional flood components

In these next cases, the SO desires to manage inundation timing and duration length. Here the SO will use the constraints thresholds proposed in Chapter 5 in two model formulations. Both formulations include the maximum stage-flood cost from previous Sto_MarketIC1. Firstly, the models will account for changes in flows with flood constraint type I and for any flow above a threshold with flood constraint type II. Secondly, the model will account for having only a rapid inundation (factor *i*) in both formulations. The implication for these market models will be shown in the next analyses.

Case 5: The first Sto_MarketIC2 model with flood constraint type I will account for the changes in exceeding flows across time, which in this case, are calculated in terms of the changes of total flows at the control points when flooding starts. Thus, the Sto_MarketIC2 clearing model penalises any increment in changing flows above 0.2 m^3/sec between times 15 and 30 (to hasten stage-flood time), and it penalises for any changes in flows below 0.2 m^3/sec between times 35 and 50 for lengthening flooding duration. A similar market clearing formulation was used to penalise rapid inundation.

Case 6: The second Sto_MarketIC2 model formulation, with flood constraints and thresholds type II, will account for any exceeding flows above a threshold of 0.2 m^3/sec, before time 25, and any exceeding flows above 0.2 m^3/sec, after time 40 at control points and storms. The type II costs the exceeding flows in those times. Thus, any participant who increases flows above these thresholds will face the cost for hastening inundation or lengthening duration. Finally, the same model penalises for hastening peak time of flood, in only the first 25 times.

The two model formulations (constraints and thresholds type I and II) penalise \$450/m^3/sec, for any flow above the thresholds at the first times (hastening the peak time of flooding) and \$5/m^3/sec at later times (lengthening inundation). The SO can shorten or lengthen the flood cost period to penalise for flood damage. Initial damage in the area varies from the previous illustration; the initial total expected damages for Sto_MarketIC2 in trading case 5.A and 5.B are \$15,342.30 and \$23,214.90, and in trading case 6.A and 6.B are \$18,855.60 and \$26,728.20 respectively. Table 7-10 shows the initial expected damage separated in each flood component using constraint type II and approximation B.

Table 7-11 and Table 7-12 show outcomes with the additional constraints and thresholds, factors *i* and *ii*, and approximation type A for cases 5.A and 6.A, and approximation type B for cases 5.B and 6.B.

Table 7-10 Initial expected flood damage estimates based on flood constraints and thresholds type II and approximation B in case 6

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$2,520.17	$1,029.55	$7.12	$3,556.84
CP-AB	$4,150.40	$2,830.21	$21.86	$7,002.48
CP-ABC	$5,130.81	$1,893.84	$107.80	$7,132.44
CP-ABCD	$5,713.80	$1,586.79	$352.50	$7,653.09
CP-D	$1,196.61	$26.47	$160.28	$1,383.35
Total Catchment	$18,711.79	$7,366.86	$649.56	$26,728.2

Table 7-11 Trading results from the application of Sto_MarketIC2 for trading case 5

Particip.	Trading case 5.A (convexified)				Trading case 5.B (non-convex)			
	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)
A1	0	150	$0	$559.71	0	150	$0	$732.99
A2	0	150	$0	$559.71	0	150	$0	$732.99
A3	0	150	$0	$559.71	0	150	$0	$732.99
A4	0	167.385	$0	$979.12	0	167.385	$0	$1,321.79
A5	150	0	$785.83	$0	100	0	$702.10	$0
A6	150	0	$785.83	$0	100	0	$702.10	$0
B1	0	50	$0	$104.01	0	50	$0	$129.21
B2	0	0	$0	$0	0	0	$0	$0
B3	0	0	$0	$0	0	0	$0	$0
B4	0	70	$0	$286.82	0	70	$0	$356.10
B5	83.333	0	$83.52	$0	83.333	0	$110.54	$0
B6	83.333	0	$83.52	$0	83.333	0	$110.54	$0
C1	514.99	0	$99.21	$0	514.99	0	$140.20	$0
C2	0	250	$0	$694.87	0	250	$0	$971.91
C3	514.99	0	$99.21	$0	514.99	0	$140.20	$0
C4	514.99	0	$99.21	$0	514.99	0	$140.20	$0
C5	0	250	$0	$414.29	0	250	$0	$582.81
C6	0	250	$0	$486.67	0	250	$0	$684.94
D1	612.83	0	$218.82	$0	612.83	0	$337.09	$0
D2	612.83	0	$32.25	$0	612.83	0	$54.93	$0
D3	612.83	0	$164.90	$0	612.83	0	$285.39	$0
D4	612.83	0	$28.19	$0	612.83	0	$47.19	$0
D5	612.83	0	$50.57	$0	612.83	0	$85.13	$0
D6	0	250	$0	$344.59	0	250	$0	$565.21
Total	5,075.79	1,737.39	$2,536.06	$4,994.50	4,975.79	1,737.39	$2,860.61	$6,815.94

After clearing the market most participants have IC allowances similar to the outcomes from Sto_MarketIC1 with case 4. Nevertheless, the final payments change, because flows violate thresholds for factors *i* and *ii*, and clearing prices account for these violations. With trading case 4.B, final prices for A1, B4 and C5 with Sto_MarketIC1 are \$3.99/ha, \$3.97/ha and \$1.97/ha. For trading case 5.B (for both flooding components *i* and *ii*), participants A1, B4, and C5 change IC in 150, 70 and 250 ha, and receive \$4.89/ha, \$5.09/ha and \$2.33/ha. For trading case 6.B, they change IC in 167.39, 83.33 and 250 ha, and receive \$7.16/ha, \$7.11/ha and \$2.47/ha. The greater prices correspond to hastening inundations and shortening durations. The last effect will be analysed next when only a flood cost for rapid inundation is considered. In trading case 4.B, with Sto_MarkeIC1, participant D3 paid only \$0.42/ha, while in trading case 5.B and 6.B, participant D3 increases allowances and pays \$0.46/ha and \$0.57/ha respectively. This rise in price is due to the flood components for rapid inundations and lengthening durations at CP-ABCD.

Table 7-12 Trading results from the application of Sto_MarketIC2 for trading case 6

Particip.	Trading case 6.A (convexified)				Trading case 6.B (non-convex)			
	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)
A1	0	167.385	$0	$1,075.00	0	167.385	$0	$1,199.12
A2	0	167.385	$0	$1,075.00	0	167.385	$0	$1,199.12
A3	0	167.385	$0	$1,075.00	0	167.385	$0	$1,199.12
A4	0	167.385	$0	$1,240.80	0	167.385	$0	$1,490.66
A5	140.702	0	$844.21	$0	100	0	$734.17	$0
A6	100	0	$600.00	$0	100	0	$734.17	$0
B1	0	50	$0	$165.19	0	50	$0	$191.29
B2	0	0	$0	$0	0	50	$0	$104.60
B3	0	0	$0	40	0	0	$0	$0
B4	0	83.333	$0	$507.68	0	83.333	$0	$592.56
B5	70	0	$310.75	$0	70	0	$331.94	$0
B6	70	0	$310.75	$0	70	0	$331.94	$0
C1	514.99	0	$123.23	$0	514.99	0	$161.22	$0
C2	0	250	$0	$752.30	0	250	$0	$1,018.75
C3	514.99	0	$123.23	$0	514.99	0	$161.22	$0
C4	514.99	0	$123.23	$0	514.99	0	$161.22	$0
C5	0	250	$0	$455.91	0	250	$0	$616.90
C6	0	250	$0	$543.37	0	250	$0	$732.47
D1	612.83	0	$222.95	$0	612.83	0	$342.88	$0
D2	612.83	0	$39.86	$0	612.83	0	$62.65	$0
D3	612.83	0	$227.78	$0	612.83	0	$348.79	$0
D4	612.83	0	$41.12	$0	612.83	0	460.20	$0
D5	612.83	0	$77.79	$0	612.83	0	$112.53	$0
D6	0	250	$0	$361.97	0	250	$0	$584.50
Total	4,989.82	1,802.87	$3,050.90	$2,661.23	4,949.12	1,852.87	$ 572.73	$8,935.09

The SO is a net payer in cases 5 and 6, resulting in flood distributions being shifted and the expected flood damage being reduced in both cases. The final exposure depends on the flood cost type and constraints, bids from participants, and the flood damage approximation used in the clearing formulation. The final expected damage under Sto_MarketIC2, with cases 5.A and 5.B, are $12,553.80 and $18,785.20, and for cases 6.A and 6.B are $13,668.80 and $19,998.90 respectively.

Table 7-13 shows the final expected flood damage components after clearing the market under case 6-B. The expected flood damage is shifted and reduced in the catchment; however, the expected flood damage at control points varies, and hence the expected damage for each flood component. For instance, the expected flood damage at CP-A and CP-AB are reduced for depth and factor *i*, and increases for factor *ii*. Thus lengthened flood duration would be present at these control points. Figure 7-6 shows the hydrograph curve at CP-AB, which illustrates the reducing effect in peak flow (depth), the delayed peak flood time, and the increment in flood duration.

Table 7-13 Final expected flood damage components for case 6-B

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$1,679.90	$437.56	$10.61	$2,128.07
CP-AB	$3,215.28	$1,684.13	$27.69	$4,927.10
CP-ABC	$3,671.79	$1,147.40	$97.29	$4,916.48
CP-ABCD	$5,074.76	$1,043.16	$374.09	$6,492.00
CP-D	$1,297.79	$34.73	$202.69	$1,535.20
Total Catchment	$14,939.52	$4,346.98	$712.37	$19,998.85

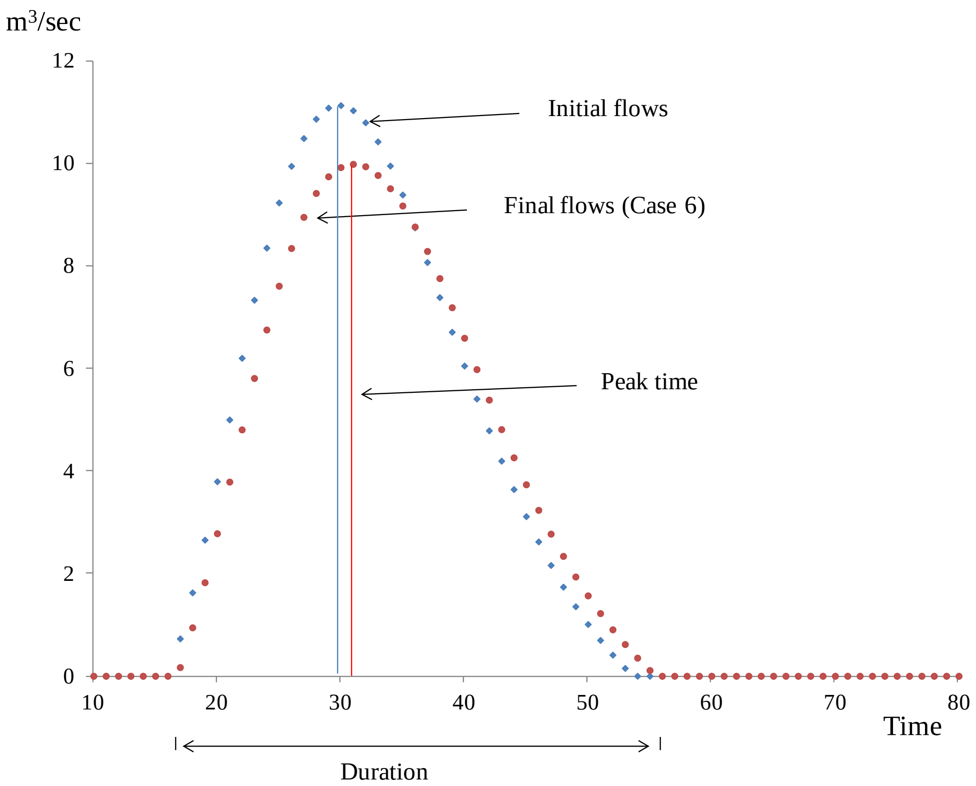


Figure 7-6 Initial and final exceeding flows above banks at control point AB under scenario 100 year and storm distribution types II.

Case 7: the SO is concerned only about rapid inundation, so the market formulation includes the first flood cost factor *i*. The market formulation accounts for both proposed constraints and threshold types I and II using linearised and non-convex flood damage cost function. To analyse the flood cost effect on prices the Sto_MarketIC2 with constraints and thresholds type II and approximation to the flood damage function type B are used. Table 7-14 shows the initial expected flood damage at control points according to the flood components. The initial expected flood damage is $26,078,64 at the catchment, decomposed by $18,711.79 for depth and $7,366.86 for hastening peak flood.

Table 7-14 Initial expected flood damage components with constraints and thresholds type II, Case 7

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$2,520.17	$1,029.55	$0	$3,549.72
CP-AB	$4,150.40	$2,830.21	$0	$6,980.62
CP-ABC	$5,130.81	$1,893.84	$0	$7,024.64
CP-ABCD	$5,713.80	$1,586.79	$0	$7,300.59
CP-D	$1,196.61	$26.47	$0	$1,223.07
Total Catchment	$18,711.79	$7,366.86	$ 0.00	$26,078.64

Table 7-15 summarises the trading outcomes. After clearing the market, participants A1, B4 and C5 receive $1,211.57 ($7.24/ha), $593.73 ($7.13/ha) and $587.56 ($2.35/ha) for reducing allowances on their properties. Final prices are higher than previous cases 5.B and 6.B (see Table 7-11 and Table 7-12) because in this case prices consider only the reducing effect of stage-flood and for hastening inundation. Previously, those final allowances also accounted for lengthening duration which penalised the final prices. Participant D3 increases IC allowances on 612.83 ha, paying $285.19 ($0.46/ha) due to increased peak flows and hastening inundation across scenarios. In this case, lengthening duration does not increase the cost to the system because it is not penalised. The SO is a net payer with $5,457,36. Table 7-16 shows the final expected flood damage, according to the flood component components using constraint and thresholds type II.

Imperviousness levels in the catchment are reduced after trading, and they could be seen as changes in the hydrographs curves at each control point, which reflect changes in the expected flood damage and flood damage components. Thus, comparing Table 7-14 and Table 7-16, the expected flood damage (depth and hastening peak flood) is reduced at CP-A, CP-AB, CP-ABC and CP-ABCD, and so the hydrographs curves. Only at CP-D, the imperviousness levels increase and the expected flood damage rises by $1,089.55.

In summary, the trading cases produced different imperviousness levels and hydrograph curves, when flood components are included or the control point is capped (case 3) in the market formulations. Figure 7-7 illustrates the initial and final hydrograph curves above channel capacity from cases 3, 4, 5, 6 and 7 at control point CP-ABC in storm scenario of 100 year. The greatest reduction in peak flow at CP-ABC is reached in case 3, but the flooding problem starts earlier (4 hours) and duration is lengthened comparing to the initial condition. In case 4, the peak flow is reduced, flooding starts one hour later, and duration is lengthened, compared to the initial hydrograph curve. Similarly, in case 5, 6 and 7, the peak flow is reduced, but the peak time is delayed and the duration is lengthened comparing with case 4. However, the final allocations and prices change in cases 5, 6 and 7.

Table 7-15 Trading result from the application of Sto_MarketIC2 for trading case 7

Particip.	Thresholds type I (non-convex)				Thresholds type II (non-convex)			
	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)
A1	0	150	$0	$735.37	0	167.385	$0	$1,211.57
A2	0	150	$0	$735.37	0	167.385	$0	$1,211.57
A3	0	150	$0	$735.37	0	167.385	$0	$1,211.57
A4	0	167.385	$0	$1,325.21	0	167.385	$0	$1,500.79
A5	100	0	$703.34	$0	100	0	$734.03	$0
A6	100	0	$703.34	$0	100	0	$734.03	$0
B1	0	50	$0	$129.51	0	50	$0	$191.81
B2	0	0	$0	$0	0	50	$0	$105.09
B3	0	0	$0	$0	0	0	$0	$0
B4	0	70	$0	$356.89	0	83.333	$0	$593.73
B5	83.333	0	$110.97	$0	70	0	$334.04	$0
B6	83.333	0	$110.97	$0	70	0	$334.04	$0
C1	514.99	0	$139.93	$0	514.99	0	$144.37	$0
C2	0	250	$0	$971.50	0	250	$0	$979.72
C3	514.99	0	$139.93	$0	514.99	0	$144.37	$0
C4	514.99	0	$139.93	$0	514.99	0	$144.37	$0
C5	0	250	$0	$582.32	0	250	$0	$587.56
C6	0	250	$0	$684.27	0	250	$0	$691.56
D1	612.83	0	$338.47	$0	612.83	0	$343.67	$0
D2	612.83	0	$54.80	$0	612.83	0	$55.41	$0
D3	612.83	0	$284.33	$0	612.83	0	$285.19	$0
D4	612.83	0	$47.05	$0	612.83	0	$47.56	$0
D5	612.83	0	$84.83	$0	612.83	0	$86.44	$0
D6	0	250	$0	$565.85	0	250	$0	$559.88
Total	4,975.79	1,737.39	$2,857.89	$6,821.66	4,949.12	1,852.87	$3,387.52	$8,844.85

Table 7-16 Final expected flood damage components with constraints and thresholds type II, Case 7

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$1,679.90	$437.56	$0	$2,117.46
CP-AB	$3,215.28	$1,684.13	$0	$4,899.41
CP-ABC	$3,671.79	$1,147.40	$0	$4,819.19
CP-ABCD	$5,074.76	$1,043.16	$0	$6,117.92
CP-D	$1,297.79	$34.73	$0	$1,332.52
Total Catchment	$14,939.52	$4,346.98	$0	$19,286.5

Participants located upstream face higher prices than those located downstream in case 7, because of the reducing effect in duration at CP-ABCD is not accounted. Participants located at CP-D and CP-ABCD notice the reduction in their settlements due to the lengthening duration component is not included in the market formulation. A further discussion about expected flood damage and revenue from the SO is presented in Section 7.4.4.

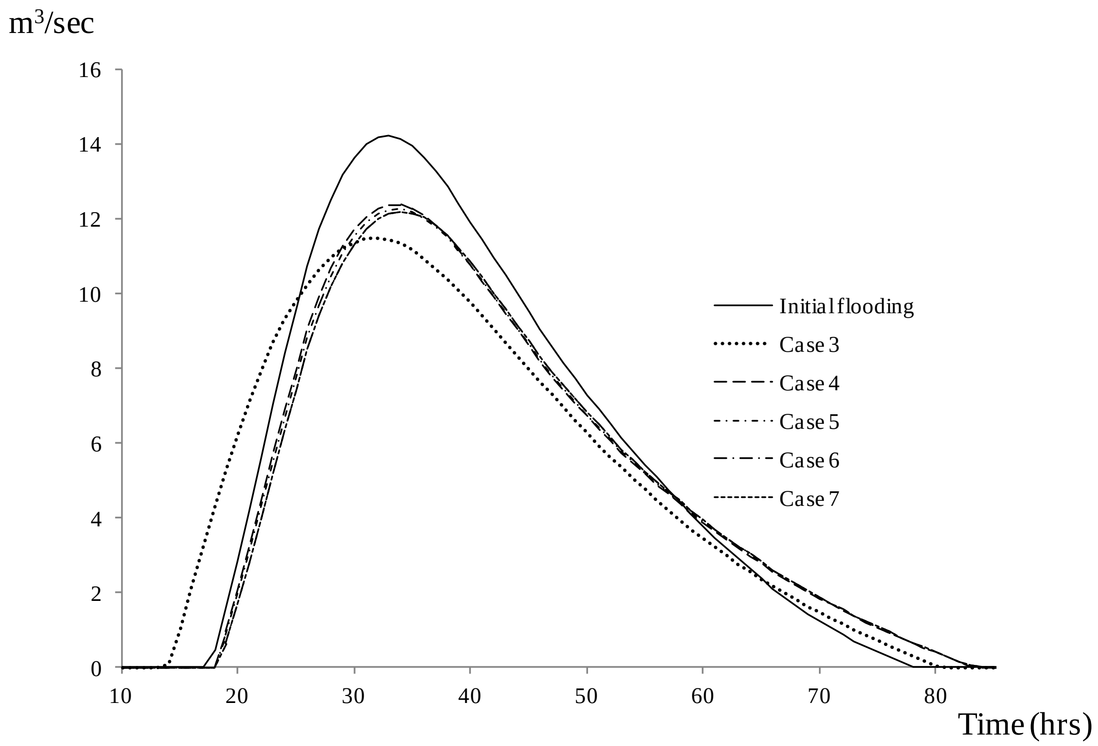


Figure 7-7 Summary of initial and final hydrographs curves for cases 3, 4, 5, 6 and 7 at control point CP-ABC with storm scenario 100 year and storm distribution type II.

7.4.3 Sto_MarketIC_Risk

Previous models considered risk neutral conditions for the SO. However, the SO could be concerned about the portfolio of IC allowances when there is a risk for a disaster if an extreme event affects the catchment. As stated in Chapter 6, the SO could hedge against changes in losses above this risk positions, via a coherent down-side risk measure such as CVaR.

A key function of the SO is to include risk positions against extreme losses under an α=0.95 (for the 5% extreme storm distribution), and so avoid expected losses above a particular loss position at each control point and at the catchment level. This position may over-constrain the market if this is below the current limit, as discussed in Chapter 6. The SO can be a net payer for reducing expected flood damage and for reaching the desired system risk position. These cases are illustrated next with the different models and flood costs. The illustrations assume that the SO uses the same CVaR values to limit risk; thus, the idea is to state and discuss the effect of price and allocation with the CVaR positions.

The SO may account for different flood components and options to estimate damage. The SO establishes different CVaR values according to the preferred flood component components and risk positions.

The SO desires to reach $\mathrm{CVaR}_{\alpha=0.95}$ values at CP-A, CP-AB, CP-ABC, CP-D, and CP-ABCD of \$90,000, \$167,000, \$137,000, \$70,000 and \$160,000 respectively, and $\mathrm{CVaR}_{\alpha=0.95}$ \$860,000 is the desired level within the catchment. These losses are assumed to correspond to the risk as desired by the community.

Let us analyse a situation where only the maximum stage-flood accounts for damage, and secondly, a situation where the SO also includes the factors for hastening peak time (factor *i*) and lengthening duration (factor *ii*). Similar CVaR damage positions will be considered at each control point for the different model formulations to emphasise the effects on prices and allowances. To illustrate and analyse these effects, the Sto_MarketIC_Risk with thresholds type II and non-convex depth damage components (approximation type B) are used.

Case 8: The SO considers only peak flow damage in the market formulation. Currently, equivalent $\mathrm{CVaR}_{\alpha=0.95}$ values with all flood components are greater than the SO desires. The CVaR constraints are not binding, so outcomes from participants are similar to those presented in Table 7-9. For example, participant A1 reduces allowances on 150 ha and receives \$599.60, and D3 increases allowances on 612.83 ha and pays \$258.90. However, the final prices and allocations change with hastening peak time of flooding and lengthening flood costs as will be shown in case 9.

Case 9: The SO considers additional flood components *i* and *ii* with constraints and flood thresholds type II and approximation type B. Table 7-18 summarises the final payments and allocations from participants. Participants A1, B4 and C5 reduce allowances by 167.39, 83.33 and 450 ha, and receive \$1,831.17 (\$10.94/ha), \$881.77 (\$10.58/ha), and

$1,355.81 ($3.01/ha) respectively. In comparison to previous illustrations with similar flood components and flood damage approximations, (see trading case 6.B in Table 7-12), participants receive higher prices for the risk-reducing effects at control points. For example, participant B4 is granted a higher price compared to the incremental proportion observed in Sto_MarketIC2 with flood components (see Table 7-12). Participant D3 increases allowances to crop by 612.83 ha and pays $414.27 ($0.67/ha). The SO is a net payer with $10,451.4, which is greater than previous net payments ($4,986.16) due to the desired risk positions being lower than the previous equivalent CVaR values at control points and at the catchment level. In the short term this could be expensive for the SO. However, in the long term, the SO is expected to be neutral, a net receiver, and the catchment remains safe from a range of storms and extreme damage.

Table 7-17 presents the final expected flood damage for each flood component. In comparison to previous trading conditions (see expected flood damage in Table 7-13), the expected flood damage is reduced in most components. For example, at CP-AB the previous final expected flood damage for depth was $3,215.28, while in case 9 was $3,084.88.

Table 7-17 Final expected flood damage components case 9

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$1,621.39	$406.95	$10.54	$2,038.87
CP-AB	$3,084.88	$1528.03	$27.86	$4,640.77
CP-ABC	$2,975.32	$949.68	$73.08	$3,998.07
CP-ABCD	$4,453.85	$887.64	$329.49	$5,670.98
CP-D	$1,297.79	$34.73	$202.67	$1,535.20
Total Catchment	$13,433.23	$3807.03	$643.64	$17,883.89

Case 10: The SO considers flood cost for flood depth and for having rapid inundations (factor *i*) with constraints and thresholds type II and approximate type B. Table 7-18 summarises the final payments and allocations. Participant A1 and B4 increase allowances on their entire properties, but participant C5 increases allowances of only 250 ha. The three participants receive $1,786.87 (10.67/ha), $908.43 ($10.91/ha) and $742.37 ($2.97/ha) respectively. Again, participant B4 is granted proportionally a lower payment, but participant A1 faces a higher price due to delaying stage-flood across control points. Participant D3 raises allowances for their property (612.83 ha) and pays $277.50

($0.45/ha), which is less than in case 9, because their lengthening duration effects are not as important at control point CP-ABCD, and also lower than in case 7 ($285.19). Cases 7 and 10 differ because the final flood distribution for participant D3 is lower in case 10 than in case 7, and because the risk positions are loose at CP-D, CP-ABCD and at the catchment level. Thus, participant D3 is facing the new reduced flood distribution; their price represents the marginal flood contribution to the new flood conditions. Finally, the SO is a net payer with $9,529.10, and proportionally pays less than in case 9.

Table 7-18 Trading outcomes from Sto_MarketIC_Risk with constraints and thresholds type II

Particip.	Trading case 9 (non-convex)				Trading case 10 (non-convex)			
	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)	Area bought (ha)	Area sold (ha)	Paid ($)	Received ($)
A1	0	167.385	$0	$1,831.17	0	167.385	$0	$1,786.87
A2	0	167.385	$0	$1,831.17	0	167.385	$0	$1,786.87
A3	0	167.385	$0	$1,831.17	0	167.385	$0	$1,786.87
A4	0	167.385	$0	$2,070.58	0	167.385	$0	$2,014.99
A5	45.144	0	$451.44	$0	100	0	$956.31	$0
A6	100	0	$1,000.00	$0	100	0	$956.31	$0
B1	0	70	$0	$406.01	0	73.5326	$0	$441.20
B2	0	50	$0	$168.07	0	50	$0	$178.60
B3	0	0	$0	$0	0	0	$0	$0
B4	0	83.333	$0	$881.77	0	83.333	$0	$908.43
B5	50	0	$395.73	$0	50	0	$427.21	$0
B6	50	0	$395.73	$0	50	0	$427.21	$0
C1	514.99	0	$209.48	$0	514.99	0	$185.05	$0
C2	0	355.776	$0	$1,778.88	0	376.33	$0	$1,881.65
C3	514.99	0	$209.48	$0	514.99	0	$185.05	$0
C4	514.99	0	$209.48	$0	514.99	0	$185.05	$0
C5	0	450	$0	$1,355.81	0	250	$0	$742.37
C6	0	450	$0	$1,621.88	0	450	$0	$1,573.26
D1	612.83	0	$427.07	$0	612.83	0	$335.86	$0
D2	612.83	0	$74.15	$0	612.83	0	$53.71	$0
D3	612.83	0	$414.27	$0	612.83	0	$277.50	$0
D4	612.83	0	$76.65	$0	612.83	0	$46.09	$0
D5	612.83	0	$143.10	$0	612.83	0	$83.75	$0
D6	0	250	$0	$681.38	0	250	$0	$547.10
Total	4,854.26	2,378.65	$4,015.58	$14,466.89	4,909.12	2,202.74	$ 0.00	$13,658.21

For the same maximum depth factor, the final expected damage in case 10 is $13,995.82, being $1,702.42 higher than the damage of $12,293.40 in case 9. The result reflects the higher level of imperviousness of the catchment in case 10. Compared to case 7, the expected damage was $19,286.50, whereas in case 10 was $17,982.11. Furthermore for case 7 the damage associated with flood depth was anticipated to be $14,939.52 compared to $13,995.82 for case 10.

The results are characterised by binding risk positions at different control points. With case 9, risk positions are priced at CP-AB with $0.012503/CVaR$_{\alpha=0.95}$ and at catchment with $0.007415/CVaR$_{\alpha=0.95}$. With case 10, risk positions are binding at CP-AB with $0.007433/CVaR$_{\alpha=0.95}$ and at CP-ABC with $0.03526/CVaR$_{\alpha=0.95}$. These differences are due to the binding risk positions changed using different flood costs in the catchment. For instance, lengthening flooding duration would be more important at CP-ABCD, while hastening peak time of flooding would be more important at CP-ABC.

Table 7-19 Final expected flood damage components for case 10

	Depth ($)	Factor *i* ($)	Factor *ii* ($)	Total Expected ($)
CP-A	$1,679.90	$437.56	$0	$2,117.46
CP-AB	$3,168.74	$1,576.61	$0	$4,745.35
CP-ABC	$3,201.47	$1,005.22	$0	$4,206.69
CP-ABCD	$4,647.92	$932.167	$0	$5,580.09
CP-D	$1,297.79	$34.73	$0	$1,332.52
Total Catchment	$13,995.82	$3,986.287	$0	$17,982.11

In case 6.B, the expected damage was $19,998.90. With the Sto_MarketIC_Risk in case 9, the expected damage was reduced to $17,883.89. However, the cost to the SO in the first trading case was $5,457.33 whereas the cost in the second trading case was $10,451.40. The real flood reductions were $6,729.30 and $8,844.30 respectively. This result means that the SO granted an extra $10.451.40−$8,844.30=$1,607.10 to participants who were reducing flood damage, and who allowed the catchment to obtain the desired risk position.

7.4.4 Summary deterministic and stochastic models

This section summarises the main outcomes from the previous deterministic and stochastic cases. Table 7-20 presents these outcomes in the expected flood damage, applied prices for two participants located upstream and downstream in the catchment, changes in flow peaks in storm scenario 100 year at CP-ABC, and the final revenue condition for the SO. Differences in the initial expected flood damage are due to the inclusion of flood damage components and CVaR positions.

Table 7-20 Summary of cases: changes in expected flood damage, applied price, changes in flow peak and the SO' revenue

Cases	Initial Expected Flood Damage	Final Expected Flood Damage	Applied price (1)		Reduction in flow peak m^3/sec (2)	Revenue position of the SO
			Particip. A1	Particip. D6		
Case 1	-	-	\$2.000/ha	\$6.140/ha	1.47 m^3/sec	\$655.71
Case 2	-	-	-	-	-	-
Case 3	-	-	\$4.014/ha	\$31.940/ha	2.77 m^3/sec	- \$18,882.80
Case 4	\$18,711.80	\$15,471.20	\$3.997/ha	\$1.899/ha	1.86 m^3/sec	-\$2,932.83
Case 5	\$23,214.90	\$18,785.20	\$4.887/ha	\$2.601/ha	1.97 m^3/sec	-\$3,955.33
Case 6	\$26,728.20	\$19,998.90	\$7.164/ha	\$2.228/ha	2.03 m^3/sec	-\$5,386.15
Case 7	\$26,078.64	\$19,286.50	\$7.238/ha	\$2.399/ha	2.03 m^3/sec	-\$5,457.36
Case 8	\$18,711.80	\$15,471.20	\$3.997/ha	\$1.899/ha	1.86 m^3/sec	-\$2,932.83
Case 9	\$26,728.20	\$17,883.89	\$10.939/ha	\$2.726/ha	3.29 m^3/sec	-\$10,451.40
Case 10	\$26,078,64	\$17,982.11	\$10.675/ha	\$2.296/ha	2.88 m^3/sec	-\$9,529.10

(1) Applied price is calculated based on the final settlement and the changed area of IC allowances (\$/ha)

(2) The reduction is estimated based on storm scenario 100 year at CP-ABC

Comparing the cases, different outcomes are noticed in the expected flood damage, the peak flows, and the SO' revenues. The flood distribution is shifted in cases 3 to 10. The lower reductions in the expected damage are in cases 4, 5 and 8 with \$3,240.6, \$4,429.7 and \$3,240.9 respectively, whereas the greatest reduction is noticed in case 9 with \$8,844.3. Regarding the peak flows at CP-ABC, in the deterministic case 3, the peak flow is reduced by 2.77 m^3/sec (initial peak flow is 14.24 m^3/sec) and the SO is a net payer with \$18,882.8. However, in case 9 the SO could reduce further the expected damage with the stochastic formulation. In this case, the peak flow is reduced by 3.29 m^3/sec and the SO is a net payer with \$10,452.4.

Between the stochastic cases (cases 4 to 8), the reducing effects in peak flows are close and vary between 1.86 and 2.03 m^3/sec; however, the applied prices change when including flood components. For instance, for participants A1 and D6 the applied prices are \$3.99/ha and \$1.89/ha, when the market accounts for depth (flows peak reduction is 1,86 m^3/sec at ABC). Those prices, increase to \$7.164/ha and \$2.228/ha respectively if the market, additionally, accounts for hastening peak flows and duration (flow peak reduction is 2.03 m^3/sec at CP-ABC).

On the other hand, the SO reaches different revenues with the deterministic and stochastic cases. In deterministic case 1, the catchment is fully allocated and the SO is a net receiver because sells impacting flows at times that previously were not binding at control points. In case 3 deterministic as well, the SO is exposed to be a net payer with \$18,882.80, and the SO reduces 2.77 m^3/sec at CP-ABC. However, a greater reduction of 3.29 m^3/sec is reached with the stochastic case 9, at equivalent storm scenario 100 year. The SO is a net payer only with $10,451.40 and could reach a better reducing position than case 3. With case 9, participants face the expected flood damage and risk position, and the SO pays mainly for the changes in the expected flood damage. With case 3, the SO pays more than the equivalent reducing effect of flooding.

Finally, the simplicity and clear application is an advantage of the deterministic method, whereas a stochastic method requires extra analysis to estimate impacting flows and flood damage across scenarios.

7.5 Final remarks and conclusion

This Chapter presented a variety of trading cases using the proposed market clearing models developed in this thesis. Participants were shown to have traded and obtained different IC allowances and prices in each model. The SO faced different revenue conditions with the different case formulations. In the trading cases, the SO was a net payer with the Sto_MarketICs and Sto_MarketIC_Risk. The SO could also be a net receiver with Det_MarketIC when the SO sold flood capacity for shifting peak times. However, the SO revenue was shown to be dependent on catchment conditions.

The SO revenue depends, in part, on the opportunity cost for changing IC allowances and shifting the flood distribution. Thus, if opportunity costs from participants to reduce allowances are lower than the opportunity cost for reducing flood damage in the catchment, the market formulation reduces allowances, and the catchment becomes pervious.

With flood costs for violating capacities for flood components in Sto_MarketIC models, the SO faces different revenue. Prices account for the marginal changes in flood distribution. Thus, for each condition, participants internalise the opportunity cost related to changes in flood distribution in the catchment.

Incorporating the flood risk that the community desires, the SO faces different revenue results using the Sto_MarketIC_Risk model. But participants who change flood distribution at the extreme tail losses would face the price for binding the CVaR risk positions. If their IC allowances bind the risk, allowances would account for the cost to the system for trying to shift the risk from the community. Consequently clearing prices would rise for participants who change the flood tail distribution and for those who reach the risk positions.

The decision of which market model to use will depend on the catchment conditions and flood conditions that the SO, the government and community desire to safely manage floods within the catchment. The formulations affect prices and allocations due to the capped points, the flood distribution, flood components and the risk positions for the flood areas. Different models will result in different revenue conditions for the SO and trade from participants.

Chapter 8

8 GROSS POOL MODEL WITH REVENUE ADJUSTMENT

Previous chapters developed net pool market models for IC allowances. The market models accounted for trading net changes in IC allowances for each property (ha), relative to the status quo allowance. With Det_MarketIC1, participants buy and sell area allowances for their properties, with a land area balance constraint for each participant. With Det_MarketIC2, Sto_MarketIC and Sto_MarketIC_Risk, participants trade for changes in IC allowances in their areas. The market models account explicitly for the difference in IC allowances from their initial to the desired allowance in the area. However, the SO may not be revenue neutral if there are changes to the calculated capacity of the catchment (to deal with runoff, or over-allocation, changes in expected flood damage, etc). This chapter will consider the following aspects:

- A gross pool formulation to clear the market. Most electricity markets use gross formulations (Alvey et al. 1998; Read and Chattopadhyay 1999) and they have been also proposed for water, nitrates and sediment (Raffensperger et al. 2009; Prabodanie 2010; Pinto et al. 2012). The allocation results should be the same as from the previous net pool formulations. Duality will help to analyse and discuss the clearing price implications.
- Introducing a “reference usage type” (land use) in order to provide a common basis for offers from participants, and a constraint transformation to deal with this reference.
- Adjusting the rights to reach revenue neutrality for the SO, as proposed by Raffensperger (2011) and Pinto et al. (2012).

8.1 Gross vs. net pool market

A market could operate using a net or a gross pool formulation. In a net pool, participants provide an offer curve to sell, and a separate bid curve to buy more. However, the demands and offers from participants in a net pool could be transformed into gross pool bids and vice-versa (Prabodanie et al. 2010). These are just two different ways of specifying the same net demand curve (Figure 8-1).

A gross pool market does not accept sell offers and ignores initial rights in the market clearing model, but deals with them in the settlement systems. The totals sold and bought are determined after clearing the market by comparing initial and final allowances.

To keep their initial IC allowance, a participant must bid high for that allowance, and low for alternative uses of that area, or else the market model could allocate another allowance to their property. This is the same as participants having high offers to sell existing IC allowances and low offers to buy alternatives in a net pool market. If a participant prefers a different IC allowance, their bids should be high enough to buy it, and with lower values for alternative IC allowances. Figure 8-1 illustrates a gross demand curve. For example, a participant currently may have 10 ha with IC allowance type ‘meadow’, but want to buy (just) 5 ha of IC allowance type ‘concrete’. Therefore, the participant should bid a high price for 5 ha of concrete, and also a high price for 5 ha of meadow, and no more, with low prices for all alternatives.

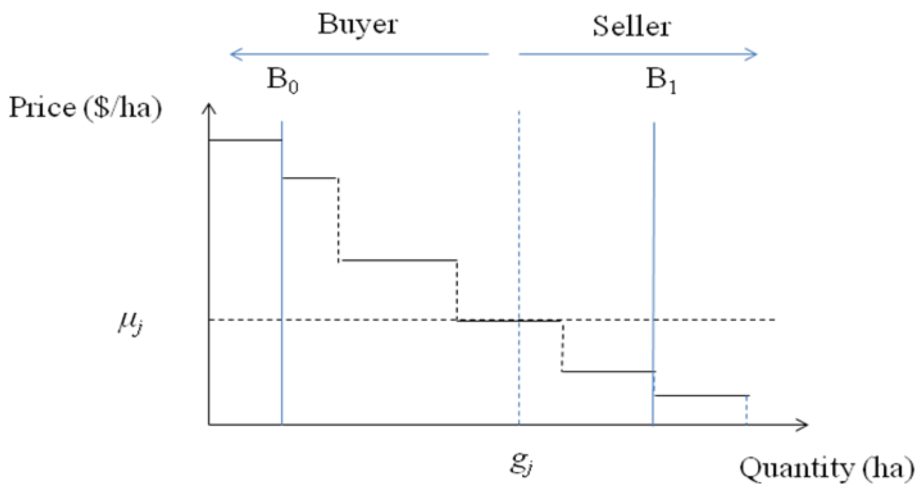


Figure 8-1 Demand curve in a gross pool market. μ_j is the applied price for IC allowance type j and g_j is the final IC allowance. The participant would be a buyer or a seller depending on whether the initial position, B, is greater or less than the final.

Setting bids in a gross pool could be easier than in a net pool market, if a participant has a complex set of IC allowances and desires. For instance, the property could be the layout in Figure 8-2. The participant currently has IC allowances type A, B and C represented by the solid lines, and desires to develop a project with IC allowances type D shown with a dashed line. The participant would need to define bids for the IC allowance for the new area A_1, for the small area represented by IC cover types A_2 and D_2, for the new B_1, for area B_2 and D_1, for the new C_1 and also for C_2 and D_3. The participant may find the process of setting these bids to be complex. However, if the SO defines a reference usage, setting bids could be simpler as will be discussed in the next sections.

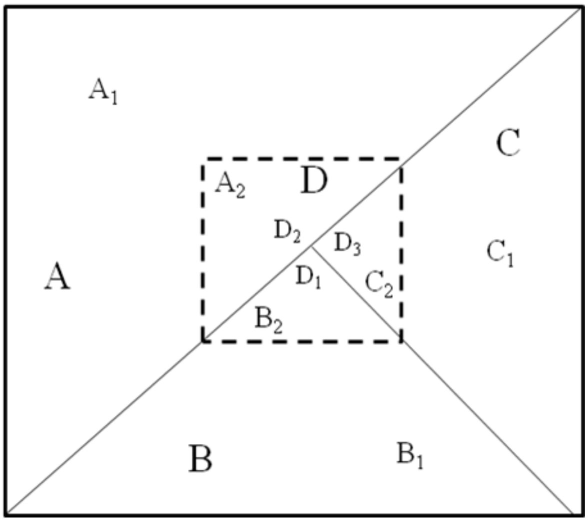


Figure 8-2 Example of a property layout of IC allowances. Original IC allowance is type A, B, and C defined by solid lines and the desired type is D defined by a dashed line. A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, and D_3 represent the areas of change.

For the gross pool formulation to function, the total area in the property must equal the total area from the IC allowances after clearing the market. This area condition requires participants to have an internal price, which will set the participant's allocations. This will be further discussed in section 1.5 and 1.6.

8.2 A gross pool formulation, Gross_MarketIC1

The TSSP model has a flood damage cost, which is assumed to be convex for incremental imperviousness. The damage depends on the stage flood level under uncertain storm events.

The gross pool model has the following new indices, parameters and variables.

Indices

i = Participant, 1,…,N.

j = Land use type (CN or imperviousness), j= 1,…,J.

b = Bid step, b=1,...,B.

t = Storm time period, t=1,…,T.

Parameters

$P_{i,j,b}$ = Demand price ($/ha) for IC allowance type j from participant i and bid step b. This is the maximum that participant i is willing to pay for an IC allowance type j and bid step b.

$D_{i,j,b}^{\max}$ = Maximum amount of IC allowance type j (ha) for participant i in bid step b at price $P_{i,j,b}$.

A_i = Total land area of participant i (ha).

$h_{0,k}^{s,t}$ = Intercept flow for the linearisation at control point k, time t and scenario s (see Figure 8-3 below). This is the flow if all the IC allowance types currently used disappear unless there is a reference usage.

$E_{i,j}$ = Initial rights of participant i and IC allowance type j. This is not used in the Gross_MarketIC1, but only in the settlement.

$G_{i,j}$ = Reference usage for participant i and IC allowance type j. This reference usage could correspond to arbitrary IC allowances for each participant, which might be established for the SO, or be the initial IC allowances for each participant.

$H_{i,j,k}^{t-u+1,s}$ = Marginal flow impact at control point k over scenario s, at the end of time t –u+1 from participant i of the land IC allowance j. u is the lag time between the storm time and the flow which reaches the control point (volume/time ha). This coefficient determines the impact of the participant's IC allowance on control points across rainfall scenarios, e.g., volume/time/ha. If participant i does not impact control point k, then $H_{i,j,k}^{t,s}$ = 0. This linear coefficient is likely to depend on the initial IC allowance conditions in the catchment, so it should be updated as IC allowances changes.

M_k = Flow capacity at channel control point k (volume/time). This corresponds to the difference between channel flow capacity and the chosen base flow. Base flow refers to the normal flows in the channel between storm events.

Variables

$qdem_{i,j,b}$ = Area in hectares of IC allowance type j and bid steps b purchased by participant i.

$g_{i,j}$ = Total hectares of IC allowance type j for participant i (ha).

Model: Gross_MarketIC1

$$\text{Maximize } \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} P_{i,j,b}\, qdem_{i,j,b} - \sum_{s=1}^{S}\phi^s \sum_{k=1}^{K} C_k\left(f_k^s\right) \qquad [8.1]$$

Subject to:

$$0 \le qdem_{i,j,b} \le D_{i,j,b}^{\max}, \forall\ i,j,b \qquad : \beta_{i,j,b}^{-}, \beta_{i,j,b}^{+} \qquad [8.2]$$

$$\sum_{b=1}^{B} qdem_{i,j,b} = g_{i,j}\ , \forall\ i,j \qquad : \mu_{i,j} \text{ (free)} \qquad [8.3]$$

$$\sum_{j=1}^{J}\sum_{b=1}^{B} qdem_{i,j,b} = A_i\ , \forall\ i \qquad : \hat{\pi}_i \text{ (free)} \qquad [8.4]$$

$$h_{0,k}^{s,t} + \sum_{i=1}^{N}\sum_{j=1}^{J} H_{i,j,k}^{t-u+1,s}\left(g_{i,j} - G_{i,j}\right) \le M_k + f_k^s\ , \forall\ t,k,s : \phi^s \lambda_{t,k}^s \qquad [8.5]$$

$$f_k^s \ge 0 \qquad : \theta_k^s \qquad [8.6]$$

$$g_{i,j} \text{ free} \qquad [8.7]$$

Explanation

[8.1] The objective function maximizes the expected total economic surplus from making IC allowances, less the expected recourse costs, due to flood damage under a finite set of rainfall events. Changes in the objective are the appropriate measure of changes in welfare (assuming the market is sufficiently competitive that bids reflect marginal costs).

[8.2] Total IC allowance allocated in each tranche is bounded by maximum demand quantities.

[8.3] The final amount allocated of IC allowance type *j* of participant *i*.

[8.4] The area for which IC allowances are allocated must equal the area owned.

[8.5] For each scenario *s*, the total flows at control point *k* in time *t* should be lower than the capacity M_k. However, flows may violate the capacity M_k and f_k^s estimates the peak flows in the channel area across scenarios. This allows for possible upstream participants that are prepared to pay the flood damage cost. This is one of the many arbitrary representations that could be employed for this constraint, which implicitly assumes that the SO has established a referential usage for IC allowances and so a point for flood flows. Figure 8-3 illustrates linearisations from this referential point in a storm scenario. This is just one dimension of a multi-dimensional surface, and $\bar{G}$ and G correspond to patterns of land cover type. The flow intercepts $h_{0,k}^{s,t}$ and $h_{0,k}^{s,t}$ mean that all the area currently used for *j* disappear unless there is a reference usage.

[8.6] Condition of non-negative exceeding flows.

[8.7] IC allowance must be non-negative. This will naturally limit the final IC allowance, $g_{i,j}$.

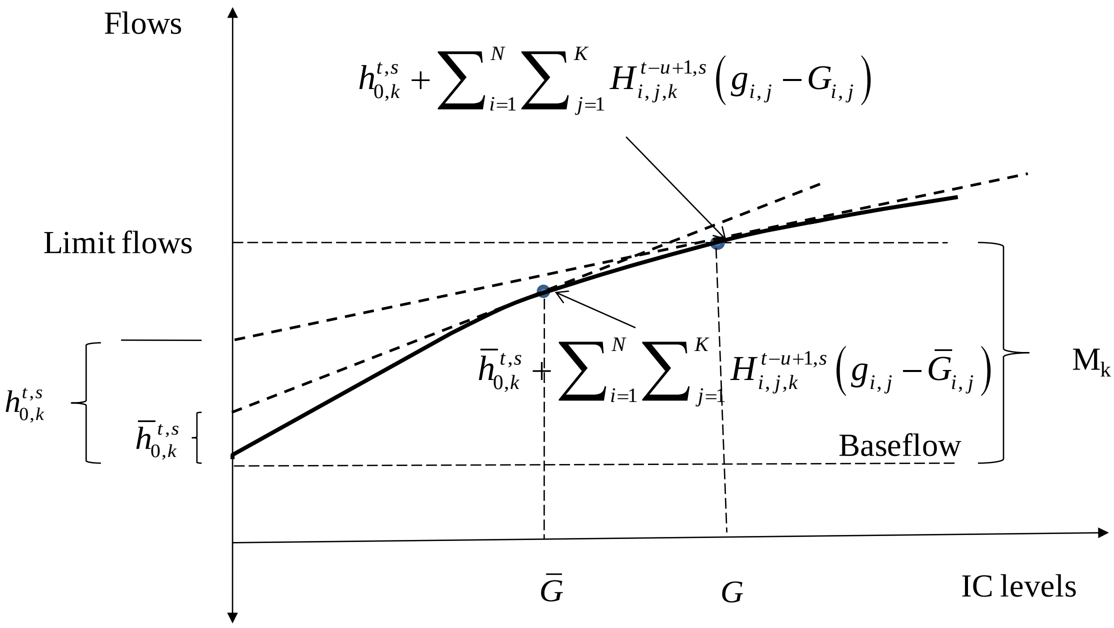


Figure 8-3 Linear approximation with a gross formulation

8.3 Price analysis

The market clearing prices associated with constraint [8.5] correspond to a set of flows received at control points, by time and by storm scenario, and will be non-zero for those flows at or above threshold capacities. Where 'exceeding flows' occur, the prices are the marginal change in the expected flood damage that may occur in the area. Otherwise, prices for binding limits will reflect the marginal cost of restricting participant allowances to void exceeding flow limits, as indicated by their offers. Each binding control point has its own dual price $\phi^s \lambda_{k,t}^s$. The dual condition from equation [4.19] in Chapter 4 represents the relationship between the clearing price and the marginal change in flood cost.

These clearing prices for binding control point capacity indirectly determine prices for participants' IC allowances of each type, which are valued according to their flow impacts at peak times. The dual price in Equation [8.3] represents this price for participant *i* and IC allowance type *j*; and may be determined as follows:

$$\mu_{i,j} = \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=u}^{T} \phi^s \lambda_{t,k}^s H_{i,j,k}^{t-u+1,s}, \ \forall\, i,j \qquad [8.8]$$

The dual price $\mu_{i,j}$ represents the economic impact of the increase in the expected flood damage, and/or reduction in other allowances, if the SO gives participant *i* another unit of IC allowance type *j*. The critical point to note is that there is no longer a price for changing from one land use to another, but a price for an extra area of land associated with that land use. In the model, only constraint [8.4] prevents this from happening, and so the shadow price on that constraint plays a critical role, as discussed in section 8.5 below.

However, participants are actually not facing only $\mu_{i,j}$. They face are $\mu_{i,j} - \mu_{i,j=ref}$. This corresponds to the price difference between the IC allowance type *j* and the reference usage *j=ref*. This reference could correspond to the initial IC allowances or a specific land use that the SO defines for each participant within the catchment. This payment will be discussed in the following Sections 8.4 and 8.5.

8.4 Settlement

The settlement for a participant depends on the reference usage. To simplify notation and the analysis, the initial IC allowance $E_{i,j=ref}$ is distinguished from the final allocated IC allowance $g_{i,j}$. (Notice that $E_{i,ref} = g_{i,j=ref}$ if participant i is not changing IC allowances.). Thus, if this reference corresponds to the initial IC allowances on their property, the final payment r_i from participant i for the changes in IC allowance is $r_i = \sum_{j=1}^{J} \mu_{i,j} \left(g_{i,j} - E_{i,j} \right)$.

If the reference usage is the initial IC allowance in the area type j=ini, which is related to initial impacting flow coefficients $\bar{H}_{i,j,k}^{t-u+1,s}$ at control points and scenarios, the settlement from each participant could be decomposed based on changes in impacting flows at peak flow time $t*$. Thus, the final payment condition is as follows:

$$r_i = \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{k=1}^{K} \phi^s \lambda_{t*,k}^{s} H_{i,j,k}^{t*,s} g_{i,j} - \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{k=1}^{K} \phi^s \lambda_{t*,k}^{s} \bar{H}_{i,j,k}^{t*,s} E_{i,j} \text{ , } \forall\ i \qquad [8.9]$$

Equation [8.9] represents the final payment from participant i. The first term represents the value of the final IC allowances after clearing the market; the second term represents the value of the initial IC allowances. If a participant is selling or buying for a particular area of land, the difference of $H_{i,j,k}^{t*,s} - \bar{H}_{i,j=ini,k}^{t*,s}$ helps to analyse the trading conditions. This could take a positive or negative value. If $\sum_{s=1}^{S} \sum_{k=1}^{K} \phi^s \lambda_{t*,k}^{s} \left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j=ini,k}^{t*,s} \right) < 0$, participant i is selling IC allowances and receives a payment from the SO, $r_i < 0$. If $\sum_{s=1}^{S} \sum_{k=1}^{K} \phi^s \lambda_{t*,k}^{s} \left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j=ini,k}^{t*,s} \right) > 0$, participant i is buying IC allowance type j and pays to the SO, $r_i > 0$.

8.5 Trading condition

A participant's clearing price may not match the demand price offered on any step of their demand curve; this may make a participant i marginal, infra or supra marginal resulting in the following:

$$\mu_{i,j} = P_{i,j,b} - \beta_{i,j,b}^{+} + \beta_{i,j,b}^{-} - \hat{\pi}_i \qquad \forall\ i,j,b \qquad : qdem_{i,j,b} \qquad [8.10]$$

If 0 < $qdem_{i,j,b}$ < $D_{i,j,b}$, then the optimal IC allowance *j* for participant *i* is in the middle of step *b*, and this makes *i* "marginal", and by complementary slackness, $\beta^{+}_{i,j,b} = \beta^{-}_{i,j,b}$ = 0 and so $\mu_{i,j} = P_{i,j,b} - \hat{\pi}_i$.

This tranche *b* for participant *i* will not generally be marginal when $\mu_{i,j} = P_{i,j,b}$. Because the opportunity cost $\hat{\pi}_i$ is not zero, a participant *i* may not trade even if $\mu_{i,j} \leq P_{i,j,b}$ because the condition is $\mu_{i,j} \geq P_{i,j,b} - \hat{\pi}_i$. Thus, even though the bid price is higher than the clearing price, an extra unit of IC allowance type *j* would not be allocated. $\hat{\pi}_i$ is the opportunity cost for a unit unbalancing of the total land. This could also be equivalent to having a clearing condition for each land area, with $\hat{\pi}_i$ equal to the marginal value of that land. Thus, if *j*=*m* is the marginal use of the IC allowance [land use], the marginal condition is $\hat{\pi}_i = P_{i,j=m,b} - \mu_{i,j=m}$. $\hat{\pi}_i$ may be high, especially in city centres.

Denoted $\vartheta_{i,m}$ as is the internal price for participant *i* for their marginal allowance *m*, the price could be represented as $\vartheta_{i,m} = \hat{\pi}_i + \mu_{i,m} = P_{i,m,b}$. The price accounts for the opportunity cost of the marginal land use and the clearing price for this usage. The participant also has internal prices for other IC allowances on their property such as $\vartheta_{i,j} = \hat{\pi}_i + \mu_{i,j}$. Thus, the participant has a relative internal price, which corresponds to $\vartheta_{i,j} = P_{i,m,b} + \mu_{i,j} - \mu_{i,m}$, i.e., a price difference between the IC allowance type *j* and the marginal usage for the property *m*. If the participant desires to change IC allowance, their bid price should be higher than the price difference between both land uses; otherwise, a participant would remain on the initial IC allowances or in the reference usage. Consequently, a participant is incentivised to disclose their opportunity cost between their land uses in their property.

If a participant knows their marginal usage *m*, then $P_{i,m,b}$ = 0 can be set and so participants can express their preference for other IC allowances according to their marginal usage. However the SO may decree that participants set their preferences related to a reference usage *ref*, which may be different than the marginal usage *m* for participant *i*.

Thus, the reference will establish referential bids[11] between participants within the catchment.

8.6 Issues with the Gross_MarketIC1

Constraint [8.4] makes the $\mu_{i,j}$ prices applied to participant allocations relative rather than absolute. Consequently, the clearing prices for flow limits, and for other participants, would not change if the $\mu_{i,j}$ prices applied to a particular participant were all shifted uniformly, so these price differences will not change. Because $\hat{\pi}_i$ is the same for the property, condition [8.10] could be used to show the relative price implication. Thus, at the equilibrium, the condition between two IC allowance types *j* and *m* are $\mu_{i,j} = P_{i,j,b} - \hat{\pi}_i$ and $\mu_{i,m} = P_{i,j=m,b} - \hat{\pi}_i$; therefore, the condition becomes $\mu_{i,j} - \mu_{i,m} = P_{i,j,b} - P_{i,j=m,b}$ for participant *i*. The participant may obtain IC allowance type *j* if $\mu_{i,j} - \mu_{i,m} \leq P_{i,j,b} - P_{i,j=m,b}$. This analysis can be extended for two participants, for which the relative condition for participant *n* is $\mu_{n,j} - \mu_{n,m} = P_{n,j,b} - P_{n,m,b}$. Thus, the relative prices are also between participants, $\mu_{i,j} - \mu_{n,j}$ and $\mu_{i,m} - \mu_{n,m}$. For example, participants could set their bids for IC allowance type 'concrete' at \$100/ha and for type 'meadow' at \$50/ha, and another participant could set their bids for type 'concrete' at \$1000/ha and for type 'meadow' at \$950, resulting in both conditions being similar $\mu_{i,j} - \mu_{i,m} \leq$ \$50/ha. If both participants are similarly located within the catchment and have the same *H* coefficients, they will face the same clearing prices for type 'concrete' and 'meadow', and both will obtain IC allowance type 'concrete'. This may initially appear peculiar, and may make it difficult to monitor abuse of market power, but it could still be conceptually correct. A similar implication was observed in the Det_MarketIC1 for the land area balance constraint. Therefore, participants could provide high bids, but the differences between the set of preferences could be small.

With this formulation, if a participant bids only once for their IC allowance, the market model may allocate independently to the bid value, because the condition [8.4] must be satisfied. Thus participants can provide low bids for their IC allowances. A user interface during bidding could be used to control possible problems with market infeasibility. Such an issue could also be avoided if a reference land use were established in the catchment.

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[11] This is analogous to trade being based on a specific currency, for instance trading only in US$.

If a catchment has no flooding problems (i.e., all constraints are loose), the clearing price $\phi^s\lambda_{k,t}^s$ is zero, and consequently the $\mu_{i,j}$ price is also zero for each participant and IC allowance *j*. However, participants face their internal price for the marginal use $\hat{\pi}_i$, which corresponds to their own opportunity cost for having an additional area for the marginal usage. On the other hand, if control points are binding, participants will be exposed to the flood cost, and also to the relative prices from the marginal IC allowance usage. The opportunity cost for marginal usage could be higher than conditions under no flood. For example, if the catchment becomes impermeable, the flood problem will be more frequent. IC allowances which were previously valued at zero, such as undeveloped areas, would now become marginal.

8.7 Setting a reference usage

To define a reference usage, the constraints [8.4] and [8.5] need to be transformed. This simplifies bids for participants and possible issues relating to absolute prices. Charles River Associates (2003) observed that a constraint transformation should deal with a physical network situation and with the commercial structure. If prices are based on a reference node, they represent marginal cost and should be correctly oriented. In the market model, the constraint can be related to a reference usage for each property and the price will be correctly oriented to the reference.

The reference usage conditions do not need to correspond to the current IC allowances. This reference usage could be defined as a particular IC allowance, and to the flows that impact at control points by time and scenario. If the SO defines an IC allowance as type *ref* for each participant, the constraints [8.4] and [8.5] are as follows:

$$\sum_{j\neq ref}^{J-1} g_{i,j} \leq A_i^0 \,,\ \forall\ i \qquad : \hat{\pi}_i \qquad [8.11]$$

$$\sum_{i=1}^{N}\sum_{j=1}^{J-1}\left(H_{i,j,k}^{t-u+1,s} - \tilde{H}_{i,j=ref,k}^{t-u+1,s}\right) g_{i,j} \leq M_k - \sum_{i=1}^{N} \tilde{H}_{i,j=ref,k}^{t-u+1,s} A_i^0 - h_{0,k}^{t,s} + f_k^s \,,$$
$$\forall\ t,k,s \qquad : \phi^s\lambda_{t,k}^s \qquad [8.12]$$

$\tilde{H}_{i,j=ref,k}^{t-u+1,s}$ corresponds to the impact coefficient at control point *k*, time *t* and scenario *s* from participant *i* with a reference usage type *ref*. The changes $\left(H_{i,j,k}^{t-u+1,s} - \tilde{H}_{i,j=ref,k}^{t-u+1,s}\right)$ can be positive or negative.

However, the market still has a degree of freedom and associated problem at a higher level, because the sum of area properties must equal the catchment area, $\sum_{i=1}^{N} A_i^0 = A^0$ and so one of the A_i^0 constraints becomes redundant. This means that the IC allowances cannot change the catchment area and that $\mu_{i,j}$ does not represent absolute prices for participants. Thus, the market model can be formulated on a referential land usage for all participants as was previously developed, and for a referential participant *i=par*. Therefore, participants will be buying IC allowances based on the referential land usage, and their bids will reflect marginal uses for new land, and $\mu_{i,j}$ will represent this price for participants. However, the price will continue to represent marginal value from the reference usage, but will be correctly oriented to represent changes in flood damage.

The constraint transformation should not compromise the final allocation, but bidding rules will require an alternative representation.

Bidding with the reference usage

The reference usage for IC allowances means that bids from participants are related to the established reference, and hence these bids should be correctly oriented to represent changes in IC allowances and be comparable between participants. Participants that agree with the reference usage should bid zero, otherwise they should bid for any available IC allowance or BMPs. If the participant desires to remain with their existing IC allowances, which could be more impervious than the reference condition, the participant should bid a high value. On the other hand, if their IC allowances are more pervious than the reference usage, the participant may bid a high negative value. For any other bids, the clearing model would consider that participants want to trade IC allowances. For instance, based on Figure 8-2, if the reference usage were A, the participant should bid only for D_1, D_2 and D_3, B_1 and B_2, and C_1 and C_2. To get IC allowance type D, the participant should bid high enough to obtain the total area needed to develop the project.

Price analysis and settlement

With the reference usage, the applied price to participants should be distinguished from the dual $\mu_{i,j}$, which is now referenced to usage type *ref* defined for the SO. The dual condition which represents the trading condition is the same Equation [8.10], and in this case $\hat{\pi}_i$ is the opportunity cost for participant *i* having extra area in the property.

However, the final trade as well as the applied price to participants could also be related to the initial IC allowances, if participants have the rights for their initial IC allowances. To calculate settlement for participants, the clearing price $\phi^{s}\lambda^{s}_{t^*,k}$ should be used for the changes in IC allowances. The applied price corresponds, in this case, to the settlement from Equation [8.9].

If the SO decides that all participants initially have only reference usage rights in their properties, any changes from the reference as well as any established land use conditions, different than the reference, should be charged or credited. In this case, for participants whose IC allowances differ from the reference usage type "*ref*", the applied charge is r_i = $\sum_{j=1}^{J-1}\mu_{i,j}\, g_{i,j}$ with $\mu_{i,j} \geq 0$ or $\mu_{i,j} \leq 0$. For those that remain in the reference usage, $\mu_{i,j}$ = 0. The settlement can also be decomposed in terms of impact coefficients and clearing prices at control points; in this case, the applied cost for participants is as follows:

$$r_i = \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{s=1}^{S}\left(H^{t-u+1,s}_{i,j,k} - \tilde{H}^{t-u+1,s}_{i,j=ref,k}\right)\phi^{s}\lambda^{s}_{t^*,k}\, g_{i,j}\ ,\ \forall\ i \qquad [8.13]$$

If the final demand curve is shifted outward, both reference usage conditions could expose the SO to be a net payer or receiver. The new demand curve may produce changes along the expected flood damage curve, which would produce revenue for the SO. Because participants may have the rights for the IC allowances, the SO is exposed to the changes in the demand curve from the initial IC allowances, and does not collect payments for any change from the reference usage across the catchment. Thus, the revenue analysis will be focused on possible surplus for shifting the demand curve from the initial IC allowances, but will also indicate changes in the expected flood damage function.

8.8 Operator revenue adequacy

As previously stated, the SO could be exposed to a possible outward shift of the demand curve, which shifts flows, the physical flood distribution, and likely the expected flood damage at control points. In this condition, the SO can be a net receiver or payer. For instance, upstream participants could desire to increase IC allowances, while other participants could desire to reduce IC allowances, but such reductions may not be sufficient to retain the same demand curve and expected flow levels. Hence the demand curve is shifted, and possible movement along the expected flood damage curve may produce revenue, $R = \sum_{i=1}^{N} r_i$, for the SO.

The revenue corresponds to the surplus for the changes in the expected flood damage. (Note that the surplus also accounts for possible extra revenue for the linearisation of the flood damage cost as previously discussed in Chapter 4.)

The final surplus and payments from the SO depend on the changes in demand and the expected marginal flood damage curves. Because the non-linear flood damage cost has a piecewise linear approximation, extra revenue may be realised in the market. The SO can adjust up and down such additional revenue, and consequently the SO continues to be exposed to changes in the expected flood damage.

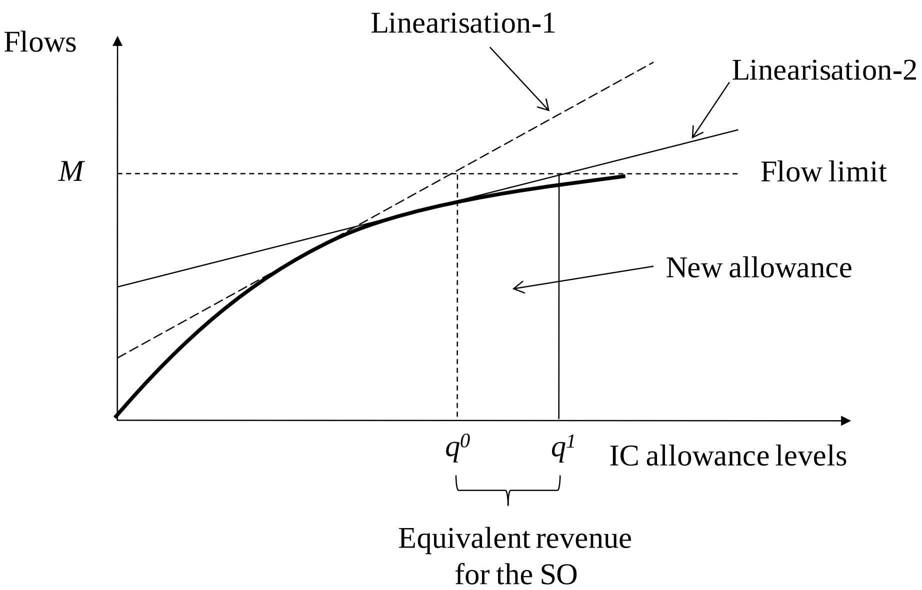

Figure 8-4 Illustration of the potential effect of linearisation and updating hydraulic coefficients on IC allowance levels. M is the flow limit, q^0 and q^1 is the IC allowance level within the catchment.

Additionally, the SO could be exposed to revenue inadequacy for the physical flow linearisation at control points (Figure 8-4). The market is initially established with parameters related to linearisation-1, which allowed an equivalent q^0 level of IC allowances. However, the SO realises that there is a need to update the hydraulic model coefficients for the market model and hence a new equivalent linearisation-2 is established.

This new condition would move allowance levels from q^0 to q^1 with equivalent revenue to the SO.

8.9 Scaling initial rights to reach revenue neutrality

The revenue could also be adjusted by scaling initial rights as proposed by Raffensperger (2011) and Pinto et al. (2012). The authors observed that when the SO capped the control points, different revenues were generated based on whether the catchment was over or under allocated. Hence they proposed to scale initial rights to reach revenue neutrality for the SO. In the case of IC allowances, the SO is exposed to changes in expected flood damage and possible physical flow conditions as previously discussed (see Figure 8-4).

Pinto et al. (2012) noted that scaling impacting rights for each constraint by the same fraction is unlikely to be revenue neutral. Thus, to preserve revenue neutrality, within the auction itself, the initial position should be re-set for each participant by scaling their constraint rights. The author also pointed out that scaling need not be linear, but assumed that the initial rights linked with constraints were all scaled proportionately. The same approach is used in the market, but in this case, scaling will be applied for each constraint *k*, time flow peak *t** and scenario *s* (up or down) to match the new flood conditions after clearing the market.

If the SO decides to scale impacting flow rights from the IC allowances up or down, the SO could use a scaling factor such as $\alpha_{k,s}$ and hence adjust initial capacity rights proportionally from Equation [8.9]. Participants' IC allowances are adjusted based on their initial set of impact flows at the new market equilibrium and hence the scaling factor will balance the revenue for the changes in the expected flood damage. The scaling factor is as follows:

$$\alpha_{k,s} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{t=1}^{T}\phi^{s}\lambda_{t^*,k}^{s}\, H_{i,j,k}^{t^*,s}\, g_{i,j}}{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{t=1}^{T}\phi^{s}\lambda_{t^*,k}^{s}\, \bar{H}_{i,j,k}^{t^*,s}\, E_{i,j}},\ \forall\ k,s \qquad [8.14]$$

The scaling factor adjusts the settlement for participants ex-post, i.e., after clearing the market, so the payments depend on the final IC allowances. Participants' rights are adjusted by receiving credits or debits in proportion to the final flood level changes. Therefore, participants may face a debit just to retain their existing rights, which will be offset from ongoing profitable trading. Thus, the participant is encouraged to express their preferences in the market. The final settlement for participants z_i^* becomes:

$$z_i^* = \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{s=1}^{S} \phi^s \lambda_{t*,k}^s H_{i,j,k}^{t*,s} g_{i,j} - \sum_{j=m}^{J} \sum_{k=1}^{K} \sum_{s=1}^{S} \alpha_{k,s} \phi^s \lambda_{t*,k}^s \bar{H}_{i,j,k}^{t*,s} E_{i,j}, \forall i \quad [8.15]$$

The SO could give credits if it is a net receiver, and debits if it is a net payer and so $\alpha_{k,s}$ would be greater or lower than 1.

If $\alpha_{k,s} > 1$, the SO gives credits to participants due to the increment in expected flooding. Thus, after clearing the market, the SO can scale up $\alpha_{k,s} > 1$ proportionally for the changes in the expected flood damage. The final payments from/to participants depend on the condition $\left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j,k}^{t*,s}\right)$.

If $0 < \alpha_{k,s} < 1$, the SO needs to cover the revenue shortfall, hence the SO charges all participants for the reduction in expected flood damage. Conditions when the SO has a revenue shortfall due to flood damage reduction are examined in the following cases.

Case 1: A participant is not changing IC allowance, so $r_i \approx 0$; however, $0 < \alpha_{k,s} < 1$ and so the final payment is $z_i^* > 0$. This means that participant *i* pays a proportional charge for the new expected flooding in the catchment. The participant needs to pay for keeping the IC allowances.

Case 2: For participants who do change IC allowance, again $0 < \alpha_{k,s} < 1$; however the payment from each participant depends on the condition $\left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j,k}^{t*,s}\right)$ before scaling. Let us see some conditions.

Case 2.1: A participant reducing imperviousness, $\left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j,k}^{t*,s}\right) < 0$, would receive a net payment $r_i < 0$. However, because a new expected flooding condition was reached in the catchment and the SO decided to scale, the participant may receive a lower compensation for the changes in imperviousness. The participant may receive $z_i^* < r_i$, or may not receive any payment for reducing imperviousness $z_i^* \approx 0$. In an extreme situation when the catchment is badly allocated, the participant may need to pay for the new condition $z_i^* > 0$.

Case 2.2: A participant increasing IC allowance, $\left(H_{i,j,k}^{t*,s} - \bar{H}_{i,j,k}^{t*,s}\right) > 0$, may pay an extra charge which increases the payment for the new imperviousness, so the participant pays z_i^* $> r_i$.

Distributional effects are observed if initial IC allowances are scaled; however this effect is outside the scope of this research. Additionally, distributional effects between upstream participants and flooding places are not considered in this analysis and research. The decision to scale is simply to remain revenue neutral for the SO. However, the SO may decide on alternative strategies to scale; for example, if the SO is a net receiver, there may be no decision to scale, and hence use the revenue for mitigation, investment in infrastructure, insurance, etc.

8.10 Example applications

This section presents two examples. The first example shows that the clearing model accounts for the relative bids rather than the absolutes for the constraint [8.4], and bids are scaled from participants to show this effect. The second example has similar trading conditions, but uses a reference usage of IC allowance in each property.

These examples assume that 5 storm scenarios approximate the expected flood damage. Channel capacities at control points (CP) CP1 and CP2 are 60 and 70 volume/time respectively. The flood damage cost at CP1 is $0.05*f^2$ and at CP2 is $0.01*f^2$, where f is the peak flow at the control points. The model has a piecewise linear approximation of the flood damage cost.

The marginal impact of each participant depends on the IC allowance and the flood area where they are impacting. Participants A and B are impacting at control points CP1 and CP2, and participant C impacts only at CP2. Table 8-1 shows the marginal flow impacts for participants under different IC allowances (land uses), scenarios and control points (CP1 and CP2). Thus, 1 ha from a participant with different IC allowance has the following flow impacts at peak flow time at control point 1 and scenario 1: $H_{i,forest,1}$= 1, $H_{i,meadow,1}$= 2, $H_{i,crop,1}$= 3 and $H_{i,concrete,1}$= 4 from A and B, and for control point 2: $H_{i,forest,2}$= 0.9, $H_{i,meadow,2}$= 1.9, $H_{i,crop,2}$= 2.9 for A or B, and $H_{C,concrete,2}$= 4.1 and $H_{C,Green\_Area,2}$= 1.6 for participant C.

Suppose three participants, A, B and C, desire to trade. Participants A, B and C each have 10 ha with IC allowance type meadow, concrete and green area respectively. Table 8-2 shows the participants' preferences. For instance, participant A desires to keep meadow in 5 ha and to increase crops in 5 ha. Participant B desires to change all 10 ha to meadow. Participant C wants to change 6 ha of green area to concrete. Participants A and C are bidding high for the 5 ha and 4 ha that they do not want to change. Participant B wants to change IC allowance, but he/she must bid more than once for their area. In this case, participant B bids for their initial IC allowance and for the desired condition. Participants also need to bid for other IC allowances.

Table 8-1 Impacting flows $H_{i,j,k}^{t*,s}$ from participant *i* at control point *k* and storm scenario

Participant A,B	Scen 1		Scen 2		Scen 3		Scen 4		Scen 5	
IC allowance	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2
Forest	1	0.9	1.3	1.17	1.8	1.62	2.5	2.25	3.4	3.1
Green Area	1.5	1.4	1.95	1.82	2.7	2.52	3.8	3.5	5.1	4.8
Meadow	2	1.9	2.6	2.47	3.6	3.42	5	4.75	6.8	6.5
Crop	3	2.9	3.9	3.77	5.4	5.22	7.5	7.25	10	9.9
Concrete	4	3.8	5.2	4.94	7.2	6.84	10	9.5	14	13
Participant C	Scen 1		Scen 2		Scen 3		Scen 4		Scen 5	
IC allowance	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2	Cp1	Cp2
Forest	-	1.1	-	1.43	-	1.98	-	2.75	-	3.7
Green Area	-	1.6	-	2.08	-	2.88	-	4	-	5.4
Meadow	-	2.1	-	2.73	-	3.78	-	5.25	-	7.1
Crop	-	3.1	-	4.03	-	5.58	-	7.75	-	11
Concrete	-	4.1	-	5.33	-	7.38	-	10.3	-	14

Example 1

If the market is cleared with Bid-1 preferences, participant A finishes with 5 ha to crops and remains with 5 ha of meadow, participant B changes to 6.24 ha meadow and remains with 3.76 ha of concrete, and participant C changes to 4 ha of concrete and remains with 6 ha of green areas.

Participant' preferences could be scaled for the condition [8.4] and the final trading solution does not change. For instance, suppose the market is cleared with Bid-2, which added a constant to participant's bids for a reference usage, forest from participant A, and for each participant subtract Bid(forest-A) = 5 from all their bids, so the new Bid(forest) = 0 for all of them (see Bid-2 in Table 8-2). The primal solution and the clearing prices are the same. Thus, if the initial IC allowance corresponds to the reference usage after trading, in both situations: participant A pays a total of \$5.20, participant B receives a total of \$12.50, and participant C pays a total of \$8.40. The SO is a net receiver with \$1.12, because expected flood damage at CP2 increases by \$2.64, but reduces by \$1.52 at CP1.

Table 8-2 Participants' initial IC allowance, required IC allowance, and bids

Participant	IC allowance	Tranche	Area	Bid-1	Bid-2
A	Forest	1	10	\$5	\$0
	Green areas	2	10	\$7	\$2
	Meadow	3	5	\$8	\$3
	Crop	4	10	\$11	\$6
	Concrete	5	10	\$11.1	\$6.1
	Meadow	6	5	\$10,000	\$9,995
B	Forest	1	10	\$4	\$0
	Green areas	2	10	\$7	\$2
	Meadow	3	10	\$10	\$5
	Crop	4	10	\$11	\$6
	Concrete	5	10	\$12	\$7
	Meadow	6	0	\$0	\$-5
C	Forest	1	10	\$4	\$0
	Green areas	2	4	\$8	\$3
	Meadow	3	10	\$9	\$4
	Crop	4	10	\$10	\$5
	Concrete	5	10	\$11	\$6
	Meadow	6	6	\$10,000	\$9,995

Suppose now the SO decides to scale proportionally to reach revenue neutrality. In this case, the SO will charge participants at CP1 with $\alpha_{k,s} < 1$ (scaling back) and gives credits to participants at CP2 with $\alpha_{k,s} > 1$ (scaling up). Table 8-3 shows the scaling factors at each control point *k* and scenario *s*. For instance, the scaling factor at CP1 and scenario 5 is $\alpha_{1,5}$=0.8755, and at CP2 and scenario 4 is $\alpha_{2,4}$=1.0432. Thus, the final adjusted payments are: participant A pays a total of \$5.03, participant B receives a total of \$12.83, and participant C pays a total of \$7.80.

Table 8-3 Scaling factor $\alpha_{k,s}$ at control point k and scenario s

Control Point	Scenario	Scaling factor ($\alpha_{k,s}$)	Control Point	Scenario	Scaling factor ($\alpha_{k,s}$)
CP1	S1	1	CP2	S1	1.043225758
	S2	0.875536055		S2	1.043226940
	S3	0.875537724		S3	1.043226347
	S4	0.875537292		S4	1.043225061
	S5	0.875535539		S5	1.043227547

Example 2

If the SO decides to establish a reference usage of IC allowance type 'meadow' for each participant, the conditions [8.11] and [8.12] are used. The reference is different than the current IC allowances. Participants that desire to remain with meadows do not need to bid. However, they need to bid if they want to keep their current conditions which are different to 'meadow' or to change these IC allowances.

Assume the same participants from the previous example, but with different preferences. Participant A desires to keep meadow in 5 ha and to increase crops in 5 ha. Participant B desires to change 10 ha to meadow. Participant C wants to change 6 ha of green area to concrete and keep 4 ha of green area. Participants should carefully establish their preferences; if not, they could finish with the reference usage established for the SO.

Participant A bids \$5/ha for the 5 ha to crop, participant B bids \$5/ha for the 10 ha of concrete and does not bid for meadow because their reference usage is meadow, and participant C bids \$0/ha for any green area (10 ha) and \$4/ha for 6 ha of concrete.

After clearing the market, using the applied price to participants from Equation [8.9], the final total payment and trade are as follows: participant A obtains 5 ha of crop and pays a total of \$13.11; participants B obtains 10 ha of meadow and receives a total of \$50.24 for their concrete; and, participant C gets 4.4 ha of concrete and 5.6 ha of green areas, and pays a total of \$24.20. The SO is a net payer with \$12.93 based on the initial IC allowances, because expected flood damage is reduced by \$6.33 and \$6.60 at CP1 and CP2 respectively.

The SO may decide to scale to reach revenue neutrality based on the initial IC allowances. In this case, the factor $\alpha_{k,s}$ should be estimated from Equation [8.14], and in both cases $\alpha_{k,s} < 0$ when floods occurs (if not $\alpha_{k,s}$=1). For instance, the SO scales back with a factor $\alpha_{1,5}$= 0.75 at CP1 and $\alpha_{2,4}$= 0.9589 at CP2 (see Table 8-4). Therefore, participants are scaled and receive debits; the final settlements after scaling are: participant A pays a total of $16.93 (previously $13.11), participant B receives a total of $42.58 (previously $50.24), and participant C pays a total of $25.64 (previously $24.20) at the end.

Table 8-4 Scaling factor $\alpha_{k,s}$ at control point k and scenario s

Control Point	Scenario	Scaling factor ($\alpha_{k,s}$)	Control Point	Scenario	Scaling factor ($\alpha_{k,s}$)
CP1	S1	1	CP2	S1	0.9589
	S2	1		S2	0.9589
	S3	0.75		S3	0.9589
	S4	0.75		S4	0.9589
	S5	0.75		S5	0.9589

8.11 Final remarks and conclusion

This chapter proposed a gross pool market formulation for trading impervious cover areas. The market allocates IC allowances efficiently among participants. Participants face a price equilibrium which will encourage the use of BMPs and IC allowances in the property to manage floods in the catchment. Additionally, this gross pool formulation allows scaling to keep the regulator revenue neutral.

The market formulation requires that participants bid for their initial IC allowance condition if they desire to remain with the same IC allowance, so the market works as if participants have no initial rights, but deals with them in the settlement systems.

Bids can be scaled due to constraint [8.4], because the market accounts for relative bid differences rather than absolute bids. Thus, bids can add a constant, and neither clearing prices nor allocations change. This could confuse participants, but still corresponds to a feasible solution, and clearing prices still account for the flood damage.

Alternatively, the SO can establish a reference usage [land use] for participants, which corresponds to particular IC allowances. This chapter presented a way to include a reference usage in the market formulation, to reduce the degree of freedom to the market formulation and to avoid problems with relative bid prices. In this case, the participants would need to bid with respect to the reference usage, but settlements are based on their initial IC allowances.

Finally, a scaling factor was proposed to reach revenue neutrality for the SO. The factor depends on the changes in expected flood damage at control points and scenarios. Scaling of final payments from participants based on their initial IC allowance was demonstrated.

Similar to the proposed Sto_MarketIC and Sto_MarketIC_Risk, the gross pool formulation can be extended for flood components and for possible risk positions that the SO desires to reach or to cap at control points and catchment levels.

Chapter 9

9 CONCLUSIONS AND FUTURE RESEARCH

This chapter presents a summary, the main conclusions of this thesis, and directions for future research.

9.1 Summary

The main contribution of this thesis is to propose market-based mechanisms for impervious cover (IC) allowances [land use], with the goal to manage extreme runoff discharges from point and non-point sources, and hence control flood problems within catchments. An IC allowance is a tradable permit to use a specific level of perviousness in a specific area (hectare).

Two deterministic market-clearing models, Det_MarketIC1 and Det_MarketIC2, were proposed as described in Chapter 3. With Det_MarketIC1, participants buy and sell IC allowances. With Det_MarketIC2, participants bid for changes in IC allowances in their area. Each market model allowed for similar outcomes and prices to be reached. The market models account for capped capacities which corresponded to peak flows and flood conditions under a given extreme storm scenario. However, the market design may have possible issues related to internalising the flood cost related to storms greater than the one used to establish the market. These deterministic models did not penalise hastening inundation and lengthening duration, but could be extended to deal with these issues.

The market signals a marginal cost for reducing flood peaks using imperviousness levels, which is connected to the dual price for capacity [3.6] and [3.22] in Det_MarketICs. These dual prices are also related to marginal increment in capacities and, in this sense, these could be related to levees, dikes, banks and temporary storage capacities. Thus, each improvement could be evaluated using the market and the solution should ensure a least cost approach. However, other control approaches such policies and physical measures are also used to control and may not be a least cost approximation. Those control evaluations are outside the scope of this thesis, but an interesting future research.

In Chapter 4, the market model Sto_MarketIC was formulated to deal with storm events where flow capacities are violated. The market model related capacity violations to flood depth damage and a TSSP market clearing model estimated flood damage via recourse flood cost across storm scenarios. The Sto_MarketIC1 model accounted for peak flow as the primary flood component across storm scenarios. With this formulation, issues related to non-convexity in the flood damage cost, the SO's revenue, and possible free riders were discussed.

In Chapter 5, the Sto_MarketIC2 model was developed to account for hastening peak time of flooding and lengthening floods in addition to the damages caused by peak flows. Additional constraints and thresholds were proposed related to these flood components and resulting flood damage. Trade conditions, allocations and prices were also analysed.

In Chapter 6, the Sto_MarketIC_Risk model was proposed to include risk of flood disaster at the extreme tail of the flood damage distribution. A downside risk measure CVaR was proposed to be used in the market model. Thus, the SO could establish risk capped positions, which represent acceptable flood risk levels that the community desires. Duality allowed interpreting prices which also accounted for the risk positions at control points and within the catchment. Applied prices for participants account for the binding risk position in their IC allowances. The Sto_MarketIC_Risk model, was shown to be constrained via CVaRs, and should be carefully evaluated, because the market can be over-constrained, and so the clearing prices can be high or resulting in an infeasible solution.

In Chapter 7, a case study simulation of the L2 catchment was analysed using hypothetical participants and property delineations. The different market models were used to simulate prices and IC allocations in the catchment. The HEC-HMS and HEC-RAS models were used to estimate runoff from properties and impact flow coefficients in the catchment for a range of storm scenarios. The market models resulted in different prices, IC allocations as well as hedging flood conditions when the market conditions were changed such as capped flows, CVaRs constraints, flood components, etc. Participants faced the cost for changing the flood distribution and the risk implied by those changes in the price of IC allowances in their properties.

In Chapter 8, a gross pool model, Gross_MarketIC, was proposed to trade IC allowances. The model accounted for uncertain storms and violated channel capacities in a similar way as Sto_MarketIC. The gross formulation is cleared as if participants do not have rights and only with demand curves. Trade and settlement for participants account for either initial IC allowances or a base line of land use defined for the SO. Additionally, the market could reach an equilibrium which does not depend on the initial allowance. A scaling factor was proposed to allow the SO to reach revenue neutrality. Additionally, the implications of establishing a reference-usage allowance, the required constraint transformation to deal with the reference usage, and the bids from participants, were discussed.

9.2 Conclusions

- The target improvements (reductions) in imperviousness levels in the catchment can be allocated according to market mechanism with a low cost for the community.
- The market design can conform to policies for managing floodplain areas, and allows targets to be reached at a low cost for the community. With the market mechanism, participants do not need to search for trading partners, nor look for contracts at each flooding place where they desire to change flood distributions.
- The market mechanisms encourage participants to use BMPs and to face the cost to the system when they change impervious cover. Consequently, flooding could be well managed.
- From duality, prices account for the cost to the system for the capped thresholds, flood components and flood costs as well as for the risk positions.
- The market establishes a marginal cost for reducing flood peak using imperviousness levels. From duality, the shadow price of Equations [3.6] and [3.22] in Det_MarketIC signals the marginal change in capacity, or the cost to the system for allowing an additional unit of capacity. The dual price from Equation [4.7] in Sto_MarketIC signals the marginal changes in the expected flood damage.
- Participants internalise the decision for shifting the flood distribution in the catchment, which can make it safe for a range of flow conditions. However, these market

mechanisms do not totally eliminate flooding in the catchment, but they rather attempt to manage flooding according to a desired flood distribution and probability of disaster.

- For the market design to work, a reference needs to be established which can correspond to a grandfathering use of previous land use, current land use, or any other reference usage that the SO desires to establish for the catchment.
- The market design reduces transaction costs, but requires setting up and monitoring; thus, the transaction cost savings would be reduced by these expenses. The SO establishes limits, but must carefully monitor during periods when flows reach critical levels. The SO will need to validate impervious levels in each property, and prosecute participants that do not follow the rules.
- The market models could be implemented in any catchment. If the main concern from the SO is stage-flood, the market could account for this flood component as in models Det_MarketIC, Sto_MarketIC, and Gross_MarketIC; other flood components could also be included. If the SO desired to keep a capped risk position (CVaR) and to transfer possible changes on it, the market model could be Sto_MarketIC_Risk.
- Non-convexity may produce market failure, because non-supporting price condition (O'Neill et al. 2005). As a result, externalities may not be internalised, the catchment may become impervious and the flood distribution will shift. Accordingly, convexification may be required if the expected flood changes are non-convex.
- Non-linearity and convexification may produce extra rents, which could mean extra payments to or from the SO. Therefore, the SO should evaluate the flood damage convexification to reduce any possible extra rent. The SO may wish to pay for reducing imperviousness and risk. The SO should evaluate a position in terms of its budget position.
- The SO could reach revenue neutrality after clearing the market with the proposed proportional scaling factor. The scaling factor could also be used by the SO to give credits and debits for participants for their trade, which adjust the final payment for their IC allowances.

9.3 Future research

9.3.1 Flood damage (cost functions) and risk measures

As stated in Chapters 4, 5, and 6, the flood damage cost functions may be non-linear and non-convex, and hence piecewise approximations are needed. These approximations affect prices and allocations. The SO should estimate accurate flood cost functions for the changes in flood distribution, and thus other cost formulations could be proposed to accurately estimate changes in flood distribution.

The risk profile could be estimated ex-post, as presented with Sto_MarketIC_Risk2. This profile affects prices and allocations. The SO could accept this risk condition, but needs to evaluate whether flood risk in the catchment could increase above accepted levels. The market also needs to be evaluated for possible relationships between risk profile positions at control points and at the catchment level as well as the possible effects of these on clearing prices.

9.3.2 Insurance for disasters

In Chapters 4 and 6, we mentioned the possible effect that implementing an IC market could have on the risk premiums from insurer companies or a governmental insurer, and for the risk of people in the flood area. Because the market could establish risk positions for the community, the risk premium and adverse selections could change for the community. With market based models, places that previously suffered adverse selection could be insured due to risk reduction. This could be noticed if the SO constrains CVaR or tries to reduce CVaR value at control points. Reducing CVaR may enable managing or even reducing the risk of flooding. This risk condition would produce that insurance prime may be reduced as well. Ermolieva and Ermoliev (2005) observed that if a flood plain considers insurance with catastrophic risk management, an insurance pooling mechanism could not be achieved without location-specific risk exposures. Partial compensation and contingent credits could be enhanced with a proposed market mechanism for IC allowances. Duncan and Myers (2000) noted that insurers may require higher risk premium with low coverage levels, and suggested a subsidy for reinsurance companies to reduce premiums. However, with the IC market, a risk position and flood distribution could be maintained for extreme events, which would also affect the risk positions of insurers. This is an interesting area for future research.

9.3.3 Transported pollutants from runoff

The market design could account for levels of pollutants such as sediments or nitrates which are transported by the runoff. The formulation would include new hydrology and hydraulic processes related to pollutants. Thus, the market model could account for accumulated deposits and pollutant levels at specific locations in the river over time. Consequently, the market would estimate prices and allocations, and participants would face the pollutant cost in their IC allowance decisions.

The market formulation for IC allowances could be extended to include externalities related to transported pollutants with runoff, e.g., sediment discharge, nitrates, and phosphorus. An additional effort would be required in modelling and computer time. The formulation would need to model the spatial and temporal effects in the catchment for the levels of pollutants that pass or are accumulated under scenarios over several years. These physical relationships are probably non-linear and non-convex, which should be evaluated for the market formulation.

Similar market mechanisms were proposed for nitrate pollution (Prabodanie et al. 2010), runoff (Raffensperger and Cochrane 2010), groundwater (Raffensperger et al. 2009), and sediment discharge (Pinto et al. 2012). These formulations assumed certain parameters for flows, leaching nitrates and sediments, but do not handle possible uncertainty as was proposed in this research. A stochastic smart market formulation could be extended for the previous resources and so uncertainty could be addressed for uncertain resource availability, process, and impacts.

9.3.4 User interface and SO simulation framework

A web based user interface could be developed which would allow users to place bids to sell and buy IC allowances at designated prices for their required changes in land use. The interface could also be designed to assist the users' decision process for changing IC allowance. Participants could evaluate possible changes in runoff from their properties and the subsequent impact on flows at control points with their changes in IC allowance. The interface would be visually appealing and would run as a web site application.

The SO would require a simulation framework allowing for capture of all bids and selling as well as for running the hydrological and hydraulic models. This framework would enable the SO to run various market scenarios and determine final prices. This interface should also be flexible enough to allow for updates in models, parameters, etc. This flexibility would also allow testing of different clearing models as presented in Chapters 3-8.

The interface would facilitate participants bid experience, and assist the SO to make decisions and transmit results to the market participants.

9.3.5 Infrastructure investment and effects in the market

The market considers an established infrastructure which retains flood and risk in hazardous places. Any flood plan considers investments for flood protections. These investments may change limits and thresholds and consequently prices and allocations. Shadow prices may signal improvement in infrastructure and reducing risk. These prices could be analysed and decomposed to evaluate improvement of infrastructure in the catchment. Real options could be used to analyse investment plans in infrastructure such as transmission in the electricity network. Real options are normally used to evaluate if an investment should be made in this period or later (Brunekreeft 2004; Boyle et al. 2006). In this case, price information from the market could be used to evaluate improvements in infrastructure. The SO could make decisions about the current period´s investment and could receive the revenue for the new infrastructure. Additionally the market could consider financial instruments as a way to reduce possible risk for changes in flood damage and to incentivise new investment in protecting potential flood areas. Future research could consider these points and use the market mechanism as a means to evaluate the effects of reducing the risk of floods and associated damage.

10 Bibliography

Acerbi, C. and D. Tasche (2002). "Expected Shortfall: a natural coherent alternative to Value at Risk." Economic Notes 31(2): 379-388.

Afualo, V. and J. McMillan (1996). Auction of rights to public property. Working Paper, University of Stanford: 14.

Ahmed, S. (2006). "Convexity and decomposition of mean-risk stochastic programs." Mathematical Programming 106(3): 433.

Alvey, T., D. Goodwin, M. Xingwang, D. Streiffert and D. Sun (1998). "A security-constrained bid-clearing system for the New Zealand wholesale electricity market." Power Systems, IEEE Transactions on 13(2): 340-346.

Arnell, N. W., M. J. Clark and A. M. Gurnell (1984). "Flood insurance and extreme events: the role of crisis in prompting changes in British institutional response to flood hazard." Applied Geography 4(2): 167-181.

Arrow, K. J. (1971). Essays in the theory of risk-bearing. Amsterdam, North Holland.

Artzner, P., F. Delbaen, J.-M. Eber and D. Heath (1999). "Coherent measures of risk." Mathematical Finance 9(3): 203-228.

Barquin, J., E. Centeno and J. Reneses (2004). Stochastic market equilibrium model for generation planning. Probabilistic Methods Applied to Power Systems, 2004 International Conference on.

Barrera, A., M. Llasat and M. Barriendos (2006). "Estimation of extreme flash flood evolution in Barcelona County from 1351 to 2005." Natural Hazards and Earth System Science 6(4): 505-518.

Beale, E. M. (1963). Two transportation problems. Third IFORS Conference, Dunod Paris and E.U.P. London.

Beale, E. M. and J. A. Tomlin (1970). Special facilities in a general mathematical programming system for non-convex problem using order set of variables. Proceedings of the fifth international conference on operations research, London, Tavistock Publications.

Birbil, Ş. İ., J. B. G. Frenk, B. Kaynar and N. Noyan (2009). Risk measures and their applications in asset management. The VaR Implementation Handbook: Financial Risk and Applications in Asset Management, Measurement, and Modeling. G. N. Gregoriou. Columbus, Ohio, The McGraw-Hill Companies: 311-338.

Birge, J. R. and F. Louveaux (1997). Introduction to stochastic programming New York, Springer.

Bjørndal, M. and K. Jörnsten (2008). "Equilibrium prices supported by dual price functions in markets with non-convexities." European Journal of Operational Research 190(3): 768-789.

Bohn, R. E., M. C. Caramanis and F. C. Schweppe (1984). "Optimal pricing in electrical networks over space and time." The RAND Journal of Economics 15(3): 360-376.

Boyle, G., G. Guthrie and R. Meade (2006). "Real options and transmission investment: the New Zealand grid investment test."

Brabec, E., S. Schulte and P. Richards (2002). "Impervious surface and water quality: a review of current literature and its implication for watershed planning." Journal of Planning Literature 16(4): 499-514.

Bradshaw, C. J., N. S. Sodhi, K. S. Peh and B. W. Brook (2007). "Global evidence that deforestation amplifies flood risk and severity in the developing world." Global Change Biology 13(11): 2379-2395.

Brown, P. D. (2007). Optimal irrigation scheduling. Civil and Natural Resources Engineering. Christchurch, University of Canterbury. Ph.D.: 184.

Brunekreeft, G. (2004). "Market-based investment in electricity transmission networks: controllable flow." Utilities Policy 12(4): 269-282.

Burton, I., R. Kates and G. White (1968). The human ecology of extreme geophysical events. FMHI Publications. Paper 78.

Calatrava, J. and A. Garrido (2005). "Modelling water market under uncertainty water supply." *European Review of Agricultural Economics* 32(2): 119-142.

Carrion, M., A. B. Philpott, A. J. Conejo and J. M. Arroyo (2007). "A stochastic programming approach to electric energy procurement for large consumers." Power Systems, IEEE Transactions on 22(2): 744-754.

Chakrabarti, B. B. and D. G. Goodwin (2008). Monitoring and measuring market power in the New Zealand electricity market. Power System Technology and IEEE Power India Conference, 2008. POWERCON 2008. Joint International Conference on.

Chang, Y. (2007). "The new electricity market of Singapore: Regulatory framework, market power and competition." Energy policy 35(1): 403-412.

Charles River Associates, C. (2003). Constraint orientation: principles and pricing implications. Melbourne, Australia, NEMMCO.

Chattopadhyay, D. (2004). "Multicommodity spatial Cournot model for generator bidding analysis." IEEE Transactions on power systems 19(1): 267-275.

Chattopadhyay, D., B. B. Chakrabarti and E. Grant Read (2003). "A spot pricing mechanism for voltage stability." Electrical Power and Energy Systems 25(9): 725-734.

Chen, J., T. D. Mount, J. S. Thorp and R. J. Thomas (2005). "Location-based scheduling and pricing for energy and reserves: a responsive reserve market proposal." Decision support systems 40(3-4): 563-577.

Chomitz, K. M. (2004). "Transferable development rights and forest protection: an exploratory analysis." International Regional Science Review 27(3): 348-373.

Chomitz, K. M., T. S. Thomas and A. Salazar (2004). "Creating markets for habitat conservation when habitats are heterogeneous." World Bank Policy Research Working Paper 3429, World Bank.

Chu, L. Y. and Z.-J. M. Shen (2007). "Trade reduction vs. multi-stage: a comparison of double auction design approaches." European Journal of Operational Research 180(2): 677-691.

Coase, R. H. (1960). "The problem of social cost." Journal of Law and Economics 3: 1-44.

Cowdin, S. (2008). Economic Analysis Guidebook. The Resources Agency, Department of Water Resources. Davis, State of California.

CSIRO (2000). Floodplain management in Australia: best practice principles and guidelines. Victoria, CSIRO.

Day, C. J., B. F. Hobbs and J. S. Pang (2002). "Oligopolistic competition in power networks: a conjectured supply function approach." IEEE Transactions on Power Systems 17(3): 597-607.

de Moel, H. and J. Aerts (2011). "Effect of uncertainty in land use, damage models and inundation depth on flood damage estimates." Natural Hazards 58(1): 407-425.

De Wolf, D. and Y. Smeers (2000) "The simplex algorithm extended to piecewise-linearly constrained problems." DOI: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.14.8090

Dinar, A., M. W. Rosegrant and R. S. Meinzen-Dick (1997). Water allocation mechanisms: principles and examples, World Bank Publications.

Doll, A., P. F. Scodari and G. Lindsey (1998). Credits as economic incentives for on-site storm water management: issues and examples. National Conference on Retrofit Opportunities for Water Resource Protection in Urban Environments, Chicago, IL, EPA.

Dougherty, M., R. L. Dymond, S. J. Goetz, C. A. Jantz and N. Goulet (2004). "Evaluation of impervious surface estimates in a rapidly urbanizing watershed." Photogrammetric Engineering and Remote Sensing 70(11): 1275-1284.

Duffie, D. and J. Pan (1997). "An overview of value at risk." The Journal of Derivatives 4(3): 7-49.

Duncan, J. and R. J. Myers (2000). "Crop insurance under catastrophic risk." American Journal of Agricultural Economics 82(4): 842-855.

Dupacová, J., A. Gaivoronski, Z. Kos and T. Szántai (1991). "Stochastic programming in water management: A case study and a comparison of solution techniques." European Journal of Operational Research 52(1): 28-44.

Dupačová, J., N. Gröwe-Kuska and W. Römisch (2003). "Scenario reduction in stochastic programming." Mathematical Programming 95(3): 493-511.

Durrans, S. R. (2010). Intensity-duration-frequency curves. Rainfall: State of the Science. Washington, DC, AGU. 191: 159-169.

Dutta, D., S. Herath and K. Musiake (2000). "Flood inundation simulation in a river basin using a physically based distributed hydrologic model." Hydrological Processes 14(3): 497-519.

Dutta, D., S. Herath and K. Musiake (2003). "A mathematical model for flood loss estimation." Journal of Hydrology 277(1-2): 24-49.

Dye, S. (1994). On a flexible model for New Zealand's hydro-thermal electricity generation system. Operations Research. New Plymonth, Massey University. Ph.D. Thesis.

E.P.A., E. P. A. (1993). Handbook: Urban runoff pollution prevention and control planning. Cincinnati, OH. United States, Diane Pub Co (March 1993).

EFTEC (2010). Flood and Coastal Erosion Risk Management: Economic Valuation of Environmental Effects. Handbook for the Environment Agency for England and Wales. London, Environment Agency for England and Wales.

Eigenraam, M., C. Beverly, G. Stoneham and J. Todd (2005). "Auctions for multiple environmental outcomes, from desk to field in Victoria, Australia." Paper presented to the Annual Conference of the Western Economic Association, 4-8 July 2005, San Francisco, California.: 30.

Ely, P. B. and J. C. Peters (1984). Probable maximum flood estimation -Easter United States. Davis, CA, US Army Corps of Engineers, Institute for Water Resources, Hydrologic Engineering Center (HEC) TP-100.

Embrechts, P. and H. Schmidli (1994). "Modelling of extremal events in insurance and finance." Mathematical methods of operations research 39(1): 1-34.

Environment, M. f. t. (2008). Meeting the challenges of future flooding in New Zealand. M. f. t. E. a. T. F. R. M. a. R. C. R. S. Group, Ministry for the Environment, New Zealand.

Eppen, G. D., R. K. Martin and L. Schrage (1989). "A scenario approach to capacity planning." Operations Research 37(4): 517-527.

Ermolieva, T. (2005). "Simulation-based optimization of social security systems under uncertainty." European Journal of Operational Research 166(3): 782-793.

Ermolieva, T. and Y. Ermoliev (2005). Catastrophic risk management: flood and seismic risk case study. Application of stochastic programming. S. W. Wallace and W. T. Ziemba. Philadelphia, USA, Society for industrial and applied mathematics, SIAM. First ed: 425-444 p.

Evenden, R. D. and A. W. Williams (2000). "Contestability: The debate and industry policy." Economic analysis and policy 30: 75-90.

Fábián, C. I. (2008). "Handling CVaR objectives and constraints in two-stage stochastic models." European Journal of Operational Research 191(3): 888-911.

Fama, E. F. (1970). "Efficient capital markets: a review of theory and empirical work." The Journal of Finance 25(2): 383-417.

Ferguson, C. E. and J. P. Gould (1980). Microeconomic theory, Irwin Homewood, IL.

Fishburn, P. C. (1977). "Mean-risk analysis with risk associated with below-target returns." The American Economic Review 67(2): 116-126.

Fishburn, P. C. (1980). "Stochastic dominance and moments of distributions." Mathematics of Operations Research 5(1): 94-100.

Ford, D. T. (1985). Flood-damage-mitigation plan selection with branch and bound enumeration. Hydrologic engineering center. TD-23, US Army Corps of Engineers: 25.

Fourer, R., D. Gay and B. W. Kernighan (2003). "AMPL Modeling Language." Mathematical Programming. Pacific Grove, CA: Thompson Learning.

Fox, J. (2008). Applied regression analysis and generalized linear models. Los Angeles, Sage Publications, Inc.

Gallien, J. and L. M. Wein (2005). "A smart market for industrial procurement with capacity constraints." Management Science 51(1): 79-91.

Georgakakos, K. P. (1986). On the design of national, real-time warning systems with capability for site-specific, flash-flood forecasts, National Emergency Training Center.

Ghosh, S. N. (1997). Flood control and drainage engineering, Taylor & Francis.

Growe-Kuska, N., H. Heitsch and W. Romisch (2003). Scenario reduction and scenario tree construction for power management problems. Power Tech Conference Proceedings, 2003 IEEE Bologna, IEEE.

Guasch, J. L. and T. Glaessner (1993). "Using auctions to allocate and price long-term credit." The World Bank Research Observer 8(2): 169.

Guganesharajah, K., D. G. Thorn, T. E. Evans and R. Harpin (1985). Mathematical modeling of the river Conon. 2nd international conference on the hydraulics of floods and flood control, Cambridge.

Haan, C. T., B. J. Barfield and J. C. Hayes (1994). Design hydrology and sedimentology for small catchments. San Diego, CA.

Haestad, M., G. Dyhouse, J. Hatchett and J. Benn (2003). Floodplain Modelling Using HECRAS. Waterbury, CT USA, Haestad Press.

Hannan, T. C. and I. C. Goulter (1985). Agricultural flood damage evaluation with multiple annual floods and time dependent flood damage susceptibility. 2nd International conference on the hydraulics of floods and flood control, Cambridge, University of Cambridge.

Harman, J., M. E. Bramley and M. Funnell (2002). "Sustainable flood defence in England and Wales." Civil Engineering 150(5): 3-9.

HEC (2008a). The hydrologic modeling system (HEC-HMS) Davis, California, Hydrologic Engineering Center (HEC), US Army Corps of Engineers.

HEC (2008b). The river analysis system (HEC-RAS) Davis, California, Hydrologic Engineering Center (HEC), US Army Corps of Engineers.

Herath, S. (2003). Flood damage estimation of an urban catchment using remote sensing and GIS. International conference on urban storm drainage, Australia.

Hill, E., S. Pugh and J. Mullen (2007). "Use of the hedonic method to estimate lake sedimentation impacts on property values in Mountain Park and Roswell, GA." Annual Meeting, July 29-August 1, 2007, Portland, Oregon: 21.

Hogan, W. W. (2002). Financial transmission right formulation. Cambridge, Massachusetts, Center for business and government. John F. Kennedy School of Government. Harvard University. 3.

Hogan, W. W., E. G. Read and B. J. Ring (1996). "Using mathematical programming for electricity spot pricing." International Transactions in Operational Research 3(3-4): 209-221.

Hollinshead, S. and J. R. Lund (2006). "Optimization of environmental water purchases with uncertainty." Water Resources Research 42(W08403, doi:10.1029/2005WR004228): 10.

Holt, C., W. Shobe, D. Burtraw, K. Palmer and J. Goeree (2007). "Auction design for selling CO2 emission allowances under the regional greenhouse gas initiative." University of Virginia, Cooper Center for Public Service and Resources for the Future, October 26.

ILOG. (2008). "ILOG AMPL CPLEX System. Version 11.0. User's Guide."

Jaffe, A. B., R. G. Newell and R. N. Stavins (2005). "A tale of two market failures: Technology and environmental policy." Ecological Economics 54(2): 164-174.

James, L. D. and R. R. Lee (1971). Economics of water resources planning. New York, McGraw-Hill, Inc.

Jarrow, R. A. and M. Larsson (2012). "The meaning of market efficiency." Mathematical Finance 22(1): 1-30.

Jason, K. L. and H. Jim (2005). "Advances in flood risk management under uncertainty." Stochastic Environmental Research and Risk Assessment 19(6): 375.

Jha, A. K., R. Bloch and J. Lamond (2012). Cities and flooding. A guide to integrated urban flood risk management for the 21st century. Washington DC, World Bank. First.

Kall, P. and S. W. Wallace (1994). Stochastic programming, Wiley New York.

Karamouz, M., O. Abesi, A. Moridi and A. Ahmadi (2009). "Development of optimization schemes for floodplain management; A case study." Water Resources Management 23: 1743-1761, doi 1710.1007/s11269-11008-19350-11269.

Kattelmann, R. (1997). "Flooding from rain-on-snow events in the Sierra Nevada." IAHS Publications-Series of Proceedings and Reports-Intern Assoc Hydrological Sciences 239: 59-66.

Kauffman, G. J. and T. Brant (2000). "The role of impervious cover as a watershed-based zoning tool to protect water quality in the Christina River Basin of Delaware, Pennsylvania, and Maryland." Proceedings of the Water Environment Federation 2000: 1656-1667.

Kazama, S., A. Sato and S. Kawagoe (2009). "Evaluating the cost of flood damage based on changes in extreme rainfall in Japan." Sustainability Science 4(1): 61-69.

Keha, A. B., I. R. de Farias and G. L. Nemhauser (2004). "Models for representing piecewise linear cost functions." Operations Research Letters 32(1): 44-48.

Kerkez, B., S. D. Glaser, J. A. Dracup and R. C. Bales (2010). A hybrid system model of seasonal snowpack water balance. Proceedings of the 13th ACM international conference on Hybrid systems: computation and control, ACM.

Knight, D. W. and A. Y. Shamseldin (2005). River basin modelling for flood risk mitigation. London, UK, Taylor & Francis/Balkema.

Konno, H. and H. Yamazaki (1991). "Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market." Management Science 37(5): 519-531.

Kopits, E., V. McConnell and M. Walls (2005). "Making markets for development rights work: What determines demand?" Discussion Paper. Resources for the Future RFF DP 05-45-REV.

Koutsoyiannis, D. (2007). A critical review of probability of extreme rainfall: Principles and models. Advances in urban flood management. R. Ashleyet al. London, UK, Taylor & Frances Group: 499.

Kreibich, H., K. Piroth, I. Seifert, H. Maiwald, U. Kunert, J. Schwarz, B. Merz and A. Thieken (2009). "Is flow velocity a significant parameter in flood damage modelling." Nat. Hazards Earth Syst. Sci 9: 1679-1692.

Krokhmal, P., S. Uryasev and G. Zrazhevsky (2005). Numerical comparison of conditional value-at-risk and conditional drawdown-at-risk approaches: application to hedge funds. Applications of Stochastic Programming. W. W. Stein and T. Z. William, Society for Industrial and Applied Mathematics: 609-631.

Kron, A. (2007). Flood damage estimation and flood risk mapping. Advances in urban flood management. R. Ashleyet al. London, UK, Taylor & Frances Group: 499.

Kron, W. (2005). "Flood risk = hazard• values• vulnerability." Water International 30(1): 58-68.

Krzysztofowicz, R. and D. R. Davis (1983a). "Category-unit loss functions for flood forecast-response system evaluation." Water resources research 19(6): 1476-1480.

Krzysztofowicz, R. and D. R. Davis (1983b). "A methodology for evaluation of flood forecast-response systems 1. Analyses and concepts." Water Resources Research 19(6): 1423-1429.

Krzysztofowicz, R. and D. R. Davis (1983c). "A methodology for evaluation of flood forecast-response systems 2. Theory." Water resources research 19(6): 1431-1440.

Krzysztofowicz, R. and D. R. Davis (1983d). "A methodology for evaluation of flood forecast-response systems 3. Case studies." Water Resources Research 19(6): 1441-1454.

Kwerel, E. (1977). "To tell the truth: imperfect information and optimal pollution control." The Review of Economic Studies 44(3): 595-601.

Laffont, J. J. and J. Tirole (1996a). "Pollution permits and compliance strategies." J. Public Econ. 62(1-2): 85-125, doi: 110.1016/0047-2727(1096)01575-01577.

Laffont, J. J. and J. Tirole (1996b). "Pollution permits and environmental innovation." J. Public Econ. 62(1-2): 127-140, doi: 110.1016/0047-2727(1096)01576-01579.

Latacz-Lohmann, U. and C. V. d. Hamsvoort (1997). "Auctioning conservation contracts: a theoretical analysis and an application." American Journal of Agricultural Economics v79(n2): p407(412).

Latacz-Lohmann, U. and S. Schilizzi (2005). Auctions for conservation contracts: a review of the theoretical and empirical literature, Scottish Executive Environment and Rural Affairs Department: 101 p.

Lawler, E. L. and D. E. Wood (1966). "Branch and bound methods. A survey." Operation Research 14: 699-719.

Lee, J. and D. Wilson (2001). "Polyhedral methods for piecewise-linear functions I: the lambda method." Discrete Applied Mathematics 108(3): 269-285.

Lee, Y. Y., R. Baldick and J. Hur (2011). "Firm-based measurements of market power in transmission-constrained electricity markets." Power Systems, IEEE Transactions on 26(4): 1962-1970.

Lesieutre, B. C. and I. A. Hiskens (2005). "Convexity of the set of feasible injections and revenue adequacy in FTR markets." Power Systems, IEEE Transactions on 20(4): 1790-1798.

Levy, H. (2006). Stochastic dominance: investment decision making under uncertainty. New York, Springer Science+Business Media.

Linsmeier, T. J. and N. D. Pearson (2000). "Value at risk." Financial Analysts Journal: 47-67.

Liu, J., C. Men, V. E. Cabrera, S. Uryasev and C. W. Fraisse (2008). "Optimizing crop insurance under climate variability." Journal of Applied Meteorology and Climatology 47(10): 2572-2580.

Liu, Z. and G. Huang (2009). "Dual-interval two-stage optimization for flood management and risk analyses." Water Resources Management 23(11): 2141-2162.

Loucks, D. P., E. V. Beek, J. R. Stedinger, J. P. Dijkman and M. T. Villars. (2005). Water resources systems planning and management: an introduction to methods, models and applications. Paris, UNESCO.

Lund, J. R. (2002). "Floodplain planning with risk-based optimization." Journal of water resources planning and management 128: 202.

Malcolm, S. A. and S. A. Zenios (1994). "Robust optimization for power systems capacity expansion under uncertainty." The Journal of the Operational Research Society 45(9): 1040-1049.

Mansini, R., W. Ogryczak and M. Speranza (2007). "Conditional value at risk and related linear programming models for portfolio optimization." Annals of Operations Research 152(1): 227-256.

Markowitz, H. (1952). "Portfolio selection." Journal of Finance 7(1): 77–91.

Mascarenhas, F. C. B. and M. G. Miguez (2005). Mathematical modelling of rural and urban floods: A hydraulic approach Flood risk simulation. M. C. B. Mascarenhaset al. Southampton, Boston, WIT press. First: 335-436.

Mascarenhas, M. C. B., K. Toda, M. G. Miguez and K. Inoue (2005). One-dimensional routing of river and dam-break flood waves. Southampton, Boston, WIT press.

Mays, L. W. (2001). Water resources engineering. New York, NY, John Wiley.

McAdams, D. (2007). "Adjustable supply in uniform price auctions: Non-commitment as a strategic tool." Economics Letters 95(1): 48-53.

McAfee, R. P. and J. McMillan (1987). "Auctions and bidding." Journal of Economic Literature 25(2): 699-738.

McCabe, K. A., S. J. Rassenti and V. L. Smith (1989). "Designing smart computer-assisted markets." Journal of Political Economy 5: 259-283.

McCabe, K. A., S. J. Rassenti and V. L. Smith (1991). "Smart computer-assisted markets." Science 254(5031): 534-538.

McCann, L., B. Colby, K. W. Easter, A. Kasterine and K. Kuperan (2005). "Transaction cost measurement for evaluating environmental policies." Ecological Economics 52(4): 527-542.

McCann, L. and K. W. Easter (1999). "Transaction costs of policies to reduce agricultural phosphorous pollution in the Minnesota River." Land Economics: 402-414.

McConnell, V., E. Kopits and M. Walls (2005). "Farmland preservation and residential density: can development rights markets affect land use?" Agricultural and Resource Economics Review 34(2): 131-144.

McConnell, V., M. Walls and E. Kopits (2006). "Zoning, TDRs and the density of development." Journal of Urban Economics 59(3): 440-457.

Merz, B., H. Kreibich, R. Schwarze and A. Thieken (2010). "Assessment of economic flood damage." Nat. Hazards Earth Syst. Sci 10: 1697-1724.

Middelmann-Fernandes, M. H. (2010). "Flood damage estimation beyond stage damage functions: an Australian example." Journal of Flood Risk Management 3: 88-96.

Midthun, K. T., M. Bjorndal and A. Tomasgard (2009). "Modeling optimal economic dispatch and system effects in natural gas networks." The Energy Journal 30(4): 155(126).

Milgrom, P. (2000). "Putting auction theory to work: the simultaneous ascending auction." The Journal of Political Economy 108(2): 245-272.

Miller, R. M. (1996). "Smart market mechanisms: From practice to theory." Journal of Economic Dynamics and Control 20(6-7): 967-978.

Montero, J.-P. (2008). "A simple auction mechanism for the optimal allocation of the commons." American Economic Review 98(1): 496.

Mujumdar, P. P. (2001). "Flood wave propagation." Resonance 6(5): 66-73.

Mulvey, J. M. and H. G. Erkan (2006). "Applying CVaR for decentralized risk management of financial companies." Journal of Banking & Finance 30(2): 627-644.

Murphy, J. J., A. Dinar, R. E. Howitt, S. J. Rassenti and V. L. Smith (2000). "The design of "smart" water market institutions using laboratory experiments." Environmental and Resource Economics 17(4): 375.

Murphy, J. J., A. Dinar, R. E. Howitt, S. J. Rassenti, V. L. Smith and M. Weinberg (2009). "The design of water markets when instream flows have value." Journal of Environmental Management 90(2): 1089-1096.

Nesbitt, D. M. and J. N. Scotcher (2009). "Spatial price and quantity relationships in world and continental commodity markets." The Energy Journal 30(SI): 21(17).

Netusil, N. R. and J. B. Braden (2001). "Transaction costs and sequential bargaining in transferable discharge permit markets." J. Environ. Manage. 61(3): 253-262, doi: 210.1006/jema.2000.0407

NIWA (2002). High intensity rainfall design system (HIRDS). Wellington, New Zealand, Climate Analysis and Applications Section. National Institute of Water and Atmospheric Research.

NIWA (2008). High intensity rainfall design systems (HIRDS). Auckland, National Institute of Water and Atmospheric Research (NIWA).

Noyan, N. (2010). "Two-stage stochastic programming involving CVaR with an application to disaster management." 2011, from http://www.optimization-online.org/DB_HTML/2010/03/2571.html.

NRCS (1975). Win TR-55 small watershed hydrology. Natural Resources Conservation Service, United State Department of Agriculture, USDA.

NRCS (2010). National Engineering Handbook. Time of Concentration. Washington, USA, United State Department of Agriculture, USDA.

O'Neill, R. P., P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf and W. R. Stewart (2005). "Efficient market-clearing prices in markets with nonconvexities." European Journal of Operational Research 164(1): 269-285.

Ogryczak, W. and A. Ruszczynski (1999). "From stochastic dominance to mean-risk models: Semideviations as risk measures." European Journal of Operational Research 116(1): 33-50.

Ogryczak, W. and A. Ruszczynski (2002). "Dual stochastic dominance and quantile risk measures." International Transactions in Operational Research 9(5): 661-680.

Olsen, J. R., P. A. Beling and J. H. Lambert (2000). "Dynamic models for floodplain management." J. Water Resour. Plng. and Mgmt. 126(3): 167-175.

Ormsbee, L. E., J. W. Delleur and M. H. Houck (1984). Development of a general planning methodology for storm water management in urban watersheds. Water. West Lafayette, Indiana, Water Resources Research Center, Purdue University: 2005.

Pappas, E. A., D. R. Smith, C. Huang, W. D. Shuster and J. V. Bonta (2008). "Impervious surface impacts to runoff and sediment discharge under laboratory rainfall simulation." CATENA 72(1): 146-152.

Parikh, P., M. A. Taylor, T. Hoagland, H. Thurston and W. Shuster (2005). "Application of market mechanisms and incentives to reduce storm water runoff: An integrated hydrologic, economic and legal approach." Environmental Science & Policy 8(2): 133-144.

Parker, D. J. (1995). "Floodplain development policy in England and Wales." Applied Geography 15(4): 341-363.

Pasche, E. (2007). Flood modelling in Urban rivers -the stage of the art and where to go Advances in urban flood management. R. Ashleyet al. London, UK, Taylor & Frances Group: 499.

Peace, J. and R. N. Stavins (2010). Meaningful and cost effective climate policy: the case for cap and trade, Pew Center on Global Climate Change.

Pearson, C. P. (1991a). "New Zealand regional flood frequency analysis using L-moments." Journal of Hydrology (NZ) 30 (2) 53: 64.

Pearson, C. P. (1991b). "Regional flood frequency analysis for small New Zealand basins. II: Flood frequency groups." Journal of hydrology. New Zealand 30(2): 77-92.

Penning-Rowsell, E., C. Johnson, S. Tunstall, S. Tapsell, J. Morris, J. Chatterton, C. Green, T. Wilson, K. Koussela and A. Fernandez-Bilbao (2005). The benefits of flood and coastal risk management: a manual of assessment techniques.

Penning-Rowsell, E. C. and J. B. Chatterton (1977). The benefits of flood alleviation: A manual of assessment techniques. Gower, Aldershot, England.

Penning-Rowsell, E. C. and C. Green (2000). "New insights into the appraisal of flood-alleviation benefits: (1) Flood damage and flood loss information." Water and Environment Journal 14(5): 347-353.

Pepper, W., B. J. Ring, E. G. Read and S. R. Starkey (2012). Implementation of a Scheduling and Pricing Model for Natural Gas. Handbook of networks in power systems II. A. Sorokinet al.

Pereira, M. V. F. and L. M. V. G. Pinto (1985). "Stochastic optimization of a multireservoir hydroelectric system: a decomposition approach." Water Resour. Res. 21(6): 779-792, doi:710.1029/WR1021i1006p00779.

Pflug, G. C. (2000). Some remarks on the value-at-risk and the conditional value-at-risk. Probabilistic Constrained Optimization: Methodology and Applications. S. Uryasev. Dordrecht, The Netherlands, Kluwer Academic Publishers.

Philbrick Jr, C. R. and P. K. Kitanidis (1999). "Limitations of deterministic optimization applied to reservoir operations." Journal of Water Resources Planning and Management 125(3): 135-142.

Philpott, A. and G. Pritchard (2004). "Financial transmission rights in convex pool markets." Operations Research Letters 32(2): 109-113.

Piantadosi, J., A. V. Metcalfe and P. G. Howlett (2008). "Stochastic dynamic programming (SDP) with a conditional value-at-risk (CVaR) criterion for management of storm-water." Journal of Hydrology 348(3-4): 320-329.

Pinto, A., T. Cochrane and J. Raffensperger (2008a). Smart market and improved management of sediment discharge. Final Report Auckland, New Zealand, Auckland Regional Council.

Pinto, A., J. Raffensperger and T. Cochrane (2008b). A proposal smart market for sediment discharge. 43rd ORSNZ annual conference, Wellington, New Zealand, Victoria University.

Pinto, A., J. Raffensperger, T. Cochrane and E. G. Read (2012). "A proposed smart market design for sediment discharge." forthcoming Journal of Water Resources Planning and Management doi:10.1061/(ASCE)WR.1943-5452.0000228.

Plagmann, M. and J. Raffensperger (2007). A smart market for ground water using the eigenmodel approach. MODSIM Conference, Christchurch, NZ.

Plessis, L. D. (2001). "The generation and use of cumulative probability distributions in flood risk assessment for the Mfolozi flood-plain." Water SA 27(1): 27-34.

Prabodanie, R. (2010). A market mechanism for the optimal control of groundwater and surface water pollution from nitrates. Department of management. Christchurch, University of Canterbury. Ph.D.: 283.

Prabodanie, R. and J. Raffensperger (2007). Cleaning the water: a smart market for nitrates. MODSIM Conference, Christchurch NZ.

Prabodanie, R. and J. Raffensperger (2009). Can market in agricultural discharge permits be competitive. 44th Annual conference ORSNZ09, University of Canterbury, Christchurch, New Zealand.

Prabodanie, R., J. Raffensperger and M. Milke (2010). "A pollution offset system for trading non-point source water pollution permits." Environmental and Resource Economics 45(4): 499-515.

Pratt, J. W. (1964). "Risk aversion in the small and in the large." Econometrica 32(1/2): 122-136.

Pritchard, G., G. Zakeri and A. Philpott (2010). "A single-settlement, energy-only electric power market for unpredictable and intermittent participants " Operations Research 58(4): 1210-1219.

Purnell, R. (2002). "Flood risk: A government perspective." Civil Engineering 150(5): 10-14.

Raffensperger, J. and T. Cochrane (2010). "A smart market for impervious cover." Water Resources Management 24(12): 3065-3083.

Raffensperger, J., M. Milke and E. G. Read (2009). "A deterministic smart market model for ground water." Operation Research 57(6): 1333-1346.

Raffensperger, J. F. (2007). "The potential use of smart markets for allocation in New Zealand." Work Document Department of Management, University of Canterbury, New Zealand.

Raffensperger, J. F. (2011). "Matching users' rights to available ground water." forthcoming in Ecological Economics, 10.1016/j.ecolecon.2011.01.015.

Ramachandra Rao, A. and K. H. Hamed (2000). Flood frequency analysis, CRC Press, Boca Raton, USA.

Read, E. G. (2010). Co-optimization of energy and ancillary service markets: handbook of power systems I. P. M. Pardaloset al., Springer Berlin Heidelberg: 307-327.

Read, E. G. and D. Chattopadhyay (1999). Electricity market models: Lessons from Australasian experience. ORSNZ 34th Annual Conference, Waikato.

Read, E. G., G. R. Drayton-Bright and B. J. Ring (1998). An integrated energy and reserve market for New Zealand. Deregulation of Electric Utilities. G. Zaccours (ed). Kluwer, Boston: 297-319.

Ribaudo, M. O., R. D. Horan and M. E. Smith (1999). Economics of water quality protection from nonpoint sources: theory and practice. Agricultural Economics Report 728. Washington, DC, U.S. Department of Agriculture, Economic Research Service: 106.

Richards, J. (2007). The English planning system and flood risk management. Advances in urban flood management. R. Ashleyet al. London, UK, Taylor & Frances Group: 499.

Riley, J. G. and W. F. Samuelson (1981). "Optimal Auctions." The American Economic Review 71(3): 381-392.

Ring, B. J. (1995). Dispatch based pricing in decentralized power system. Department of management. Christchurch, New Zealand, University of Canterbury. Ph.D. thesis: 254.

Roche, K. M., K. J. McAneney and R. C. Van Den Honert (2010). "Policy options for managing flood insurance." Environmental Hazards 9(4): 369-378.

Rockafellar, R. T. and S. Uryasev (2000). "Optimization of conditional value-at-risk." Journal of Risk 2(3): 21-42.

Rockafellar, R. T. and S. Uryasev (2001). Conditional value-at-risk for general loss distribution. Gainesville, Center for applied optimization, University of Florida. 2: 34.

Rockafellar, R. T. and S. Uryasev (2002). "Conditional value-at-risk for general loss distributions." Journal of Banking and Finance 26(7): 1443-1471.

Rogers, G. O. and B. B. DeFee II (2005). "Long-term impact of development on a watershed: early indicators of future problems." Landscape and Urban Planning 73: 215-233.

Rothkopf, M. H. (2007). "Thirteen reasons why the Vickrey-Clarke-Groves process is not practical." Operations Research 55(2): 191-197.

Rothkopf, M. H., T. J. Teisberg and E. P. Kahn (1990). "Why are Vickrey auctions rare?" Journal of Political Economy 98(1): 94-109.

Sarykalin, S., G. Serraino and S. Uryasev (2008). "Value-at-Risk vs. Conditional Value-at-risk in risk management and optimization." 2008 Tutorials in Operations Research: State-of-the-Art Decision-Making Tools in the Information-Intensive Age.

Sayers, P. B., J. W. Hall and I. C. Meadowcroft (2002). "Towards risk-based flood hazard management in the UK." Civil Engineering 150(5): 36-42.

Schielen, R. M. J. (2009). Flood Management. Handbook of Catchment Management. R. C. F. a. A. Jenkins, Wiley-Blackwell: 51-76.

Schummer, J. and R. V. Vohra (2003). "Auctions for procuring options." Operations Research 51(1): 41-51.

Scott, H., J. DeAngulo, M. Daniels and L. Wood (1989). "Flood duration effects on soybean growth and yield." Agronomy Journal 81(4): 631-636.

SCS (1985). National engineering handbook. Soil conservation service, section 4-hydrology. Soil Conservation Service, SCS, Washington.

Sharpe, W. F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk." The Journal of Finance 19(3): 425-442.

Sheffrin, A. (2002). Predicting market power using the residual supply index. FERC Market Monitoring Workshop, December, www.caiso. com/docs/2002/12/05/2002120508555221628.pdf.

Sheffrin, A. Y., J. Chen and B. F. Hobbs (2004). "Watching watts to prevent abuse of power." Power and Energy Magazine, IEEE 2(4): 58-65.

Shortle, J. S. and R. D. Horan (2001). "The economics of nonpoint pollution control." J. Econ. Surv. 15(3): 255-289, doi: 210.1111/1467-6419.00140.

Shortle, J. S. and R. D. Horan (2008). "The economics of water quality trading." Int Rev Env Res Econ 2(2): 101-133, http:/dx.doi.org/110.1561/1101.00000014.

Singh, V. P. and W. G. Strupczewski (2002). "On the status of flood frequency analysis." Hydrological Processes 16(18): 3737-3740.

Smith, D. I. (1994). "Flood damage estimation - A review of urban stage-damage curves and loss functions." Water SA 20(3): 231-238.

Stavins, R. (2008). "A meaningful US cap-and-trade system to address climate change."

Stedinger, J. R. and V. W. Griffis (2008). "Flood frequency analysis in the United States: Time to update." Journal of Hydrologic Engineering 13(4): 199-204.

Stoft, S. (2002). Power system economics, designing markets for electricity. New York Wiley-Interscience and IEEE.

Strappazzon, L., A. Ha, M. Eigenraam, C. Duke and G. Stoneham (2003). "Efficiency of alternative property right allocations when farmers produce multiple environmental goods under the condition of economies of scope." Australian Journal of Agricultural and Resource Economics 47(1): 1-27.

Street, A. (2010). "On the Conditional Value-at-Risk probability-dependent utility function." Theory and Decision 68(1): 49-68.

Tang, Z., B. A. Engel, B. C. Pijanowski and K. J. Lim (2005). "Forecasting land use change and its environmental impact at a watershed scale." Journal of Environmental Management 76(1): 35-45.

Thieken, A. H., M. Müller, H. Kreibich and B. Merz (2005). "Flood damage and influencing factors: New insights from the August 2002 flood in Germany." Water Resources Research 41(12): W12430.

Thurston, H., M. Taylor, A. Roy, M. Morrison, W. Shuster, J. Templeton, M. Clagett and H. Cabezas (2008). "Applying a reverse auction to reduce storm water runoff." Ambio 37(4): 326.

Thurston, H. W., H. C. Goddard, D. Szlag and B. Lemberg (2003). "Controlling storm-water runoff with tradable allowances for impervious surfaces." Journal of Water Resources Planning and Management 129(5): 409-418.

Tilmant, A., D. Pinte and Q. Goor (2008). "Assessing marginal water values in multipurpose multireservoir system via stochastic programming." Water Resources Research 44(W12341, doi:10.1029/2008WR007024): 17.

Tineke De Jonge, M. K. and M. Hogeweg (1996). Modelling floods and damage assessment using GIS, International Association of Hydrological Sciences.

Transpower (2008). Wind generation investigation project: Effect of wind generation on reactive power contribution and dynamic voltage responses Investigation 9, WGIP: INV. S. Operator, Transpower New Zealand Ltd: 47p.

Treby, E. J., M. J. Clark and S. J. Priest (2006). "Confronting flood risk: Implications for insurance and risk transfer." Journal of Environmental Management 81(4): 351-359.

Trindade, A., S. Uryasev, A. Shapiro and G. Zrazhevsky (2007). "Financial prediction with constrained tail risk." Journal of Baking and Finance 31(11): 3524-3538.

Tung, Y.-K., B. C. Yen and C. S. Melching (2006). Hydrosystems engineering reliability assessment and risk analysis. New York, McGraw-Hill Professional.

Uryasev, S. (2000a). Conditional value-at-risk: optimization algorithms and applications. Computational Intelligence for Financial Engineering, 2000. (CIFEr) Proceedings of the IEEE/IAFE/INFORMS 2000 Conference on.

Uryasev, S. (2000b). Introduction to the theory of probabilistic functions and percentiles (value at risk). Probabilistic Constrained Optimization: Methodology and Applications. S. Uryasev. Dordrecht, The Netherlands, Kluwer Academic Publishers: 307 p.

Uryasev, S., U. A. Theiler and G. Serraino (2010). "Risk-return optimization with different risk-aggregation strategies." The journal of risk finance 11(2): 129-146.

USDA-SWAT (2008). Soil and water assessment tool (SWAT). Temple, Texas, USA, Grassland, Soil and Water Research Laboratory. Agricultural Research Service, United States Department of Agriculture (USDA)

USDA-WEPP (2008). The water erosion prediction project (WEPP). Washington, DC, Agricultural Research Service, United States Department of Agriculture (USDA); National Soil Erosion Research Lab & Purdue University.

van der Vlerk, M. H. (2002). On multiple simple recourse models. Research Report, University of Groningen, Research Institute SOM (Systems, Organisations and Management).

van der Vlerk, M. H. (2004). "Convex approximations for complete integer recourse models." Mathematical Programming 99(2): 297-310.

Varian, H. R. (1994). "A solution to the problem of externalities when agents are well-informed." The American Economic Review 84(5): 1278-1293.

Veerbeek, W. (2007). Flood induced indirect hazard loss estimation models. Advances in urban flood management. R. Ashleyet al. London, UK, Taylor & Frances Group: 499.

Vickrey, W. (1961). "Counterspeculation, Auctions, and Competitive Sealed Tenders." Journal of Finance 16(1): 8-37.

Von Neumann, J. and O. Morgenstern (1947). Theory of games and economic behavior. New Jersey, Princeton : Princeton University Press.

Walls, M. and V. McConnell (2004). Incentive-based land use policies and water quality in the Chesapeake Bay, Resources For the Future.

Walls, M. and V. McConnell (2007). Transfer of development: rights in U.S. communities. Washington D.C., Resources for the Future.

Walter, M. T., E. S. Brooks, D. K. McCool, L. G. King, M. Molnau and J. Boll (2005). "Process-based snowmelt modeling: does it require more input data than temperature-index modeling?" Journal of Hydrology 300(1): 65-75.

Westra, J. V., J. K. H. Zimmerman and B. Vondracek (2005). "Bioeconomic analysis of selected conservation practices on soil erosion and freshwater fisheries." Journal of the American Water Resources Association 41(2): 309.

Wets, R. (1998). Stochastic programs with chance constraints: generalized convexity and approximation issues. Generalized Convexity, Generalized Monotonicity: Recent Results. J.-P. Crouzeixet al., Kluwer: 61-74.

White, G. (1994). A perspective on reducing losses from natural hazards. Conferencia interamericana sobre reducción de los desastres naturales, Cartagena de Indias, Colombia.

White, G. F. (2011). "Decision or procrastination in floodplain management." Journal of Contemporary Water Research and Education 97(1): 11.

Wilson, R. (1979). "Auctions of shares." The Quarterly Journal of Economics 93(4): 675 - 689.

Woodward, R. T. and R. A. Kaiser (2002). "Market Structures for U.S. Water Quality Trading." Review of Agricultural Economics 24(2): 366-383.

Yamada, F., R. Kakimoto, M. Yamamoto, T. Fujimi and N. Tanaka (2010). "Implementation of community flood risk communication in Kumamoto, Japan." Journal of Advanced Transportation DOI: 10.1002/atr.119: n/a-n/a.

Yang, M. and E. G. Read (1999). A constructive dual DP for a reservoir model with correlation. WMRG-WP-99-01. Christchurch, Department of Management, University of Canterbury: 22.

Zheng, H. (2009). "Efficient frontier of utility and CVaR." Mathematical Methods of Operations Research 70(1): 129-148.

Appendix A

This appendix presents the dual formulations of the different market model. Models are deterministic, based on an extreme storm event. Det_MarketIC1 model accounts for participants that sell and buy, while a land area balance is kept. Additionally a primal formulation for a variation of the Det_MarketIC1 model is presented. The Det_MarketIC2 model considers participants bidding for the desired land conditions (changing IC allowance), while the model accounts explicitly for the changes in impacting flows for these changes across control points. The dual formulations for Sto_MarketIC1, Sto_MarketIC2 and Sto_MarketIC_Risk models are presented.

Dual formulation Det_MarketIC1 model

Minimize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\beta_{i,j,b}^{+}D_{i,j,b}^{\max}+\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\gamma_{i,j,b}^{+}S_{i,j,b}^{\max}-\sum_{i=1}^{N}\sum_{j=1}^{J}\mu_{i,j}C_{i,j}$

$$+\sum_{t=1}^{T}\sum_{k=1}^{K}\lambda_{k,t}\,L_k$$

Subject to

$\beta_{i,j,b}^{+}+\mu_{i,j}-\beta_{i,j,b}^{-}+\nu_i = P_{i,j,b}^{D}$, $\forall$ *i,j,b* : $qbuy_{i,j,b}$

$\gamma_{i,j,b}^{+}-\mu_{i,j}-\gamma_{i,j,b}^{-}-\nu_i = -P_{i,j,b}^{S}$, $\forall$ *i,j,b* : $qsell_{i,j,b}$

$-\mu_{i,j}+\sum_{k=1}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t-u+1}\,\lambda_{t,k} = 0$, $\forall$ *i,j* : $g_{i,j}$

$\beta_{i,j,b}^{+}$, $\beta_{i,j,b}^{-}$, $\gamma_{i,j,b}^{+}$, $\gamma_{i,j,b}^{-}$, $\lambda_{t,k}\geq 0$, $\mu_{i,j}$ (free)

Primal formulation Det_MarketIC1.1 model

Maximize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}P_{i,j,b}^{D}\;qbuy_{i,j,b}-\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}P_{i,j,b}^{S}\;qsell_{i,j,b}$

Subject to

$$0 \leq qbuy_{i,j,b} \leq D^{\max}_{i,j,b}, \ \forall \ i,j,b \qquad : \ \beta^{-}_{i,j,b}, \beta^{+}_{i,j,b}$$

$$0 \leq qsell_{i,j,b} \leq S^{\max}_{i,j,b}, \ \forall \ i,j,b \qquad : \ \gamma^{-}_{i,j,b}, \gamma^{+}_{i,j,b}$$

$$g_{i,j} = \sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{b=1}^{B} qsell_{i,j,b} + C_{i,j}, \ \forall \ i,j \qquad : \ \mu_{i,j} \ \text{(free)}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{J} H^{t-u+1}_{i,j,k} \, g_{i,j} \leq L^{t}_{k}, \ \forall \ t,k \qquad : \ \lambda_{t,k}$$

$$\sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{b=1}^{B} qsell_{i,j,b} \geq 0, \ \forall \ i \qquad : \ \nu^{+}_{i}$$ [12]

$$\sum_{b=1}^{B} qbuy_{i,j,b} - \sum_{b=1}^{B} qsell_{i,j,b} \leq 0, \ \forall \ i \qquad : \ \nu^{-}_{i}$$

$g_{i,j}$ free

Dual formulation Det_MarketIC1.1 model

Minimize: $$\sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{b=1}^{B} \beta^{+}_{i,j,b} D^{\max}_{i,j,b} + \sum_{i=1}^{N} \sum_{j=1}^{J} \sum_{b=1}^{B} \gamma^{+}_{i,j,b} S^{\max}_{i,j,b}$$

$$- \sum_{i=1}^{N} \sum_{j=1}^{J} \mu_{i,j} C_{i,j} + \sum_{t=1}^{T} \sum_{k=1}^{K} \lambda_{k,t} \, L_{k}$$

Subject to

$$\beta^{+}_{i,j,b} + \mu_{i,j} - (\nu^{+}_{i} - \nu^{-}_{i}) - \beta^{-}_{i,j,b} = P^{D}_{i,j,b}, \ \forall \ i,j,b \qquad : \ qbuy_{i,j,b}$$

$$\gamma^{+}_{i,j,b} - \mu_{i,j} - (\nu^{-}_{i} - \nu^{+}_{i}) - \gamma^{-}_{i,j,b} = -P^{S}_{i,j,b}, \ \forall \ i,j,b \qquad : \ qsell_{i,j,b}$$

$$-\mu_{i,j} + \sum_{k=1}^{K} \sum_{t=u}^{T} H^{t-u+1}_{i,j,k} \ \lambda_{t,k} = 0, \ \forall \ i,j \qquad : \ g_{i,j}$$

$$\beta^{+}_{i,j,b}, \ \beta^{-}_{i,j,b}, \ \gamma^{+}_{i,j,b}, \gamma^{-}_{i,j,b}, \ \nu^{+}_{i}, \nu^{-}_{i}, \lambda_{t,k} \geq 0, \ \mu_{i,j} \ \text{(free)}$$

Dual formulation Det_MarketIC1.2 model (Subcatchment with connected control points)

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[12] By definition ν^{+}_{i} has a negative dual value, but this dual will be represented in a canonical way with a positive sign and so $-\nu^{+}_{i} \geq 0$. Positive ν^{+}_{i} will be conveyed.

Minimize: $$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\beta^{+}_{i,j,b}D^{\max}_{i,j,b}+\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\gamma^{+}_{i,j,b}S^{\max}_{i,j,b}-\sum_{i=1}^{N}\sum_{j=1}^{J}\mu_{i,j}C_{i,j}$$
$$+\sum_{t=1}^{T}\sum_{k=1}^{K}\lambda_{k,t}\,L_k+\sum_{k=z}^{Z\in K}\sum_{t=1}^{T}\eta_{t,k}\bar{L}_k$$

Subject to

$$\beta^{+}_{i,j,b}+\mu_{i,j}-\beta^{-}_{i,j,b}+\nu_i=P^{D}_{i,j,b}\ ,\ \forall\ i,j,b \qquad :\ qbuy_{i,j,b}$$

$$\gamma^{+}_{i,j,b}-\mu_{i,j}-\gamma^{-}_{i,j,b}-\nu_i=-P^{S}_{i,j,b},\ \forall\ i,j,b \qquad :\ qsell_{i,j,b}$$

$$-\mu_{i,j}+\sum_{k=1}^{K}\sum_{t=u}^{T}H^{t-u+1}_{i,j,k}\ \lambda_{t,k}+\sum_{k=z}^{Z\in K}\sum_{t=s}^{T}\overline{H}^{t-s+1}_{i,j,k}\,\eta_{t,k}=0,\ \forall\ i,j \qquad :\ g_{i,j}$$

$$\beta^{+}_{i,j,b},\ \beta^{-}_{i,j,b},\ \gamma^{+}_{i,j,b},\ \gamma^{-}_{i,j,b}\geq 0,\ \mu_{i,j}\ \text{(free)}$$

Dual formulation Det_MarketIC2 model

Minimize: $$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\beta^{+}_{i,j,b}D^{\max}_{i,j,b}+\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\gamma^{+}_{i,j,b}S^{\max}_{i,j,b}+$$
$$\sum_{t=1}^{T}\sum_{k=1}^{K}\lambda_{k,t}\,L_k+\sum_{i=1}^{N}\sum_{j=1}^{J}\nu^{D}_{i,j}\,A^{0}_{i,j}+\sum_{i=1}^{N}\sum_{j=1}^{J}\nu^{S}_{i,j}\,A^{0}_{i,j}$$

Subject to

$$\beta^{+}_{i,j,b}+\mu^{D}_{i,j}-\beta^{-}_{i,j,b}+\nu^{D}_{i,j}=P^{D}_{i,j,b}\ ,\ \forall\ i,j,b \qquad :\ qbuy_{i,j,b}$$

$$\gamma^{+}_{i,j,b}+\mu^{S}_{i,j}-\gamma^{-}_{i,j,b}+\nu^{S}_{i,j}=-P^{S}_{i,j,b},\ \forall\ i,j,b \qquad :\ qsell_{i,j,b}$$

$$-\mu^{D}_{i,j}+\sum_{k=1}^{K}\sum_{t=u}^{T}H^{t-u+1}_{i,j,k}\,\lambda_{t,k}=0,\ \forall\ i,j \qquad :\ g^{D}_{i,j}$$

$$-\ \mu^{S}_{i,j}+\sum_{k=1}^{K}\sum_{t=u}^{T}H^{t-u+1}_{i,j,k}\,\lambda_{t,k}=0\forall\ i,j \qquad :\ g^{S}_{i,j}$$

$$\beta^{+}_{i,j,b},\ \beta^{-}_{i,j,b}\ ,\ \gamma^{+}_{i,j,b},\ \gamma^{-}_{i,j,b},\ \upsilon^{D}_{i,j},\ \upsilon^{S}_{i,j}\ \text{and}\ \lambda_{t,k}\geq 0\ ;\ \mu^{D}_{i,j},\mu^{S}_{i,j}\ \text{(free)}$$

Dual formulation Sto_MarketIC1 model

Minimize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\beta_{i,j,b}^{+}D_{i,j,b}^{\max}+\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\gamma_{i,j,b}^{+}S_{i,j,b}^{\max}+$

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\nu_{i,j}^{D}\,A_{i,j}^{0}+\sum_{i=1}^{N}\sum_{j=1}^{J}\nu_{i,j}^{S}\,A_{i,j}^{0}+\sum_{s=1}^{S}\phi^{s}\sum_{t=1}^{T}\sum_{k=1}^{K}\lambda_{t,k}^{s}M_{k}$$

Subject to

$$\beta_{i,j,b}^{+}+\mu_{i,j}^{D}-\beta_{i,j,b}^{-}+\nu_{i,j}^{D}=P_{i,j,b}^{D},\ \forall\ i,j,b \qquad :\ qbuy_{i,j,b}$$

$$\gamma_{i,j,b}^{+}+\mu_{i,j}^{S}-\gamma_{i,j,b}^{-}+\nu_{i,j}^{S}=-P_{i,j,b}^{S},\ \forall\ i,j,b \qquad :\ qsell_{i,j,b}$$

$$-\mu_{i,j}^{D}+\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t-u+1,s}\,\lambda_{t,k}^{s}=0,\ \forall\ i,j \qquad :\ g_{i,j}^{D}$$

$$-\mu_{i,j}^{S}+\sum_{s=1}^{S}\phi^{s}\sum_{k=1}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t-u+1,s}\,\lambda_{t,k}^{s}=0,\ \forall\ i,j \qquad :\ g_{i,j}^{S}$$

$$-\sum_{t=1}^{T}\phi^{s}\lambda_{k,t}^{s}-\theta_{k}^{s}=-\phi^{s}\frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}},\ \forall\ k,s \qquad :\ f_{k}^{s}$$

$\beta_{i,j,b}^{+}$, $\beta_{i,j,b}^{-}$, $\gamma_{i,j,b}^{+}$, $\gamma_{i,j,b}^{-}$, $\phi^{s}\lambda_{t,k}^{s}$, $\theta_{k}^{s}\geq 0$; $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ (free)

Dual formulation Sto_MarketICD2 model

Minimize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\beta_{i,j,b}^{+}D_{i,j,b}^{\max}+\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B}\gamma_{i,j,b}^{+}S_{i,j,b}^{\max}+\sum_{i=1}^{N}\sum_{j=1}^{J}\nu_{i,j}^{D}\,A_{i,j}^{0}$

$$+\sum_{i=1}^{N}\sum_{j=1}^{J}\nu_{i,j}^{S}\,A_{i,j}^{0}+\sum_{s=1}^{S}\phi^{s}\sum_{t=1}^{T}\sum_{k=1}^{K}\lambda_{t,k}^{s}M_{k}+\sum_{s}^{S}\phi^{s}\sum_{t=1}^{T}\sum_{k=1}^{K}\varphi_{k,t}^{s}\,M_{k}-$$

$$\sum_{s}^{S}\phi^{s}\sum_{t}^{T}\sum_{k}^{K}\chi_{k,t}^{s-}\ \Delta_{k,t}^{s-}+\sum_{s}^{S}\phi^{s}\sum_{t}^{T}\sum_{k}^{K}\chi_{k,t}^{s+}\ \Delta_{k,t}^{s+}$$

Subject to:

$$\beta_{i,j,b}^{+}+\mu_{i,j}^{D}-\beta_{i,j,b}^{-}+\nu_{i,j}^{D}=P_{i,j,b}^{D},\ \forall\ i,j,b \qquad :\ qbuy_{i,j,b}$$

$$\gamma_{i,j,b}^{+}+\mu_{i,j}^{S}-\gamma_{i,j,b}^{-}+\nu_{i,j}^{S}=-P_{i,j,b}^{S},\ \forall\ i,j,b \qquad :\ qsell_{i,j,b}$$

$$-\mu_{i,j}^{D}+\sum_{s}^{S}\phi^{s}\sum_{k}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t-u+1,s}\lambda_{k,t}^{s}+\sum_{s}^{S}\phi^{s}\sum_{k}^{K}\sum_{t=u}^{T}H_{i,j,k}^{t-u+1,s}\varphi_{k,t}^{s}=0,$$

$$\forall\ i,j \qquad :\ g_{i,j}^{D}$$

$$-\ \mu_{i,j}^{S} + \sum_{s}^{S} \phi^{s} \sum_{k}^{K} \sum_{t=u}^{T} H_{i,j,k}^{t-u+1,s} \lambda_{k,t}^{s} + \sum_{s}^{S} \phi^{s} \sum_{k}^{K} \sum_{t=u}^{T} H_{i,j,k}^{t-u+1,s} \varphi_{k,t}^{s} = 0, \quad \forall\ i,j \qquad :\ g_{i,j}^{S}$$

$$-\sum_{t=1}^{T} \phi^{s} \lambda_{k,t}^{s} - \theta_{k}^{s} = -\phi^{s} \frac{\partial C_{k}^{f}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}},\ \forall\ k,s \qquad :\ f_{k}^{s}$$

$$-\phi^{s} \varphi_{k,t}^{s} + \phi^{s} \chi_{k,t}^{s+} - \phi^{s} \chi_{k,t+1}^{s+} + \phi^{s} \chi_{k,t}^{s-} - \phi^{s} \chi_{k,t+1}^{s-} - \varpi_{k,t}^{s} = 0,\ \forall\ k,s,t \qquad :\ z_{k,t}^{s}$$

$$-\phi^{s} \chi_{k,t}^{s+} - \vartheta_{k,t}^{s+} = -\phi^{s} \frac{\partial\ C_{k}^{v+}\left(Vz_{k,t}^{s+}\right)}{\partial Vz_{k,t}^{s+}},\ \forall\ k,s,t \qquad :\ Vz_{k,t}^{s+}$$

$$-\phi^{s} \chi_{k,t}^{s-} - \vartheta_{k,t}^{s-} = -\phi^{s} \frac{\partial\ C_{k}^{v-}\left(Vz_{k,t}^{s-}\right)}{\partial Vz_{k,t}^{s-}},\ \forall\ k,s,t \qquad :\ Vz_{k,t}^{s-}$$

$\beta_{i,j,b}^{+}$, $\beta_{i,j,b}^{-}$, $\gamma_{i,j,b}^{+}$, $\gamma_{i,j,b}^{-}$, $\phi^{s}\lambda_{t,k}^{s}$, $\phi^{s}\varphi_{k,t}^{s}$, $\phi^{s}\chi_{k,t}^{s+}$, $\phi^{s}\chi_{k,t}^{s-}$ $\vartheta_{k,t}^{s+}$, $\vartheta_{k,t}^{s-}$ and $\theta_{k}^{s} \geq 0$; $\mu_{i,j}^{D}$ and $\mu_{i,j}^{S}$ (free)

Dual formulation Sto_MarketIC_Risk

Minimize: $\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} \beta_{i,j,b}^{+} D_{i,j,b}^{\max} + \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{b=1}^{B} \gamma_{i,j,b}^{+} S_{i,j,b}^{\max} +$

$$\sum_{i=1}^{N}\sum_{j=1}^{J} \nu_{i,j}^{D} A_{i,j}^{0} + \sum_{i=1}^{N}\sum_{j=1}^{J} \nu_{i,j}^{S} A_{i,j}^{0} + \sum_{s=1}^{S} \phi^{s} \sum_{t=1}^{T}\sum_{k=1}^{K} \lambda_{t,k}^{s} M_{k} + \sum_{k}^{K} W_{k}^{\alpha} \sigma_{k}^{\alpha}$$

$$+ \bar{W}_{\alpha} \bar{\sigma}^{\alpha}$$

Subject to

$$\beta_{i,j,b}^{+} + \mu_{i,j}^{D} - \beta_{i,j,b}^{-} + \nu_{i,j}^{D} = P_{i,j,b}^{D},\ \forall\ i,j,b \qquad :\ qbuy_{i,j,b}$$

$$\gamma_{i,j,b}^{+} + \mu_{i,j}^{S} - \gamma_{i,j,b}^{-} + \nu_{i,j}^{S} = -P_{i,j,b}^{S},\ \forall\ i,j,b \qquad :\ qsell_{i,j,b}$$

$$-\mu_{i,j}^{D} + \sum_{s}^{S} \phi^{s} \sum_{k}^{K} \sum_{t=u}^{T} H_{i,j,k}^{t-u+1,s} \ \lambda_{t,k}^{s} = 0,\ \forall\ i,j \qquad :\ g_{i,j}^{D}$$

$$-\mu_{i,j}^{S} + \sum_{s}^{S} \phi^{s} \sum_{k}^{K} \sum_{t=u}^{T} H_{i,j,k}^{t-u+1,s} \ \lambda_{t,k}^{s} = 0,\ \forall\ i,j \qquad :\ g_{i,j}^{S}$$

$$-\sum_{t=1}^{T} \phi^{s} \lambda_{k,t}^{s} + \phi^{s} \Phi_{k}^{s} \frac{\partial C_{k}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}} + \phi^{s} \bar{\Phi}^{s} \frac{\partial C_{k}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}} - \theta_{k}^{s} = -\phi^{s} \frac{\partial C_{k}\left(f_{k}^{s}\right)}{\partial f_{k}^{s}},\ \forall\ k,s \qquad :\ f_{k}^{s}$$

$$-\phi^s\Phi_k^s + \frac{\phi^s\sigma_k^\alpha}{1-\alpha} - \Theta_k^s = 0,\ \forall\ k,s \qquad :\upsilon_k^s$$

$$-\sum_s^S \phi^s\Phi_k^s + \sigma_k^\alpha = 0,\ \forall\ k \qquad :\ \eta_\alpha^k$$

$$-\phi^s\bar{\Phi}^s + \frac{\phi^s\bar{\sigma}^\alpha}{1-\alpha} - \bar{\Theta}^s = 0,\ \forall\ s \qquad :\ \bar{\upsilon}^s$$

$$-\sum_s^S \phi^s\Phi^s + \bar{\sigma}^\alpha = 0 \qquad :\ \bar{\eta}_\alpha$$

$\beta_{i,j,b}^{+}$, $\beta_{i,j,b}^{-}$, $\gamma_{i,j,b}^{+}$, $\gamma_{i,j,b}^{-}$, $\phi^s\lambda_{t,k}^s$, $\phi^s\Phi_k^s$, $\phi^s\bar{\Phi}^s$, σ_k^α, $\bar{\sigma}^\alpha$ and $\theta_k^s \geq 0$; $\mu_{i,j}^D$, and $\mu_{i,j}^S$ (free)

Appendix B

These sections extend the convexification introduced in Section 4.9.2 of Chapter 4, with discussions about the flood damage approximations and SOS2 method for the market model. These approximations affect the SO revenue and the final IC in the catchment. The first section presents the possible externality in design with a non-convex damage function, when the design uses one extreme storm scenario. Then, this section illustrates similar conditions with several scenarios. Similar assumptions to examples in Chapter 4 are taken about timeframe, competitiveness, and damage estimation. The second section introduces flood damage with lower, middle and upper bounds approximations. Prices, allocations, imperviousness and final revenue from the SO are discussed. The extra rents correspond to the net surplus that the SO could face for the approximation and also for the differences between changes in damage and the total payment for the changes in flood damage.

B.1 Linearised non-convex flood damage cost function, example

This section illustrates a piece-wise approximation to a hypothetical flood damage cost function (see Figure B- 1) for the section 4.10 in Chapter 4. The idea is to analyse the cost function of a non-uniform or topographical sectional shape and see its implication for trading.

The flooding area has two levees. The first protects up to 70 m^3/time, and then the second levee protects up to flows of 335 m^3/time (equivalent to exceeding peak flow 265 m^3/time). Figure B- 1 shows the flood damage cost related to the exceeding flows with both levees. The final ICs and storm scenarios shift the flood distribution and prices.

Ten participants are in the catchment with one control point. Each participant has 10 ha with different IC allowances, and they desire to change IC allowances in 9 ha each. For instance, participant 2 has 10 ha with meadows, and desires to increase to crops, which would raise peak flow by 2 m^3/time. Some participants desire to reduce IC and bid extremely high. Some participants desire to increase IC, but do not bid high enough, so their bids are not accepted. Table B- 1Table B- 1 shows the initial IC allowances from participants who desire to change, and the participant preferred prices and areas.

Table B- 1 Participants' preferences for example

Particip.	Initial IC	Initial trading area (ha)	Impact at control point	Option 1			Option 2			Option 3		
				ha	IC	$/ha	ha	IC	$/ha	Ha	IC	$/ha
1	Forest (F)	9	1	5	M	$8	3	M	$7	1	M	$6
2	Meadow (M)	9	1	5	Cr	-	3	Cr	-	1	Cr	-
3	Meadow (M)	9	1	5	Cn	$9	3	Cn	$7	1	Cn	$5
4	Meadow (M)	9	1	5	Cn	$10	3	Cn	$8	1	Cn	$6
5	Crop (Cr)	9	1	5	Cn	$11	3	Cn	$8	1	Cn	$7
6	Meadow (M)	9	1	5	F	$2	3	F	$8	1	F	$9
7	Concrete (Cn)	9	1	5	M	$4	3	M	$7	1	M	$10
8	Concrete (Cn)	9	1	5	Cr	$5	3	Cr	$8	1	Cr	$9
9	Crop (Cr)	9	1	5	M	$7	3	M	$10	1	M	$12
10	Meadow (M)	9	1	5	F	$1	3	F	$10	1	F	$15

Figure B- 1 depicts the flood damage from peak flows at the control point. The segmented red line represents a rough convexification of the damage function. The convexification ensures participants who desire to increase peak flows will face increasing prices, but prices and allocations could be far from the optimum.

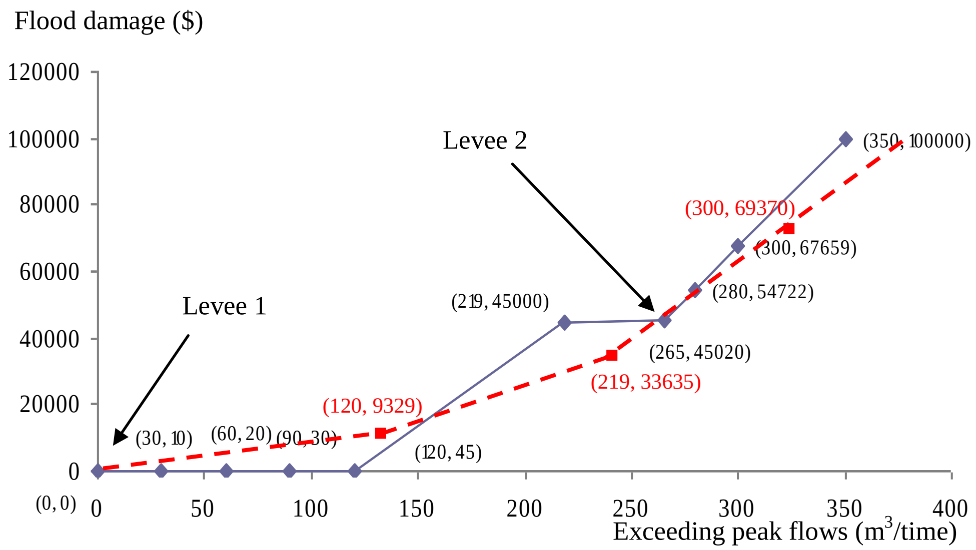

Figure B- 1 Hypothetical flood damage cost related to peak flow. Levee 1 limits up to 70 m^3/time, and levee 2 limits the exceeding peak flow up to 265 m^3/time.

Consider an extreme situation with a single storm scenario. The current flood condition under this scenario has an exceeding peak flow of 218 m^3/time at the flooding area with damage of $44,545.9. The market model includes SOS2 conditions.

Participant 2 might have strategic power if he/she has knowledge of the flood damage and flow levels. The participant could bid high enough to raise the exceeding peak flow above 219 m^3/time; but pay only $0.09, while flooding increased to 220.16 m^3/time and the damage increased by $454.4 to $45,000.3. The difference would be assumed by the SO or by society. This externality could be observed across several scenarios when flooding has similar conditions.

Extending modelling flood damage with SOS2 conditions, suppose the SO uses five storm scenarios, which all produce flood problems. The fifth scenario increases flood intensity, but does not change the stage-flood time. Probabilities of storm scenarios are 0.5, 0.3, 0.1, 0.07 and 0.03. The initial expected flood damage is $30,049.9. Assume participants have the preferences of the previous example, and participant 2 bids high enough to increase their IC and to change the flood distribution. At the end, participant 2 pays $903.8 and changes 9 ha to "crop". The expected flood damage increases by $949.2 and the difference between the payment and the increment in the expected damage, $45.4, would be assumed by the SO or society.

Modelling flood damage by convexification

The SO could convexify the flood damage as a way to force a revenue target. (Alternative convexification will be discussed in the next sections.) Convexifying the flood damage cost as illustrated in Figure B- 1, the estimated initial expected damage is $26,034.3 rather than $30,049.9. The SO could be aware of it, and may accept the differences in the estimated expected damage, but he/she may be more concerned about the change in damage rather than the estimated damage. Under this new damage approximation, if participant 2 offers a high price to increase imperviousness, after clearing the market, the final damage is $26,253.7 and the total payment to the SO is $632.6. Notice that without convexification and using the SOS2, the damage increases by $949.2.

B.2 Approximations to the flood costs function

Section 4.10 showed that convex approximations to the flood damage and linearisation with non-convex approximation produce different allocations, prices and SO revenue. In some cases, low, middle and upper bound convex approximations could produce final trade and prices that may encourage increasing or reducing the impervious levels in the catchment. A low bound approximation could signal participants to increase IC allowances, and upper bound to reduce them. The approximations also affect the SO extra rent in expectancy.

Figure B- 2 illustrates cost approximations with over and under damage estimation in two scenarios. Over and under estimation could correspond to upper and lower bound approximations in a storm scenario respectively. In scenario M1, assuming *A* is the real flood damage, and *A'* the estimated damage, any increase in IC would produce a marginal increase in runoff and damage. The clearing price will become $\lambda_A^{'}$ being $\lambda_A^{'} \geq \lambda_A$, and this price accounts for the marginal changes in flood damage. However, in underestimating damage such as *A''* (lower bound approximation), prices will be lower $\lambda_A^{''} \leq \lambda_A$ and coincide with a rent neutrality for the SO in the scenario.

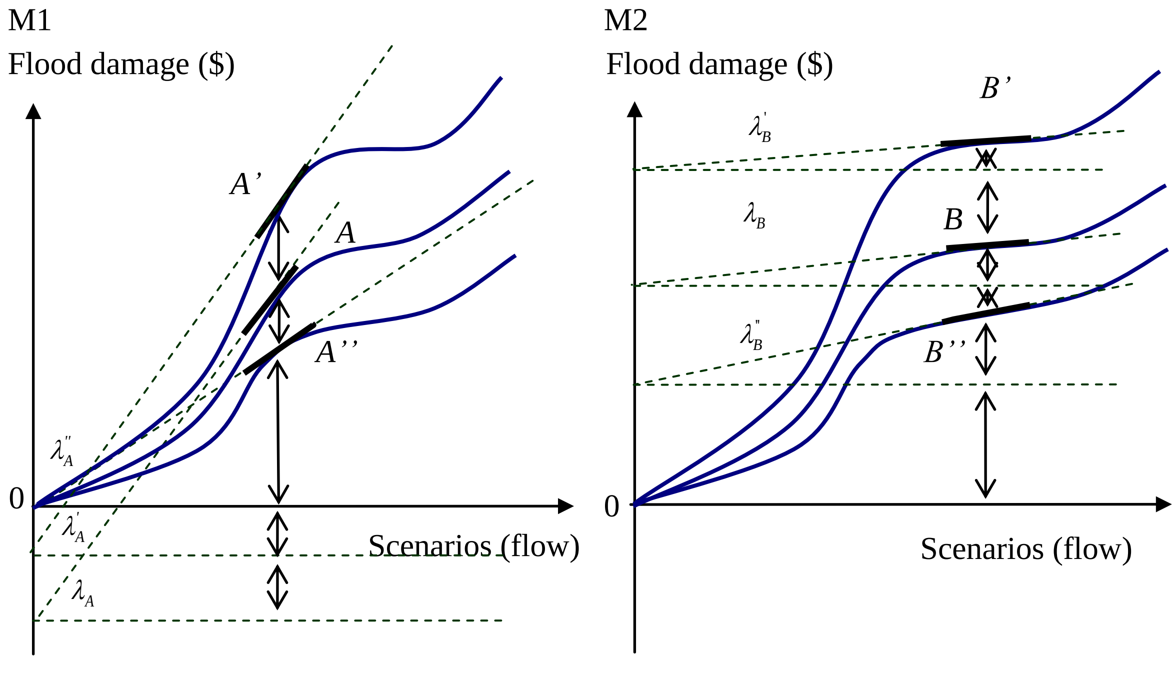

Figure B- 2 Estimated flood damage; M1 and M2 are flood damage approximations with two flooding levels.

In scenario M2, though the total damage *B'* is greater than the real damage *B*, the price $\lambda_B^{'}$ is lower than the marginal damage λ_B. In M2, the SO will be a net extra payer, and probably the final cost would not be paid by those participants who are producing the damage in this scenario. Approximation *B''* implies that marginal flood changes prices $\lambda_B^{''} \geq \lambda_B$. Paradoxically, the marginal damage (price) would be higher than the real marginal damage *B*, and so the SO could be a net payer in *B''* scenario. However, as was stated in previous chapters, the changes in expected damage could be convex.

Section B.2.1 illustrates a flood damage cost, where different convexifications produce different price effects and allocations; thus, the final IC in the catchment may be higher or lower at the end. Section B.2.2 extends linearised convexification.

B.2.1 Example flood damage approximations

This example considers a hypothetical small catchment with ten participants. Table B- 2 presents their preferences for increasing or reducing IC allowances. Each participant has 10 ha, but their demand and supply prices are different from the previous example (see Table B- 1), because we desire to stress effects of convexification, revenue and extra rents for the SO.

The market model uses nine storm scenarios, and the damage function is related to maximum peak. The flood area has three levees, with exceeding peak flows 110, 160 and 210 m^3/time. Figure B- 3 illustrates the damage function and the approximations (low, middle and upper bounds). Storm probabilities are 0.35, 0.2, 0.15, 0.2, 0.07, 0.05, 0.04, 0.03 and 0.01 and all these storm scenarios produce flood problems.

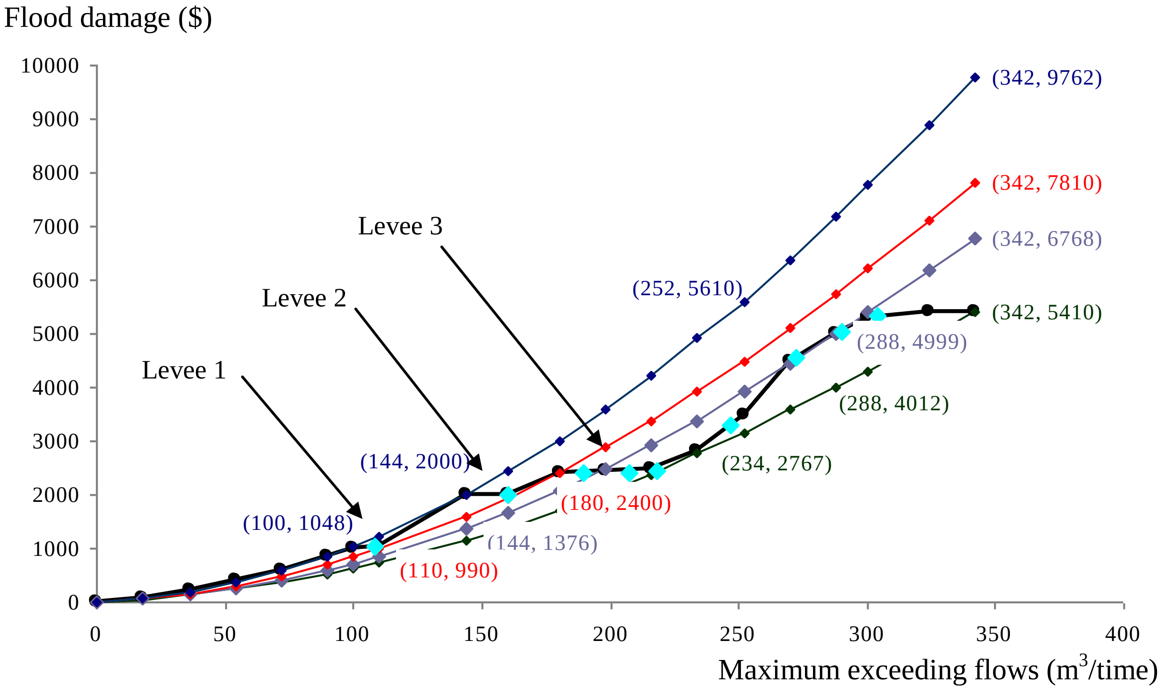

Figure B- 3 Flood damage cost and four approximations

We use the linearised and non-convex damage function and participant's preferences in Table B- 2. After clearing the market, the catchment becomes impervious and the flood distribution is shifted (Table B- 4 and Table B- 5). The expected flood damage rises by $59.11 (A-B). Figure B- 4 illustrates the different final flooding with the approximation.

Table B- 2 Participants' preferences

Particip.	Initial IC	Initial trading area (ha)	Impact at control point	Option 1			Option 2			Option 3		
				ha	IC	$/ha	ha	IC	$/ha	ha	IC	$/ha
1	Forest (F)	9	1	5	M	$6	3	M	$5	1	M	$4
2	Meadow (M)	9	1	5	Cr	$15	3	Cr	$12	1	Cr	$10
3	Meadow (M)	9	1	5	Cn	$15	3	Cn	$12	1	Cn	$8
4	Meadow (M)	9	1	5	Cn	$14	3	Cn	$12	1	Cn	$10
5	Crop (Cr)	9	1	5	Cn	$11	3	Cn	$8	1	Cn	$7
6	Meadow (M)	9	1	5	F	$10	3	F	$11	1	F	$12
7	Concrete (Cn)	9	1	5	M	$15	3	M	$16	1	M	$18
8	Concrete (Cn)	9	1	5	Cr	$12	3	Cr	$13	1	Cr	$15
9	Crop (Cr)	9	1	5	M	$7	3	M	$10	1	M	$12
10	Meadow (M)	9	1	5	F	$5	3	F	$10	1	F	$15

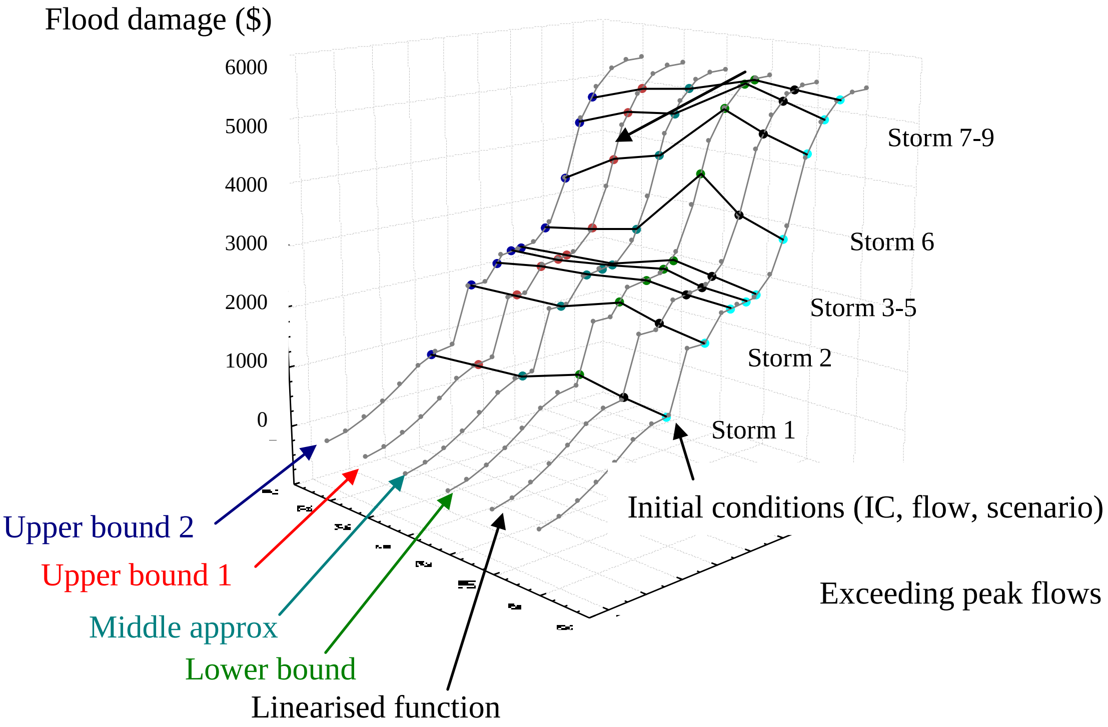

Figure B- 4 Flood damage and final damage after trading. Dotted colour points correspond to the final flood damage reached in a particular storm scenario after trading with the different approximations.

Using the lower bound approximation, the catchment becomes more impervious comparing to previous outcome's approximation (linearised). The expected damage increased by $187.45 with the approximation. The real damage rose by $186.34 (see Table B- 3). (The real damage corresponds to the damage estimates with the non-convex damage function under the final IC allowances.) With the upper approximation, the expected damage would be reduced by $423.62 and the equivalent expected damage (E) by only $154.97. However, the SO faces different revenue (C) and extra rent (D).

With the linearised non-convex damage function, the SO allows increasing IC allowances and receives $73.43, of which $14.31 is extra rent due to the linearisation.

With the lower bound approximation, the SO again allows increasing IC allowances and receives $192.715, of which $5.26 is extra rent due to the approximation.

With the middle approximations, the SO is a net payer, reduces IC allowances, and pays $126.26. Comparing to the equivalent real reducing damage, the equivalent real expected damage is reduced by only \$58.11. The SO receives an extra rent almost zero (\$0.65) due to the middle approximation.

With the upper bound 2 approximation, the SO reduces IC allowances and pays \$426.52, of which \$2.9 is extra rent payment given approximation. However, the estimated expected damage is reduced by \$423.62. In addition, the SO pays more than the equivalent real reducing damage by \$2,106.99 – \$1,952.01 = \$154.97.

Table B- 3 Expected damage, payment and extra rent of the SO

	Linearised flooding function (\$)	Lower bound approx (\$)	Middle approx (\$)	Upper bound 1 approx (\$)	Upper bound 2 approx (\$)
Initial expected damage	\$2,106.99 (A)	\$1,661.04	\$2,020.26	\$2,341.98	\$2,927.72
Final expected damage (B)	\$2,166.10	\$1,848.49	\$1,893.35	\$2,133.00	\$2,504.10
Difference (A-B)	\$59.11	\$187.45	-\$126.91	-\$208.98	-\$423.62
Payment SO (C)	-\$73.425	-\$192.715	\$126.264	\$201.797	\$426.523
Extra rent (D)	\$14.31	\$5.26	\$0.65	\$7.19	-\$2.90
Final equivalent real expected damage (E)		\$2,293.33	\$2,048.88	\$2,023.46	\$1,952.01
Difference (A-E)		\$186.34	-\$58.11	-\$83.53	-\$154.97

Table B- 4 Trading and final payment under different flood damage approximation and participant 2 demands with a low price

Part.	Initial area (ha)	Area to trade (ha)	Linearised damage			Lower bound approx			Middle approx			Upper bound approx 1			Upper bound approx 2		
			Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)
1	10	9	9	0	$33.58	9	0	$32.66	8	0	$33.47	8	0	$39.82	5	0	$28.78
2	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	10	9	2.67	0	$40.11	5	0	$72.79	0	0	0	0	0	0	0	0	0
4	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	10	9	0	0	0	5	0	$54.7	0	0	0	0	0	0	0	0	0
6	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	8	-$1.97
7	10	9	0	0	0	0	0	0	0	8	-$34.24	0	9	-$179.68	0	9	-$207.80
8	10	9	0	0	0	0	0	0	0	5	-$3.041	0	8.19	-$122.82	0	9	-$156.15
9	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	5	-$28.78
Total	100	90	11.67	0	$0	19	0	$0	8	13	0	8	17.1	0	5	31	-28.78

Table B- 5 Trading and final payment under different flood damage approximation and participant 2 demands with a high price

Part.	Initial area (ha)	Area to trade (ha)	Linearised damage			Lower bound approx			Middle approx			Upper bound approx 1			Upper bound approx 2		
			Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)	Area bought (ha)	Area sold (ha)	Payment ($)
1	10	9	9	0	$33.58	9	0	$32.66	8	0	$33.47	8	0	$37.34	4.10	0	$24.62
2	10	9	9	0	$33.48	9	0	$32.57	9	0	$37.55	9	0	$41.88	9	0	$53.79
3	10	9	0.42	0	$6.36	5	0	$72.79	0	0	0	0	0	0	0	0	0
4	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	10	9	0	0	0	5	0	$54.67	0	0	0	0	0	0	0	0	0
6	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	8	-$95.81
7	10	9	0	0	0	0	0	0	0	8	-$134.24	0	9	-$168.49	0	9	-$216.45
8	10	9	0	0	0	0	0	0	0	5	-$63.04	0	8	-$112.54	0	9	-$162.67
9	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	5	-$30.00
Total	100	90	18.42	0	$0	28	0	$0	17	13	0	17	17	0	13.1	31	-30

Consider the extreme storm events and their relation with the scenario selections in Figure B- 3. When a linearised but non-convex function is used to model flood damage, the damage almost does not change above flow level 300 m^3/time. For instance, in scenario 9, the peak flow reaches 303.75 m^3/time with an estimated flood damage \$6,031.2, and probable damage is 0.01*\$6,031.2 =\$60.31. The clearing value is \$0.0833/area m^3/time, almost zero in the scenario. Thus, any scenario more extreme than scenario 9 would not significantly change damage, so the marginal damage would be almost zero. Consequently, any probable marginal damage would be lower, and the dual would be lower and close to zero. So the SO should evaluate this condition to decide which storm scenarios should be included to reduce computational time and avoid possible issues with dimensionality.

B.2.2 Extending convexification

We can extend the convexification previously analysed to allow rent neutrality. To do this, we fit the linear damage function with lower and upper bounds, and a middle linear (average) approximation. The final revenue for the SO will still depend on the market strength. Figure B- 5 shows these damage function approximations for the previous example.

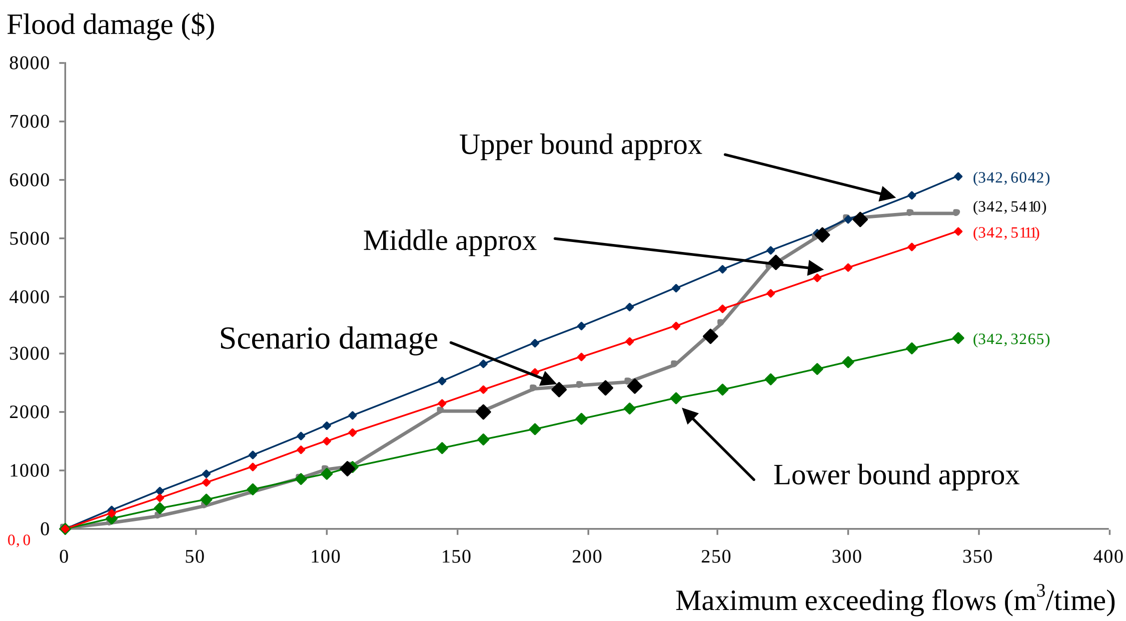

Figure B- 5 Lower, middle and upper linear flood damage cost approximation

The initial condition of expected damage is $2,106.99. With the approximations, the expected damages are $1,616.26, $2,530.35 and $2,991.38 for the lower, middle and upper linear approximations. Keeping assumptions about participants' preferences, catchment conditions and storm probabilities, a final trade is as follows:

- Lower bound approximation: Participants 1-5 change IC allowance level, each participant changes 9 ha and pays $17, $17, $68, $68 and $51 respectively. The final expected damage increases to $1,838.7 by $222.44 and the SO receives $222.44. The SO is a net receiver due to selling flood capacity at the control point and does not receive any extra rent for convexification. The final expected real damage is $2,547.5.

- Middle bound approximation: Participants 1-5 change IC allowance level in 9, 9, 8, 8 and 5 ha, paying $26.67, $26.62, $95.27, $95.27 and $44.75 respectively. The final expected damage increases to $2,818.95 by $288.6 and the SO receives $288.6. The SO is a net receiver, having sold flood capacity at the control point. The final expected real damage is $2,483.75.

- Upper bound approximation: Participants 1, 2, 3 and 5 change IC allowance in 9, 9, 5, and 5 ha and the payment is $31.53, $31.47, $70.4 and $52.91 respectively. The final expected damage increases to $3,177.7 by $186.31, and the SO receives $186.31. The SO is a net receiver, having sold flood capacity at the control point. The final expected real damage is $2,298.4.

Appendix C

This appendix presents the data used for the illustration in chapter 7. Any additional data could be asked to the author.

Appendix C.1 Impervious condition from participants

Table C- 1 Initial and desired conditions of participants A1 to A6 in the subcatchment A

Initial condition subcatchment	A1	A2	A3	A4	A5	A6
Area (ha)	167.39	167.39	167.39	167.39	167.39	167.39
IC (*CN*)	85	85	85	80	74	74
Hydraulic length (m)	1293.77	1293.77	1293.77	1293.77	1293.77	1293.77
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	65	65	65	65	9	9
SCS Lag Time (hrs)	14.28	14.28	14.28	16.85	20.13	20.13

Desired condition subcatchment	A1	A2	A3	A4	A5	A6
Area (ha)	167.39	167.39	167.39	167.39	167.39	167.39
IC (*CN*)	80	80	80	74	80	80
Hydraulic length (m)	1293.77	1293.77	1293.77	1293.77	1293.77	1293.77
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	60	60	60	60	9	9
SCS Lag Time (hrs)	16.85	16.85	16.85	20.13	16.85	16.85

Table C- 2 Initial and desired conditions of participants B1 to B6 in the subcatchment B

Initial condition subcatchment	B1	B2	B3	B4	B5	B6
Area (ha)	83.33	83.33	83.33	83.33	83.33	83.33
IC (*CN*)	78	82	75	78	80	80
Hydraulic length (m)	912.87	912.87	912.87	912.87	912.87	912.87
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	10.5	10.5	10.5	10.5	10.5	10.5
SCS Lag Time (hrs)	13.55	11.96	14.80	13.55	12.74	12.74

Desired condition subcatchment	B1	B2	B3	B4	B5	B6
Area (ha)	83.33	83.33	83.33	83.33	83.33	83.33
IC (*CN*)	74	80	74	70	85	85
Hydraulic length (m)	912.87	912.87	912.87	912.87	912.87	912.87
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	10.5	10.5	10.5	10.5	10.5	10.5
SCS Lag Time (hrs)	15.23	12.74	15.23	17.01	10.81	10.81

Table C- 3 Initial and desired conditions of participants C1 to C6 in the subcatchment C

Initial condition subcatchment	C1	C2	C3	C4	C5	C6
Area (ha)	514.99	514.99	514.99	514.99	514.99	514.99
IC (*CN*)	53	78	53	53	74	74
Hydraulic length (m)	2269.34	2269.34	2269.34	2269.34	2269.34	2269.34
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	5	9	5	5	9	9
SCS Lag Time (hrs)	54.55	28.08	54.55	54.55	31.55	31.55

Desired condition subcatchment	C1	C2	C3	C4	C5	C6
Area (ha)	514.99	514.99	514.99	514.99	514.99	514.99
IC (*CN*)	60	60	60	60	60	50
Hydraulic length (m)	2269.34	2269.34	2269.34	2269.34	2269.34	2269.34
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	5	9	5	5	9	9
SCS Lag Time (hrs)	45.72	45.72	45.72	45.72	45.72	58.86

Table C- 4 Initial and desired conditions of participants D1 to D6 in the subcatchment D

Initial condition subcatchment	D1	D2	D3	D4	D5	D6
Area (ha)	612.83	612.83	612.83	612.83	612.83	612.83
IC (*CN*)	78	53	53	47	47	78
Hydraulic length (m)	2475.55	2475.55	2475.55	2475.55	2475.55	2475.55
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	65	5	5	5	5	65
SCS Lag Time (hrs)	30.10	58.49	58.49	68.15	68.15	30.10

Final condition subcatchment	D1	D2	D3	D4	D5	D6
Area (ha)	612.83	612.83	612.83	612.83	612.83	612.83
IC (*CN*)	80	53	70	53	60	60
Hydraulic length (m)	2475.55	1984.30	3426.40	2475.55	3087.23	1787.16
Av. Slope (m/m)	0.01	0.01	0.01	0.01	0.01	0.01
Imperviousness (%)	65	5	5	5	5	65
SCS Lag Time (hrs)	28.31	49	49	58.49	58.48	37.77

Appendix C.2

This section presents the node points for the flood damage cost approximations at control points. The cost is related to the exceeding peak flows at control points. The linear convexified cost function corresponds to an approximation to the total damage function rather than approximations for each scenario.

Control point A

Linear non-convex		Linear convexified	
Node (flow)	Damage ($/m^3/sec)	Node (flow)	Damage ($/m^3/sec)
0	0	0	0
5	9,643.8	5	5,977.4
10	30,369.1	10	17,932.1
15	60,645.4	15	35,864.2
20	120,434.0	20	59,773.6
25	156,664.0	25	89,660.4
30	171,822.0	30	125,525.0
35	195,660.0	35	167,366.0
40	194,654.0	40	215,185.0
45	196,807.0	45	268,981.0
46.5	197,136.0	46.5	286,913.0

Control point AB

Linear non-convex		Linear convexified	
Node (flow)	Damage ($/m^3/sec)	Node (flow)	Damage ($/m^3/sec)
0	0	0	0
5	5,415.4	5	4,659.0
10	19,564.0	10	18,196.4
15	47,366.2	15	55,833.7
20	95,628.5	20	112,912.0
25	178,890.0	25	189,431.0
30	290,865.0	30	285,391.0
35	455,820.0	35	400,792.0
40	511,314.0	40	535,634.0
42	528,914.0	42	601,235.0

Control point ABC

Linear non-convex		Linear convexified	
Node (flow)	Damage ($/m^3/sec)	Node (flow)	Damage ($/m^3/sec)
0	0	0	0
5	6,261.8	5	4,294.5
10	17,257.1	10	12,883.5
15	33,503.8	15	25,767.0
20	55,089.7	20	42,945.1
25	91,664.2	25	64,417.6
30	110,024.0	30	90,184.6
35	117,303.0	35	120,246.0
40	129,951.0	40	154,602.0
42	135,292.0	42	170,921.0

Control point D

Linear non-convex		Linear convexified	
Node (flow)	Damage ($/m^3/sec)	Node (flow)	Damage ($/m^3/sec)
0	0	0	0
5	1,300.9	5	3,397.1
10	3,902.6	10	7,866.9
15	7,805.1	15	13,868.8
20	13,008.5	20	20,445.9
25	19,512.7	25	29,224.1
30	27,317.8	30	34,575.3
35	36,423.8	35	38,161.2
40	46,830.6	40	42,523.1
45	58,538.2	45	47,111.2
45.5	59,839.0	45.5	47,631.0

Control point ABCD

Linear non-convex		Linear convexified	
Node (flow)	Damage ($/m^3/sec)	Node (flow)	Damage ($/m^3/sec)
0	0	0	0
5	2,007.9	5	4,328.7
10	6,023.8	10	11,290.8
15	12,047.5	15	20,219.8
20	20,079.2	20	30,941.0
25	30,118.8	25	46,405.0
30	42,166.3	30	50,971.1
35	56,221.8	35	54,659.8
40	72,285.2	40	59,289.1
42.5	81,521.6	42.5	61,817.0

Appendix C.3 Flood damage related to peak flow at control points

Control point A		Control point AB		Control point ABC		Control point D		Control point ABCD	
Flow (m^3)	Damage ($/m^3/sec)	Flow (m^3)	Damage ($/m^3/sec)	Flow (m^3)	Damage ($/m^3/sec)	Flow (m^3)	Damage ($/m^3/sec)	Flow (m^3)	Damage ($/m^3/sec)
0	0	0	0	0	0	0	0	0	0
0.5	0	0.5	0	0.5	0	0.5	0	0.5	0
1	0	1	0	1	0	1	0	1	0
1.5	0	1.5	0	1.5	0	1.5	0	1.5	0
2	0	2	0	2	0	2	0	2	0
2.5	0	2.5	0	2.5	0	2.5	0	2.5	0
3	0	3	0	3	0	3	0	3	0
3.5	0	3.5	0	3.5	0	3.5	0	3.5	0
4	359	4	0	4	0	4	0	4	0
4.5	1,126	4.5	0	4.5	0	4.5	0	4.5	0
5	1,938	5	0	5	0	5	314	5	0
5.5	2,842	5.5	0	5.5	0	5.5	631	5.5	0
6	3,859	6	0	6	0	6	936	6	0
6.5	5,008	6.5	0	6.5	0	6.5	1,246	6.5	0
7	6,310	7	0	7	0	7	1,482	7	0
7.5	7,828	7.5	0	7.5	0	7.5	1,994	7.5	0
8	9,326	8	0	8	0	8	2,377	8	161
8.5	10,861	8.5	488	8.5	518	8.5	2,756	8.5	724
9	12,454	9	947	9	1,136	9	3,137	9	1,192
9.5	14,119	9.5	1,422	9.5	1,504	9.5	3,523	9.5	1,644
10	15,863	10	1,931	10	2,315	10	3,916	10	2,097
10.5	17,696	10.5	2,485	10.5	2,928	10.5	4,317	10.5	2,559
11	19,622	11	3,089	11	3,569	11	4,728	11	3,035
11.5	21,649	11.5	3,749	11.5	4,242	11.5	5,148	11.5	3,526
12	23,782	12	4,471	12	4,952	12	5,580	12	4,036
12.5	26,027	12.5	5,261	12.5	5,749	12.5	6,023	12.5	4,567

13	28,390	13	6,124	13	6,963	13	6,479	13	5,118
13.5	30,876	13.5	7,066	13.5	7,403	13.5	6,947	13.5	5,693
14	33,491	14	8,092	14	8,420	14	7,429	14	6,293
14.5	36,242	14.5	9,210	14.5	9,457	14.5	7,963	14.5	6,918
15	39,134	15	10,426	15	10,523	15	8,507	15	7,669
15.5	42,173	15.5	11,746	15.5	11,622	15.5	9,090	15.5	8,420
16	45,367	16	13,180	16	12,758	16	9,672	16	9,172
16.5	48,721	16.5	15,496	16.5	13,934	16.5	10,256	16.5	9,933
17	52,243	17	19,086	17	15,152	17	10,845	17	10,704
17.5	55,939	17.5	18,239	17.5	16,414	17.5	11,439	17.5	11,489
18	59,817	18	20,289	18	17,724	18	12,040	18	12,289
18.5	63,884	18.5	22,509	18.5	19,082	18.5	12,648	18.5	13,104
19	68,147	19	24,845	19	20,491	19	13,265	19	13,937
19.5	72,616	19.5	27,312	19.5	21,952	19.5	13,890	19.5	14,788
20	77,299	20	29,919	20	23,468	20	14,524	20	15,659
20.5	82,202	20.5	32,676	20.5	25,040	20.5	15,168	20.5	16,549
21	87,336	21	35,593	21	26,671	21	15,822	21	17,460
21.5	92,709	21.5	38,677	21.5	28,362	21.5	16,486	21.5	18,392
22	98,332	22	41,939	22	30,115	22	17,161	22	19,346
22.5	104,212	22.5	45,388	22.5	31,934	22.5	17,847	22.5	20,323
23	110,361	23	49,033	23	33,818	23	18,544	23	21,324
23.5	116,791	23.5	52,884	23.5	35,771	23.5	19,253	23.5	22,348
24	123,508	24	56,952	24	37,795	24	19,974	24	23,397
24.5	130,526	24.5	61,246	24.5	39,891	24.5	20,707	24.5	24,471
25	137,855	25	65,779	25	42,063	25	21,452	25	25,571
25.5	144,602	25.5	70,560	25.5	44,312	25.5	22,210	25.5	26,697
26	145,890	26	75,602	26	46,641	26	22,979	26	27,850
26.5	147,233	26.5	80,918	26.5	49,052	26.5	23,765	26.5	29,032
27	148,634	27	86,519	27	51,547	27	24,563	27	30,241
27.5	150,096	27.5	92,420	27.5	54,129	27.5	25,374	27.5	31,480
28	151,621	28	98,634	28	56,801	28	26,199	28	32,748
28.5	153,209	28.5	105,176	28.5	59,565	28.5	27,041	28.5	34,046
29	154,865	29	112,058	29	62,424	29	27,904	29	35,374
29.5	156,591	29.5	119,300	29.5	65,380	29.5	28,766	29.5	36,735
30	158,389	30	126,915	30	68,437	30	29,629	30	38,128
30.5	160,262	30.5	134,919	30.5	71,597	30.5	31,207	30.5	39,553
31	162,232	31	143,333	31	74,862	31	31,593	31	41,011
31.5	164,260	31.5	152,171	31.5	78,238	31.5	31,978	31.5	42,504
32	166,323	32	164,187	32	81,724	32	32,361	32	44,031
32.5	168,430	32.5	171,136	32.5	85,326	32.5	32,744	32.5	47,236
33	170,586	33	180,980	33	89,046	33	33,127	33	47,581
33.5	172,793	33.5	191,051	33.5	92,889	33.5	33,512	33.5	47,927
34	175,055	34	201,390	34	100,018	34	33,897	34	48,273
34.5	177,374	34.5	212,027	34.5	102,204	34.5	34,284	34.5	48,623
35	179,752	35	222,975	35	103,029	35	34,672	35	48,975
35.5	182,191	35.5	234,256	35.5	103,858	35.5	35,062	35.5	49,330
36	184,694	36	245,890	36	104,697	36	35,454	36	49,689
36.5	187,262	36.5	257,881	36.5	105,547	36.5	35,848	36.5	50,052
37	189,897	37	270,253	37	106,409	37	36,245	37	50,419
37.5	193,778	37.5	283,012	37.5	107,285	37.5	36,644	37.5	50,790
38	193,876	38	296,171	38	108,176	38	37,045	38	51,166

38.5	193,978	38.5	309,750	38.5	109,083	38.5	37,449	38.5	51,546
39	194,082	39	323,758	39	110,005	39	37,856	39	51,931
39.5	194,188	39.5	338,213	39.5	110,944	39.5	38,266	39.5	52,321
40	194,298	40	353,118	40	111,900	40	38,678	40	52,716
40.5	194,410	40.5	368,493	40.5	112,873	40.5	39,094	40.5	53,116
41	194,525	41	384,359	41	113,865	41	39,512	41	53,521
41.5	194,644	41.5	400,713	41.5	114,876	41.5	39,933	41.5	53,931
42	194,765	42	417,587	42	115,905	42	40,358	42	54,347
42.5	194,889	42.5	434,980	42.5	116,953	42.5	40,786	42.5	54,769
43	195,017	43	452,926	43	118,022	43	41,217	43	55,196
43.5	195,148	43.5	471,203	43.5	119,110	43.5	41,652	43.5	55,628
44	195,282	44	474,693	44	120,220	44	42,090	44	56,067
44.5	195,420	44.5	478,321	44.5	121,351	44.5	42,532	44.5	56,511
45	195,561	45	482,090	45	122,502	45	42,976	45	56,962
45.5	195,706	45.5	486,010	45.5	123,676	45.5	43,425	45.5	57,419
46	195,854	46	490,081	46	124,872	46	43,877	46	57,881
46.5	196,006	46.5	494,311	46.5	126,091	46.5	44,333	46.5	58,350
47	196,162	47	498,706	47	127,333	47	44,793	47	58,826
47.5	196,322	47.5	503,269	47.5	128,597	47.5	45,256	47.5	59,308
48	196,486	48	508,008	48	129,888	48	45,723	48	59,796
48.5	196,654	48.5	512,929	48.5	131,202	48.5	46,194	48.5	60,291
49	196,826	49	518,031	49	132,541	49	46,670	49	60,793
49.5	197,002	49.5	523,331	49.5	133,904	49.5	47,148	49.5	61,301
50	197,183	50	528,828	50	135,294	50	47,631	50	61,817

Appendix C.4 Peak flow and depth levels at different sections at control points

Nomenclature uses in the tables.

ht = Depth at the centre of the channel (m).

hc1 = Bank depth at the right side of the channel (m).

hc2 = Bank depth at the left side of the channel (m).

hl1 = Depth at the left side in the first flood section (m).

hl2 = Depth at the left side in the second flood section (m).

hl3 = Depth at the left side in the third flood section (m).

hr1 = Depth at the right side in the first flood section (m).

hr2 = Depth at the right side in the second flood section (m).

hr3 = Depth at the right side in the third flood section (m).

Control point A: peak flow and flood depth at sections

Flow (m^3)	ht (m)	hc1 (m)	hc2 (m)	hl1 (m)	hr1 (m)	hl2 (m)	hr2 (m)	hl3 (m)	hr3 (m)
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.255	0.255	0.255	0.000	0.000	0.000	0.000	0.000	0.000

1	0.390	0.390	0.390	0.000	0.000	0.000	0.000	0.000	0.000
1.5	0.501	0.501	0.501	0.000	0.000	0.000	0.000	0.000	0.000
2	0.600	0.600	0.600	0.000	0.000	0.000	0.000	0.000	0.000
2.5	0.691	0.691	0.691	0.000	0.000	0.000	0.000	0.000	0.000
3	0.775	0.775	0.775	0.000	0.000	0.000	0.000	0.000	0.000
3.5	0.855	0.855	0.855	0.000	0.000	0.000	0.000	0.000	0.000
4	0.920	0.920	0.900	0.000	0.020	0.000	0.000	0.000	0.000
4.5	0.951	0.951	0.900	0.000	0.051	0.000	0.000	0.000	0.000
5	0.975	0.975	0.900	0.000	0.075	0.000	0.000	0.000	0.000
5.5	0.997	0.997	0.900	0.000	0.097	0.000	0.000	0.000	0.000
6	1.016	1.016	0.900	0.000	0.116	0.000	0.000	0.000	0.000
6.5	1.033	1.033	0.900	0.000	0.133	0.000	0.000	0.000	0.000
7	1.050	1.050	0.900	0.000	0.150	0.000	0.000	0.000	0.000
7.5	1.064	1.064	0.900	0.000	0.164	0.000	0.014	0.000	0.000
8	1.076	1.076	0.900	0.000	0.176	0.000	0.026	0.000	0.000
8.5	1.086	1.086	0.900	0.000	0.186	0.000	0.036	0.000	0.000
9	1.096	1.096	0.900	0.000	0.196	0.000	0.046	0.000	0.000
9.5	1.105	1.105	0.900	0.000	0.205	0.000	0.055	0.000	0.000
10	1.114	1.114	0.900	0.000	0.214	0.000	0.064	0.000	0.000
10.5	1.123	1.123	0.900	0.000	0.223	0.000	0.073	0.000	0.000
11	1.131	1.131	0.900	0.000	0.231	0.000	0.081	0.000	0.000
11.5	1.139	1.139	0.900	0.000	0.239	0.000	0.089	0.000	0.000
12	1.146	1.146	0.900	0.000	0.246	0.000	0.096	0.000	0.000
12.5	1.153	1.153	0.900	0.000	0.253	0.000	0.103	0.000	0.000
13	1.160	1.160	0.900	0.000	0.260	0.000	0.110	0.000	0.000
13.5	1.167	1.167	0.900	0.000	0.267	0.000	0.117	0.000	0.000
14	1.174	1.174	0.900	0.000	0.274	0.000	0.124	0.000	0.000
14.5	1.181	1.181	0.900	0.000	0.281	0.000	0.131	0.000	0.000
15	1.187	1.187	0.900	0.000	0.287	0.000	0.137	0.000	0.000
15.5	1.194	1.194	0.900	0.000	0.294	0.000	0.144	0.000	0.000
16	1.200	1.200	0.900	0.000	0.300	0.000	0.150	0.000	0.000
16.5	1.206	1.206	0.900	0.000	0.306	0.000	0.156	0.000	0.000
17	1.212	1.212	0.900	0.000	0.312	0.000	0.162	0.000	0.000
17.5	1.218	1.218	0.900	0.000	0.318	0.000	0.168	0.000	0.000
18	1.224	1.224	0.900	0.000	0.324	0.000	0.174	0.000	0.000
18.5	1.229	1.229	0.900	0.000	0.329	0.000	0.179	0.000	0.000
19	1.235	1.235	0.900	0.000	0.335	0.000	0.185	0.000	0.000
19.5	1.240	1.240	0.900	0.000	0.340	0.000	0.190	0.000	0.000
20	1.246	1.246	0.900	0.000	0.346	0.000	0.196	0.000	0.000
20.5	1.251	1.251	0.900	0.000	0.351	0.000	0.201	0.000	0.000
21	1.257	1.257	0.900	0.000	0.357	0.000	0.207	0.000	0.000
21.5	1.262	1.262	0.900	0.000	0.362	0.000	0.212	0.000	0.000
22	1.267	1.267	0.900	0.000	0.367	0.000	0.217	0.000	0.000
22.5	1.272	1.272	0.900	0.000	0.372	0.000	0.222	0.000	0.000
23	1.277	1.277	0.900	0.000	0.377	0.000	0.227	0.000	0.000
23.5	1.282	1.282	0.900	0.000	0.382	0.000	0.232	0.000	0.000
24	1.287	1.287	0.900	0.000	0.387	0.000	0.237	0.000	0.000
24.5	1.292	1.292	0.900	0.000	0.392	0.000	0.242	0.000	0.000
25	1.297	1.297	0.900	0.000	0.397	0.000	0.247	0.000	0.000
25.5	1.302	1.302	0.900	0.000	0.402	0.000	0.252	0.000	0.000
26	1.307	1.307	0.900	0.000	0.407	0.000	0.257	0.000	0.000
26.5	1.311	1.311	0.900	0.000	0.411	0.000	0.261	0.000	0.000
27	1.316	1.316	0.900	0.000	0.416	0.000	0.266	0.000	0.000
27.5	1.321	1.321	0.900	0.000	0.421	0.000	0.271	0.000	0.000
28	1.325	1.325	0.900	0.000	0.425	0.000	0.275	0.000	0.000
28.5	1.330	1.330	0.900	0.000	0.430	0.000	0.280	0.000	0.000

29	1.334	1.334	0.900	0.000	0.434	0.000	0.284	0.000	0.000
29.5	1.339	1.339	0.900	0.000	0.439	0.000	0.289	0.000	0.000
30	1.343	1.343	0.900	0.000	0.443	0.000	0.293	0.000	0.000
30.5	1.348	1.348	0.900	0.000	0.448	0.000	0.298	0.000	0.000
31	1.352	1.352	0.900	0.000	0.452	0.000	0.302	0.000	0.002
31.5	1.356	1.356	0.900	0.000	0.456	0.000	0.306	0.000	0.006
32	1.360	1.360	0.900	0.000	0.460	0.000	0.310	0.000	0.010
32.5	1.364	1.364	0.900	0.000	0.464	0.000	0.314	0.000	0.014
33	1.368	1.368	0.900	0.000	0.468	0.000	0.318	0.000	0.018
33.5	1.372	1.372	0.900	0.000	0.472	0.000	0.322	0.000	0.022
34	1.376	1.376	0.900	0.000	0.476	0.000	0.326	0.000	0.026
34.5	1.379	1.379	0.900	0.000	0.479	0.000	0.329	0.000	0.029
35	1.383	1.383	0.900	0.000	0.483	0.000	0.333	0.000	0.033
35.5	1.387	1.387	0.900	0.000	0.487	0.000	0.337	0.000	0.037
36	1.390	1.390	0.900	0.000	0.490	0.000	0.340	0.000	0.040
36.5	1.394	1.394	0.900	0.000	0.494	0.000	0.344	0.000	0.044
37	1.397	1.397	0.900	0.000	0.497	0.000	0.347	0.000	0.047
37.5	1.400	1.400	0.900	0.000	0.500	0.000	0.350	0.000	0.050
38	1.404	1.404	0.900	0.000	0.504	0.000	0.354	0.000	0.054
38.5	1.407	1.407	0.900	0.000	0.507	0.000	0.357	0.000	0.057
39	1.410	1.410	0.900	0.000	0.510	0.000	0.360	0.000	0.060
39.5	1.414	1.414	0.900	0.000	0.514	0.000	0.364	0.000	0.064
40	1.417	1.417	0.900	0.000	0.517	0.000	0.367	0.000	0.067
40.5	1.420	1.420	0.900	0.000	0.520	0.000	0.370	0.000	0.070
41	1.423	1.423	0.900	0.000	0.523	0.000	0.373	0.000	0.073
41.5	1.426	1.426	0.900	0.000	0.526	0.000	0.376	0.000	0.076
42	1.430	1.430	0.900	0.000	0.530	0.000	0.380	0.000	0.080
42.5	1.433	1.433	0.900	0.000	0.533	0.000	0.383	0.000	0.083
43	1.436	1.436	0.900	0.000	0.536	0.000	0.386	0.000	0.086
43.5	1.439	1.439	0.900	0.000	0.539	0.000	0.389	0.000	0.089
44	1.442	1.442	0.900	0.000	0.542	0.000	0.392	0.000	0.092
44.5	1.445	1.445	0.900	0.000	0.545	0.000	0.395	0.000	0.095
45	1.448	1.448	0.900	0.000	0.548	0.000	0.398	0.000	0.098
45.5	1.451	1.451	0.900	0.000	0.551	0.000	0.401	0.000	0.101
46	1.454	1.454	0.900	0.000	0.554	0.000	0.404	0.000	0.104
46.5	1.457	1.457	0.900	0.000	0.557	0.000	0.407	0.000	0.107
47	1.459	1.459	0.900	0.000	0.559	0.000	0.409	0.000	0.109
47.5	1.462	1.462	0.900	0.000	0.562	0.000	0.412	0.000	0.112
48	1.465	1.465	0.900	0.000	0.565	0.000	0.415	0.000	0.115
48.5	1.468	1.468	0.900	0.000	0.568	0.000	0.418	0.000	0.118
49	1.471	1.471	0.900	0.000	0.571	0.000	0.421	0.000	0.121
49.5	1.474	1.474	0.900	0.000	0.574	0.000	0.424	0.000	0.124
50	1.476	1.476	0.900	0.000	0.576	0.000	0.426	0.000	0.126

Control point AB: peak flow and flood depth at sections

Flow (m^3)	ht (m)	hc1 (m)	hc2 (m)	hl1 (m)	hr1 (m)	hl2 (m)	hr2 (m)	hl3 (m)	hr3 (m)
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.294	0.294	0.294	0.000	0.000	0.000	0.000	0.000	0.000
1	0.452	0.452	0.452	0.000	0.000	0.000	0.000	0.000	0.000
1.5	0.583	0.583	0.583	0.000	0.000	0.000	0.000	0.000	0.000
2	0.699	0.699	0.699	0.000	0.000	0.000	0.000	0.000	0.000
2.5	0.807	0.807	0.807	0.000	0.000	0.000	0.000	0.000	0.000
3	0.908	0.908	0.908	0.000	0.000	0.000	0.000	0.000	0.000

3.5	1.003	1.003	1.003	0.000	0.000	0.000	0.000	0.000	0.000
4	1.095	1.095	1.095	0.000	0.000	0.000	0.000	0.000	0.000
4.5	1.183	1.183	1.183	0.000	0.000	0.000	0.000	0.000	0.000
5	1.269	1.269	1.269	0.000	0.000	0.000	0.000	0.000	0.000
5.5	1.352	1.352	1.352	0.000	0.000	0.000	0.000	0.000	0.000
6	1.433	1.433	1.433	0.000	0.000	0.000	0.000	0.000	0.000
6.5	1.512	1.512	1.512	0.000	0.000	0.000	0.000	0.000	0.000
7	1.590	1.590	1.590	0.000	0.000	0.000	0.000	0.000	0.000
7.5	1.666	1.666	1.666	0.000	0.000	0.000	0.000	0.000	0.000
8	1.741	1.741	1.741	0.000	0.000	0.000	0.000	0.000	0.000
8.5	1.770	1.750	1.770	0.020	0.000	0.000	0.000	0.000	0.000
9	1.783	1.750	1.783	0.033	0.000	0.000	0.000	0.000	0.000
9.5	1.794	1.750	1.794	0.044	0.000	0.000	0.000	0.000	0.000
10	1.804	1.750	1.804	0.054	0.000	0.000	0.000	0.000	0.000
10.5	1.812	1.750	1.812	0.062	0.000	0.000	0.000	0.000	0.000
11	1.820	1.750	1.820	0.070	0.000	0.000	0.000	0.000	0.000
11.5	1.828	1.750	1.828	0.078	0.000	0.000	0.000	0.000	0.000
12	1.835	1.750	1.835	0.085	0.000	0.000	0.000	0.000	0.000
12.5	1.842	1.750	1.842	0.092	0.000	0.000	0.000	0.000	0.000
13	1.848	1.750	1.848	0.098	0.000	0.000	0.000	0.000	0.000
13.5	1.854	1.750	1.854	0.104	0.000	0.000	0.000	0.000	0.000
14	1.860	1.750	1.860	0.110	0.000	0.000	0.000	0.000	0.000
14.5	1.866	1.750	1.866	0.116	0.000	0.000	0.000	0.000	0.000
15	1.872	1.750	1.872	0.122	0.000	0.000	0.000	0.000	0.000
15.5	1.877	1.750	1.877	0.127	0.000	0.000	0.000	0.000	0.000
16	1.883	1.750	1.883	0.133	0.000	0.000	0.000	0.000	0.000
16.5	1.890	1.624	1.675	0.140	0.000	0.000	0.000	0.000	0.000
17	1.900	1.750	1.900	0.150	0.000	0.000	0.000	0.000	0.000
17.5	1.898	1.750	1.898	0.148	0.000	0.000	0.000	0.000	0.000
18	1.903	1.750	1.903	0.153	0.000	0.003	0.000	0.000	0.000
18.5	1.907	1.750	1.907	0.157	0.000	0.007	0.000	0.000	0.000
19	1.912	1.750	1.912	0.162	0.000	0.012	0.000	0.000	0.000
19.5	1.916	1.750	1.916	0.166	0.000	0.016	0.000	0.000	0.000
20	1.920	1.750	1.920	0.170	0.000	0.020	0.000	0.000	0.000
20.5	1.924	1.750	1.924	0.174	0.000	0.024	0.000	0.000	0.000
21	1.928	1.750	1.928	0.178	0.000	0.028	0.000	0.000	0.000
21.5	1.932	1.750	1.932	0.182	0.000	0.032	0.000	0.000	0.000
22	1.935	1.750	1.935	0.185	0.000	0.035	0.000	0.000	0.000
22.5	1.939	1.750	1.939	0.189	0.000	0.039	0.000	0.000	0.000
23	1.943	1.750	1.943	0.193	0.000	0.043	0.000	0.000	0.000
23.5	1.946	1.750	1.946	0.196	0.000	0.046	0.000	0.000	0.000
24	1.950	1.750	1.950	0.200	0.000	0.050	0.000	0.000	0.000
24.5	1.953	1.750	1.953	0.203	0.000	0.053	0.000	0.000	0.000
25	1.956	1.750	1.956	0.206	0.000	0.056	0.000	0.000	0.000
25.5	1.960	1.750	1.960	0.210	0.000	0.060	0.000	0.000	0.000
26	1.963	1.750	1.963	0.213	0.000	0.063	0.000	0.000	0.000
26.5	1.966	1.750	1.966	0.216	0.000	0.066	0.000	0.000	0.000
27	1.969	1.750	1.969	0.219	0.000	0.069	0.000	0.000	0.000
27.5	1.973	1.750	1.973	0.223	0.000	0.073	0.000	0.000	0.000
28	1.976	1.750	1.976	0.226	0.000	0.076	0.000	0.000	0.000
28.5	1.979	1.750	1.979	0.229	0.000	0.079	0.000	0.000	0.000
29	1.982	1.750	1.982	0.232	0.000	0.082	0.000	0.000	0.000
29.5	1.985	1.750	1.985	0.235	0.000	0.085	0.000	0.000	0.000
30	1.988	1.750	1.988	0.238	0.000	0.088	0.000	0.000	0.000
30.5	1.991	1.750	1.991	0.241	0.000	0.091	0.000	0.000	0.000
31	1.993	1.750	1.993	0.243	0.000	0.093	0.000	0.000	0.000

31.5	1.996	1.750	1.996	0.246	0.000	0.096	0.000	0.000	0.000
32	2.000	1.750	2.000	0.250	0.000	0.100	0.000	0.000	0.000
32.5	2.002	1.750	2.002	0.252	0.000	0.102	0.000	0.002	0.000
33	2.005	1.750	2.005	0.255	0.000	0.105	0.000	0.005	0.000
33.5	2.007	1.750	2.007	0.257	0.000	0.107	0.000	0.007	0.000
34	2.010	1.750	2.010	0.260	0.000	0.110	0.000	0.010	0.000
34.5	2.012	1.750	2.012	0.262	0.000	0.112	0.000	0.012	0.000
35	2.015	1.750	2.015	0.265	0.000	0.115	0.000	0.015	0.000
35.5	2.017	1.750	2.017	0.267	0.000	0.117	0.000	0.017	0.000
36	2.019	1.750	2.019	0.269	0.000	0.119	0.000	0.019	0.000
36.5	2.021	1.750	2.021	0.271	0.000	0.121	0.000	0.021	0.000
37	2.024	1.750	2.024	0.274	0.000	0.124	0.000	0.024	0.000
37.5	2.026	1.750	2.026	0.276	0.000	0.126	0.000	0.026	0.000
38	2.028	1.750	2.028	0.278	0.000	0.128	0.000	0.028	0.000
38.5	2.030	1.750	2.030	0.280	0.000	0.130	0.000	0.030	0.000
39	2.032	1.750	2.032	0.282	0.000	0.132	0.000	0.032	0.000
39.5	2.034	1.750	2.034	0.284	0.000	0.134	0.000	0.034	0.000
40	2.036	1.750	2.036	0.286	0.000	0.136	0.000	0.036	0.000
40.5	2.038	1.750	2.038	0.288	0.000	0.138	0.000	0.038	0.000
41	2.040	1.750	2.040	0.290	0.000	0.140	0.000	0.040	0.000
41.5	2.042	1.750	2.042	0.292	0.000	0.142	0.000	0.042	0.000
42	2.044	1.750	2.044	0.294	0.000	0.144	0.000	0.044	0.000
42.5	2.046	1.750	2.046	0.296	0.000	0.146	0.000	0.046	0.000
43	2.048	1.750	2.048	0.298	0.000	0.148	0.000	0.048	0.000
43.5	2.050	1.750	2.050	0.300	0.000	0.150	0.000	0.050	0.000
44	2.052	1.750	2.052	0.302	0.000	0.152	0.000	0.052	0.000
44.5	2.054	1.750	2.054	0.304	0.000	0.154	0.000	0.054	0.000
45	2.056	1.750	2.056	0.306	0.000	0.156	0.000	0.056	0.000
45.5	2.058	1.750	2.058	0.308	0.000	0.158	0.000	0.058	0.000
46	2.059	1.750	2.059	0.309	0.000	0.159	0.000	0.059	0.000
46.5	2.061	1.750	2.061	0.311	0.000	0.161	0.000	0.061	0.000
47	2.063	1.750	2.063	0.313	0.000	0.163	0.000	0.063	0.000
47.5	2.065	1.750	2.065	0.315	0.000	0.165	0.000	0.065	0.000
48	2.067	1.750	2.067	0.317	0.000	0.167	0.000	0.067	0.000
48.5	2.068	1.750	2.068	0.318	0.000	0.168	0.000	0.068	0.000
49	2.070	1.750	2.070	0.320	0.000	0.170	0.000	0.070	0.000
49.5	2.072	1.750	2.072	0.322	0.000	0.172	0.000	0.072	0.000
50	2.074	1.750	2.074	0.324	0.000	0.174	0.000	0.074	0.000

Control point ABC: peak flow and flood depth at sections

Flow (m^3)	ht (m)	hc1 (m)	hc2 (m)	hl1 (m)	hr1 (m)	hl2 (m)	hr2 (m)	hl3 (m)	hr3 (m)
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.294	0.294	0.294	0.000	0.000	0.000	0.000	0.000	0.000
1	0.452	0.452	0.452	0.000	0.000	0.000	0.000	0.000	0.000
1.5	0.583	0.583	0.583	0.000	0.000	0.000	0.000	0.000	0.000
2	0.699	0.699	0.699	0.000	0.000	0.000	0.000	0.000	0.000
2.5	0.807	0.807	0.807	0.000	0.000	0.000	0.000	0.000	0.000
3	0.908	0.908	0.908	0.000	0.000	0.000	0.000	0.000	0.000
3.5	1.003	1.003	1.003	0.000	0.000	0.000	0.000	0.000	0.000
4	1.095	1.095	1.095	0.000	0.000	0.000	0.000	0.000	0.000
4.5	1.183	1.183	1.183	0.000	0.000	0.000	0.000	0.000	0.000
5	1.269	1.269	1.269	0.000	0.000	0.000	0.000	0.000	0.000
5.5	1.352	1.352	1.352	0.000	0.000	0.000	0.000	0.000	0.000

6	1.433	1.433	1.433	0.000	0.000	0.000	0.000	0.000	0.000
6.5	1.512	1.512	1.512	0.000	0.000	0.000	0.000	0.000	0.000
7	1.590	1.590	1.590	0.000	0.000	0.000	0.000	0.000	0.000
7.5	1.666	1.666	1.666	0.000	0.000	0.000	0.000	0.000	0.000
8	1.741	1.741	1.741	0.000	0.000	0.000	0.000	0.000	0.000
8.5	1.785	1.770	1.785	0.015	0.000	0.000	0.000	0.000	0.000
9	1.800	1.770	1.800	0.030	0.000	0.000	0.000	0.000	0.000
9.5	1.807	1.618	1.589	0.037	0.000	0.000	0.000	0.000	0.000
10	1.821	1.770	1.821	0.051	0.000	0.000	0.000	0.000	0.000
10.5	1.830	1.770	1.830	0.060	0.000	0.000	0.000	0.000	0.000
11	1.838	1.770	1.838	0.068	0.000	0.000	0.000	0.000	0.000
11.5	1.846	1.770	1.846	0.076	0.000	0.000	0.000	0.000	0.000
12	1.853	1.770	1.853	0.083	0.000	0.000	0.000	0.000	0.000
12.5	1.860	1.638	1.640	0.090	0.000	0.000	0.000	0.000	0.000
13	1.870	1.770	1.870	0.100	0.000	0.000	0.000	0.000	0.000
13.5	1.873	1.770	1.873	0.103	0.000	0.003	0.000	0.000	0.000
14	1.878	1.770	1.878	0.108	0.000	0.008	0.000	0.000	0.000
14.5	1.884	1.770	1.884	0.114	0.000	0.014	0.000	0.000	0.000
15	1.889	1.770	1.889	0.119	0.000	0.019	0.000	0.000	0.000
15.5	1.894	1.770	1.894	0.124	0.000	0.024	0.000	0.000	0.000
16	1.898	1.770	1.898	0.128	0.000	0.028	0.000	0.000	0.000
16.5	1.903	1.770	1.903	0.133	0.000	0.033	0.000	0.000	0.000
17	1.907	1.770	1.907	0.137	0.000	0.037	0.000	0.000	0.000
17.5	1.911	1.770	1.911	0.141	0.000	0.041	0.000	0.000	0.000
18	1.916	1.770	1.916	0.146	0.000	0.046	0.000	0.000	0.000
18.5	1.920	1.770	1.920	0.150	0.000	0.050	0.000	0.000	0.000
19	1.924	1.770	1.924	0.154	0.000	0.054	0.000	0.000	0.000
19.5	1.927	1.770	1.927	0.157	0.000	0.057	0.000	0.000	0.000
20	1.931	1.770	1.931	0.161	0.000	0.061	0.000	0.000	0.000
20.5	1.935	1.770	1.935	0.165	0.000	0.065	0.000	0.000	0.000
21	1.939	1.770	1.939	0.169	0.000	0.069	0.000	0.000	0.000
21.5	1.942	1.770	1.942	0.172	0.000	0.072	0.000	0.000	0.000
22	1.946	1.770	1.946	0.176	0.000	0.076	0.000	0.000	0.000
22.5	1.949	1.770	1.949	0.179	0.000	0.079	0.000	0.000	0.000
23	1.953	1.770	1.953	0.183	0.000	0.083	0.000	0.000	0.000
23.5	1.956	1.770	1.956	0.186	0.000	0.086	0.000	0.000	0.000
24	1.959	1.770	1.959	0.189	0.000	0.089	0.000	0.000	0.000
24.5	1.962	1.770	1.962	0.192	0.000	0.092	0.000	0.000	0.000
25	1.966	1.770	1.966	0.196	0.000	0.096	0.000	0.000	0.000
25.5	1.969	1.770	1.969	0.199	0.000	0.099	0.000	0.000	0.000
26	1.972	1.770	1.972	0.202	0.000	0.102	0.000	0.000	0.000
26.5	1.975	1.770	1.975	0.205	0.000	0.105	0.000	0.000	0.000
27	1.978	1.770	1.978	0.208	0.000	0.108	0.000	0.000	0.000
27.5	1.981	1.770	1.981	0.211	0.000	0.111	0.000	0.000	0.000
28	1.984	1.770	1.984	0.214	0.000	0.114	0.000	0.000	0.000
28.5	1.987	1.770	1.987	0.217	0.000	0.117	0.000	0.000	0.000
29	1.990	1.770	1.990	0.220	0.000	0.120	0.000	0.000	0.000
29.5	1.993	1.770	1.993	0.223	0.000	0.123	0.000	0.000	0.000
30	1.996	1.770	1.996	0.226	0.000	0.126	0.000	0.000	0.000
30.5	1.999	1.770	1.999	0.229	0.000	0.129	0.000	0.000	0.000
31	2.002	1.770	2.002	0.232	0.000	0.132	0.000	0.000	0.000
31.5	2.004	1.770	2.004	0.234	0.000	0.134	0.000	0.000	0.000
32	2.007	1.770	2.007	0.237	0.000	0.137	0.000	0.000	0.000
32.5	2.010	1.770	2.010	0.240	0.000	0.140	0.000	0.000	0.000
33	2.013	1.770	2.013	0.243	0.000	0.143	0.000	0.000	0.000
33.5	2.015	1.770	2.015	0.245	0.000	0.145	0.000	0.000	0.000

34	2.020	1.770	2.020	0.250	0.000	0.150	0.000	0.000	0.000
34.5	2.021	1.770	2.021	0.251	0.000	0.151	0.000	0.001	0.000
35	2.023	1.770	2.023	0.253	0.000	0.153	0.000	0.003	0.000
35.5	2.026	1.770	2.026	0.256	0.000	0.156	0.000	0.006	0.000
36	2.028	1.770	2.028	0.258	0.000	0.158	0.000	0.008	0.000
36.5	2.030	1.770	2.030	0.260	0.000	0.160	0.000	0.010	0.000
37	2.033	1.770	2.033	0.263	0.000	0.163	0.000	0.013	0.000
37.5	2.035	1.770	2.035	0.265	0.000	0.165	0.000	0.015	0.000
38	2.037	1.770	2.037	0.267	0.000	0.167	0.000	0.017	0.000
38.5	2.039	1.770	2.039	0.269	0.000	0.169	0.000	0.019	0.000
39	2.042	1.770	2.042	0.272	0.000	0.172	0.000	0.022	0.000
39.5	2.044	1.770	2.044	0.274	0.000	0.174	0.000	0.024	0.000
40	2.046	1.770	2.046	0.276	0.000	0.176	0.000	0.026	0.000
40.5	2.048	1.770	2.048	0.278	0.000	0.178	0.000	0.028	0.000
41	2.050	1.770	2.050	0.280	0.000	0.180	0.000	0.030	0.000
41.5	2.052	1.770	2.052	0.282	0.000	0.182	0.000	0.032	0.000
42	2.054	1.770	2.054	0.284	0.000	0.184	0.000	0.034	0.000
42.5	2.056	1.770	2.056	0.286	0.000	0.186	0.000	0.036	0.000
43	2.058	1.770	2.058	0.288	0.000	0.188	0.000	0.038	0.000
43.5	2.060	1.770	2.060	0.290	0.000	0.190	0.000	0.040	0.000
44	2.062	1.770	2.062	0.292	0.000	0.192	0.000	0.042	0.000
44.5	2.064	1.770	2.064	0.294	0.000	0.194	0.000	0.044	0.000
45	2.065	1.770	2.065	0.295	0.000	0.195	0.000	0.045	0.000
45.5	2.067	1.770	2.067	0.297	0.000	0.197	0.000	0.047	0.000
46	2.069	1.770	2.069	0.299	0.000	0.199	0.000	0.049	0.000
46.5	2.071	1.770	2.071	0.301	0.000	0.201	0.000	0.051	0.000
47	2.073	1.770	2.073	0.303	0.000	0.203	0.000	0.053	0.000
47.5	2.075	1.770	2.075	0.305	0.000	0.205	0.000	0.055	0.000
48	2.076	1.770	2.076	0.306	0.000	0.206	0.000	0.056	0.000
48.5	2.078	1.770	2.078	0.308	0.000	0.208	0.000	0.058	0.000
49	2.080	1.770	2.080	0.310	0.000	0.210	0.000	0.060	0.000
49.5	2.082	1.770	2.082	0.312	0.000	0.212	0.000	0.062	0.000
50	2.083	1.770	2.083	0.313	0.000	0.213	0.000	0.063	0.000

Control point D: peak flow and flood depth at sections

Flow (m^3)	ht (m)	hc1 (m)	hc2 (m)	hl1 (m)	hr1 (m)	hl2 (m)	hr2 (m)	hl3 (m)	hr3 (m)
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.412	0.412	0.412	0.000	0.000	0.000	0.000	0.000	0.000
1	0.639	0.639	0.639	0.000	0.000	0.000	0.000	0.000	0.000
1.5	0.830	0.830	0.830	0.000	0.000	0.000	0.000	0.000	0.000
2	1.002	1.002	1.002	0.000	0.000	0.000	0.000	0.000	0.000
2.5	1.162	1.162	1.162	0.000	0.000	0.000	0.000	0.000	0.000
3	1.313	1.313	1.313	0.000	0.000	0.000	0.000	0.000	0.000
3.5	1.457	1.457	1.457	0.000	0.000	0.000	0.000	0.000	0.000
4	1.596	1.596	1.596	0.000	0.000	0.000	0.000	0.000	0.000
4.5	1.731	1.731	1.731	0.000	0.000	0.000	0.000	0.000	0.000
5	1.795	1.795	1.768	0.000	0.027	0.000	0.000	0.000	0.000
5.5	1.816	1.816	1.768	0.000	0.049	0.000	0.000	0.000	0.000
6	1.834	1.834	1.768	0.000	0.066	0.000	0.000	0.000	0.000
6.5	1.848	1.848	1.768	0.000	0.081	0.000	0.000	0.000	0.000
7	1.858	1.858	1.750	0.000	0.091	0.000	0.000	0.000	0.000
7.5	1.872	1.858	1.768	0.014	0.105	0.000	0.000	0.000	0.000
8	1.881	1.858	1.768	0.023	0.114	0.000	0.000	0.000	0.000

8.5	1.890	1.858	1.768	0.031	0.122	0.000	0.000	0.000	0.000
9	1.897	1.858	1.768	0.039	0.130	0.000	0.000	0.000	0.000
9.5	1.905	1.858	1.768	0.046	0.137	0.000	0.000	0.000	0.000
10	1.911	1.858	1.768	0.053	0.144	0.000	0.000	0.000	0.000
10.5	1.918	1.858	1.768	0.060	0.150	0.000	0.000	0.000	0.000
11	1.924	1.858	1.768	0.066	0.157	0.000	0.000	0.000	0.000
11.5	1.930	1.858	1.768	0.072	0.163	0.000	0.000	0.000	0.000
12	1.936	1.858	1.768	0.078	0.169	0.000	0.000	0.000	0.000
12.5	1.942	1.858	1.768	0.083	0.174	0.000	0.000	0.000	0.000
13	1.947	1.858	1.768	0.089	0.180	0.000	0.000	0.000	0.000
13.5	1.953	1.858	1.768	0.094	0.185	0.000	0.000	0.000	0.000
14	1.958	1.858	1.768	0.100	0.191	0.000	0.000	0.000	0.000
14.5	1.963	1.858	1.768	0.105	0.196	0.005	0.000	0.000	0.000
15	1.968	1.858	1.768	0.109	0.200	0.009	0.000	0.000	0.000
15.5	1.972	1.858	1.768	0.114	0.205	0.014	0.005	0.000	0.000
16	1.977	1.858	1.768	0.118	0.209	0.018	0.009	0.000	0.000
16.5	1.981	1.858	1.768	0.123	0.213	0.023	0.013	0.000	0.000
17	1.985	1.858	1.768	0.127	0.218	0.027	0.017	0.000	0.000
17.5	1.989	1.858	1.768	0.131	0.221	0.031	0.021	0.000	0.000
18	1.993	1.858	1.768	0.134	0.225	0.034	0.025	0.000	0.000
18.5	1.996	1.858	1.768	0.138	0.229	0.038	0.029	0.000	0.000
19	2.000	1.858	1.768	0.142	0.233	0.042	0.033	0.000	0.000
19.5	2.004	1.858	1.768	0.145	0.236	0.045	0.036	0.000	0.000
20	2.007	1.858	1.768	0.149	0.240	0.049	0.040	0.000	0.000
20.5	2.011	1.858	1.768	0.152	0.243	0.052	0.043	0.000	0.000
21	2.014	1.858	1.768	0.156	0.246	0.056	0.046	0.000	0.000
21.5	2.017	1.858	1.768	0.159	0.250	0.059	0.050	0.000	0.000
22	2.020	1.858	1.768	0.162	0.253	0.062	0.053	0.000	0.000
22.5	2.024	1.858	1.768	0.165	0.256	0.065	0.056	0.000	0.000
23	2.027	1.858	1.768	0.168	0.259	0.068	0.059	0.000	0.000
23.5	2.030	1.858	1.768	0.171	0.262	0.071	0.062	0.000	0.000
24	2.033	1.858	1.768	0.175	0.265	0.075	0.065	0.000	0.000
24.5	2.036	1.858	1.768	0.178	0.268	0.078	0.068	0.000	0.000
25	2.039	1.858	1.768	0.181	0.271	0.081	0.071	0.000	0.000
25.5	2.042	1.858	1.768	0.183	0.274	0.083	0.074	0.000	0.000
26	2.045	1.858	1.768	0.186	0.277	0.086	0.077	0.000	0.000
26.5	2.048	1.858	1.768	0.189	0.280	0.089	0.080	0.000	0.000
27	2.050	1.858	1.768	0.192	0.283	0.092	0.083	0.000	0.000
27.5	2.053	1.858	1.768	0.195	0.286	0.095	0.086	0.000	0.000
28	2.056	1.858	1.768	0.198	0.289	0.098	0.089	0.000	0.000
28.5	2.059	1.858	1.768	0.200	0.291	0.100	0.091	0.000	0.000
29	2.061	1.858	1.768	0.203	0.294	0.103	0.094	0.003	0.000
29.5	2.064	1.858	1.768	0.206	0.296	0.106	0.096	0.006	0.000
30	2.067	1.858	1.768	0.208	0.299	0.108	0.099	0.008	0.000
30.5	2.069	1.858	1.768	0.211	0.301	0.111	0.101	0.011	0.001
31	2.071	1.858	1.768	0.213	0.304	0.113	0.104	0.013	0.004
31.5	2.074	1.858	1.768	0.215	0.306	0.115	0.106	0.015	0.006
32	2.076	1.858	1.768	0.217	0.308	0.117	0.108	0.017	0.008
32.5	2.078	1.858	1.768	0.220	0.311	0.120	0.111	0.020	0.011
33	2.080	1.858	1.768	0.222	0.313	0.122	0.113	0.022	0.013
33.5	2.082	1.858	1.768	0.224	0.315	0.124	0.115	0.024	0.015
34	2.084	1.858	1.768	0.226	0.317	0.126	0.117	0.026	0.017
34.5	2.086	1.858	1.768	0.228	0.319	0.128	0.119	0.028	0.019
35	2.088	1.858	1.768	0.230	0.321	0.130	0.121	0.030	0.021
35.5	2.090	1.858	1.768	0.232	0.323	0.132	0.123	0.032	0.023
36	2.092	1.858	1.768	0.234	0.325	0.134	0.125	0.034	0.025

36.5	2.094	1.858	1.768	0.236	0.327	0.136	0.127	0.036	0.027
37	2.096	1.858	1.768	0.238	0.329	0.138	0.129	0.038	0.029
37.5	2.098	1.858	1.768	0.240	0.331	0.140	0.131	0.040	0.031
38	2.100	1.858	1.768	0.241	0.332	0.141	0.132	0.041	0.032
38.5	2.102	1.858	1.768	0.243	0.334	0.143	0.134	0.043	0.034
39	2.104	1.858	1.768	0.245	0.336	0.145	0.136	0.045	0.036
39.5	2.105	1.858	1.768	0.247	0.338	0.147	0.138	0.047	0.038
40	2.107	1.858	1.768	0.249	0.340	0.149	0.140	0.049	0.040
40.5	2.109	1.858	1.768	0.250	0.341	0.150	0.141	0.050	0.041
41	2.111	1.858	1.768	0.252	0.343	0.152	0.143	0.052	0.043
41.5	2.112	1.858	1.768	0.254	0.345	0.154	0.145	0.054	0.045
42	2.114	1.858	1.768	0.256	0.346	0.156	0.146	0.056	0.046
42.5	2.116	1.858	1.768	0.257	0.348	0.157	0.148	0.057	0.048
43	2.117	1.858	1.768	0.259	0.350	0.159	0.150	0.059	0.050
43.5	2.119	1.858	1.768	0.261	0.351	0.161	0.151	0.061	0.051
44	2.121	1.858	1.768	0.262	0.353	0.162	0.153	0.062	0.053
44.5	2.122	1.858	1.768	0.264	0.355	0.164	0.155	0.064	0.055
45	2.124	1.858	1.768	0.265	0.356	0.165	0.156	0.065	0.056
45.5	2.125	1.858	1.768	0.267	0.358	0.167	0.158	0.067	0.058
46	2.127	1.858	1.768	0.269	0.359	0.169	0.159	0.069	0.059
46.5	2.129	1.858	1.768	0.270	0.361	0.170	0.161	0.070	0.061
47	2.130	1.858	1.768	0.272	0.363	0.172	0.163	0.072	0.063
47.5	2.132	1.858	1.768	0.273	0.364	0.173	0.164	0.073	0.064
48	2.133	1.858	1.768	0.275	0.366	0.175	0.166	0.075	0.066
48.5	2.135	1.858	1.768	0.276	0.367	0.176	0.167	0.076	0.067
49	2.136	1.858	1.768	0.278	0.369	0.178	0.169	0.078	0.069
49.5	2.138	1.858	1.768	0.279	0.370	0.179	0.170	0.079	0.070
50	2.139	1.858	1.768	0.281	0.372	0.181	0.172	0.081	0.072

Control point ABCD: peak flow and flood depth at sections

Flow (m^3)	ht (m)	hc1 (m)	hc2 (m)	hl1 (m)	hr1 (m)	hl2 (m)	hr2 (m)	hl3 (m)	hr3 (m)
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.291	0.291	0.291	0.000	0.000	0.000	0.000	0.000	0.000
1	0.445	0.445	0.445	0.000	0.000	0.000	0.000	0.000	0.000
1.5	0.572	0.572	0.572	0.000	0.000	0.000	0.000	0.000	0.000
2	0.685	0.685	0.685	0.000	0.000	0.000	0.000	0.000	0.000
2.5	0.788	0.788	0.788	0.000	0.000	0.000	0.000	0.000	0.000
3	0.884	0.884	0.884	0.000	0.000	0.000	0.000	0.000	0.000
3.5	0.974	0.974	0.974	0.000	0.000	0.000	0.000	0.000	0.000
4	1.061	1.061	1.061	0.000	0.000	0.000	0.000	0.000	0.000
4.5	1.144	1.144	1.144	0.000	0.000	0.000	0.000	0.000	0.000
5	1.224	1.224	1.224	0.000	0.000	0.000	0.000	0.000	0.000
5.5	1.302	1.302	1.302	0.000	0.000	0.000	0.000	0.000	0.000
6	1.377	1.377	1.377	0.000	0.000	0.000	0.000	0.000	0.000
6.5	1.450	1.450	1.450	0.000	0.000	0.000	0.000	0.000	0.000
7	1.522	1.522	1.522	0.000	0.000	0.000	0.000	0.000	0.000
7.5	1.592	1.592	1.592	0.000	0.000	0.000	0.000	0.000	0.000
8	1.655	1.650	1.650	0.005	0.005	0.000	0.000	0.000	0.000
8.5	1.671	1.650	1.650	0.021	0.021	0.000	0.000	0.000	0.000
9	1.681	1.650	1.650	0.031	0.031	0.000	0.000	0.000	0.000
9.5	1.690	1.650	1.650	0.040	0.040	0.000	0.000	0.000	0.000
10	1.698	1.650	1.650	0.048	0.048	0.000	0.000	0.000	0.000
10.5	1.705	1.650	1.650	0.055	0.055	0.000	0.000	0.000	0.000

11	1.712	1.650	1.650	0.062	0.062	0.000	0.000	0.000	0.000
11.5	1.718	1.650	1.650	0.068	0.068	0.000	0.000	0.000	0.000
12	1.724	1.650	1.650	0.074	0.074	0.000	0.000	0.000	0.000
12.5	1.729	1.650	1.650	0.079	0.079	0.000	0.000	0.000	0.000
13	1.735	1.650	1.650	0.085	0.085	0.000	0.000	0.000	0.000
13.5	1.740	1.650	1.650	0.090	0.090	0.000	0.000	0.000	0.000
14	1.745	1.650	1.650	0.095	0.095	0.000	0.000	0.000	0.000
14.5	1.750	1.650	1.650	0.100	0.100	0.000	0.000	0.000	0.000
15	1.754	1.650	1.650	0.104	0.104	0.004	0.004	0.000	0.000
15.5	1.758	1.650	1.650	0.108	0.108	0.008	0.008	0.000	0.000
16	1.762	1.650	1.650	0.112	0.112	0.012	0.012	0.000	0.000
16.5	1.766	1.650	1.650	0.116	0.116	0.016	0.016	0.000	0.000
17	1.770	1.650	1.650	0.120	0.120	0.020	0.020	0.000	0.000
17.5	1.773	1.650	1.650	0.123	0.123	0.023	0.023	0.000	0.000
18	1.776	1.650	1.650	0.126	0.126	0.026	0.026	0.000	0.000
18.5	1.780	1.650	1.650	0.130	0.130	0.030	0.030	0.000	0.000
19	1.783	1.650	1.650	0.133	0.133	0.033	0.033	0.000	0.000
19.5	1.786	1.650	1.650	0.136	0.136	0.036	0.036	0.000	0.000
20	1.789	1.650	1.650	0.139	0.139	0.039	0.039	0.000	0.000
20.5	1.792	1.650	1.650	0.142	0.142	0.042	0.042	0.000	0.000
21	1.795	1.650	1.650	0.145	0.145	0.045	0.045	0.000	0.000
21.5	1.798	1.650	1.650	0.148	0.148	0.048	0.048	0.000	0.000
22	1.800	1.650	1.650	0.150	0.150	0.050	0.050	0.000	0.000
22.5	1.803	1.650	1.650	0.153	0.153	0.053	0.053	0.000	0.000
23	1.806	1.650	1.650	0.156	0.156	0.056	0.056	0.000	0.000
23.5	1.808	1.650	1.650	0.158	0.158	0.058	0.058	0.000	0.000
24	1.811	1.650	1.650	0.161	0.161	0.061	0.061	0.000	0.000
24.5	1.814	1.650	1.650	0.164	0.164	0.064	0.064	0.000	0.000
25	1.816	1.650	1.650	0.166	0.166	0.066	0.066	0.000	0.000
25.5	1.819	1.650	1.650	0.169	0.169	0.069	0.069	0.000	0.000
26	1.821	1.650	1.650	0.171	0.171	0.071	0.071	0.000	0.000
26.5	1.824	1.650	1.650	0.174	0.174	0.074	0.074	0.000	0.000
27	1.826	1.650	1.650	0.176	0.176	0.076	0.076	0.000	0.000
27.5	1.828	1.650	1.650	0.178	0.178	0.078	0.078	0.000	0.000
28	1.831	1.650	1.650	0.181	0.181	0.081	0.081	0.000	0.000
28.5	1.833	1.650	1.650	0.183	0.183	0.083	0.083	0.000	0.000
29	1.835	1.650	1.650	0.185	0.185	0.085	0.085	0.000	0.000
29.5	1.838	1.650	1.650	0.188	0.188	0.088	0.088	0.000	0.000
30	1.840	1.650	1.650	0.190	0.190	0.090	0.090	0.000	0.000
30.5	1.842	1.650	1.650	0.192	0.192	0.092	0.092	0.000	0.000
31	1.844	1.650	1.650	0.194	0.194	0.094	0.094	0.000	0.000
31.5	1.846	1.650	1.650	0.196	0.196	0.096	0.096	0.000	0.000
32	1.848	1.650	1.650	0.198	0.198	0.098	0.098	0.000	0.000
32.5	1.851	1.650	1.650	0.201	0.201	0.101	0.101	0.001	0.001
33	1.853	1.650	1.650	0.203	0.203	0.103	0.103	0.003	0.003
33.5	1.855	1.650	1.650	0.205	0.205	0.105	0.105	0.005	0.005
34	1.857	1.650	1.650	0.207	0.207	0.107	0.107	0.007	0.007
34.5	1.858	1.650	1.650	0.208	0.208	0.108	0.108	0.008	0.008
35	1.860	1.650	1.650	0.210	0.210	0.110	0.110	0.010	0.010
35.5	1.862	1.650	1.650	0.212	0.212	0.112	0.112	0.012	0.012
36	1.864	1.650	1.650	0.214	0.214	0.114	0.114	0.014	0.014
36.5	1.866	1.650	1.650	0.216	0.216	0.116	0.116	0.016	0.016
37	1.867	1.650	1.650	0.217	0.217	0.117	0.117	0.017	0.017
37.5	1.869	1.650	1.650	0.219	0.219	0.119	0.119	0.019	0.019
38	1.871	1.650	1.650	0.221	0.221	0.121	0.121	0.021	0.021
38.5	1.873	1.650	1.650	0.223	0.223	0.123	0.123	0.023	0.023

39	1.874	1.650	1.650	0.224	0.224	0.124	0.124	0.024	0.024
39.5	1.876	1.650	1.650	0.226	0.226	0.126	0.126	0.026	0.026
40	1.877	1.650	1.650	0.227	0.227	0.127	0.127	0.027	0.027
40.5	1.879	1.650	1.650	0.229	0.229	0.129	0.129	0.029	0.029
41	1.881	1.650	1.650	0.231	0.231	0.131	0.131	0.031	0.031
41.5	1.882	1.650	1.650	0.232	0.232	0.132	0.132	0.032	0.032
42	1.884	1.650	1.650	0.234	0.234	0.134	0.134	0.034	0.034
42.5	1.885	1.650	1.650	0.235	0.235	0.135	0.135	0.035	0.035
43	1.887	1.650	1.650	0.237	0.237	0.137	0.137	0.037	0.037
43.5	1.888	1.650	1.650	0.238	0.238	0.138	0.138	0.038	0.038
44	1.890	1.650	1.650	0.240	0.240	0.140	0.140	0.040	0.040
44.5	1.891	1.650	1.650	0.241	0.241	0.141	0.141	0.041	0.041
45	1.893	1.650	1.650	0.243	0.243	0.143	0.143	0.043	0.043
45.5	1.894	1.650	1.650	0.244	0.244	0.144	0.144	0.044	0.044
46	1.896	1.650	1.650	0.246	0.246	0.146	0.146	0.046	0.046
46.5	1.897	1.650	1.650	0.247	0.247	0.147	0.147	0.047	0.047
47	1.898	1.650	1.650	0.248	0.248	0.148	0.148	0.048	0.048
47.5	1.900	1.650	1.650	0.250	0.250	0.150	0.150	0.050	0.050
48	1.901	1.650	1.650	0.251	0.251	0.151	0.151	0.051	0.051
48.5	1.903	1.650	1.650	0.253	0.253	0.153	0.153	0.053	0.053
49	1.904	1.650	1.650	0.254	0.254	0.154	0.154	0.054	0.054
49.5	1.905	1.650	1.650	0.255	0.255	0.155	0.155	0.055	0.055
50	1.907	1.650	1.650	0.257	0.257	0.157	0.157	0.057	0.057